University of California at Berkeley

Dept. of Electrical Engineering and Computer Sciences

EE 105:

Microelectronic Devices and Circuits

Fall 2000

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A Motivating Example

* An analog-to-digital converter for data transmission -- the analog voltage is converted into a 13 bit digital word at 5 Msamples /sec.
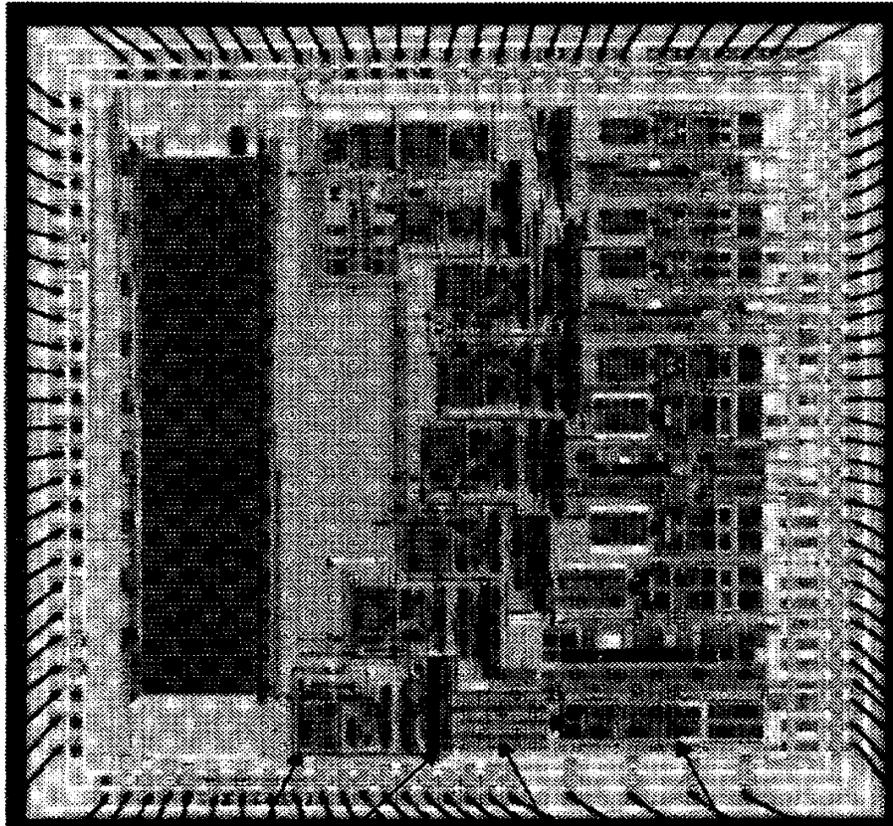
stage 0 comparators   stage 0 sampling capacitors   stage 0 opamp

stage 0 sampling switches

Fig. 7.   Die photograph of the prototype ADC.

ref. D. W. Cline and P. R. Gray, *IEEE J. Solid-State Circuits,* **31**, March 1996, pp. 294-303. © 1996 IEEE. Used by permission.

# *Course Goal*: **Build a Solid Foundation for Designing the Analog Subsystems in ICs**

Major course sections

* Semiconductor physics and integrated-circuit fabrication (Weeks ~1-2)
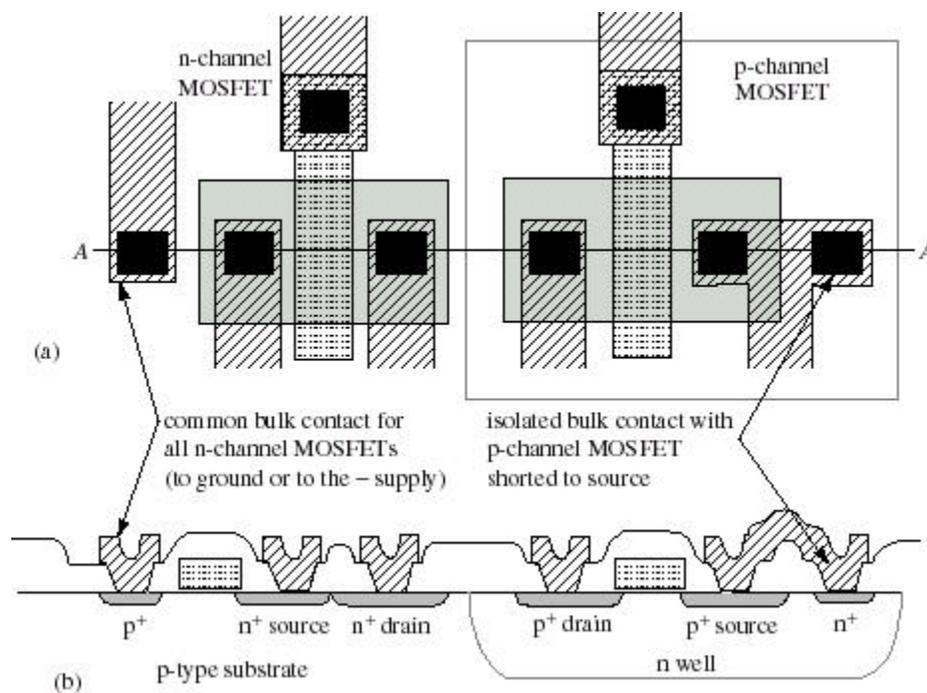
# Course Overview

* Device physics for analog circuit design (Weeks ~3-7)
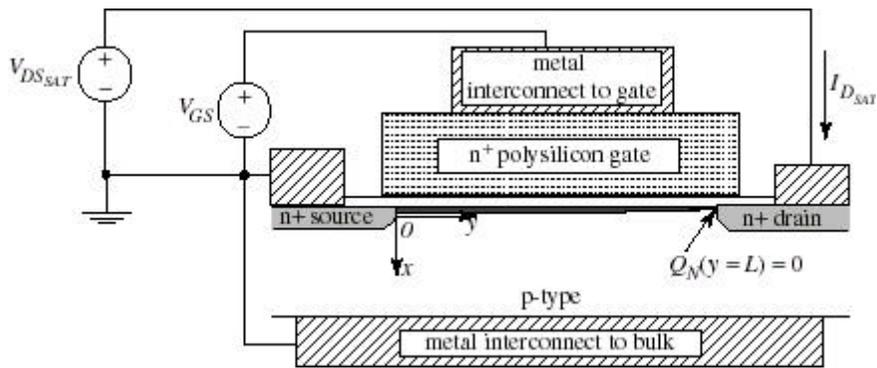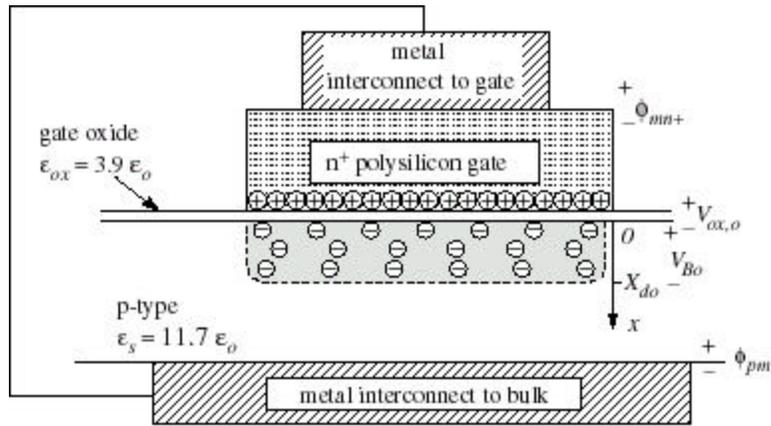
Course Overview
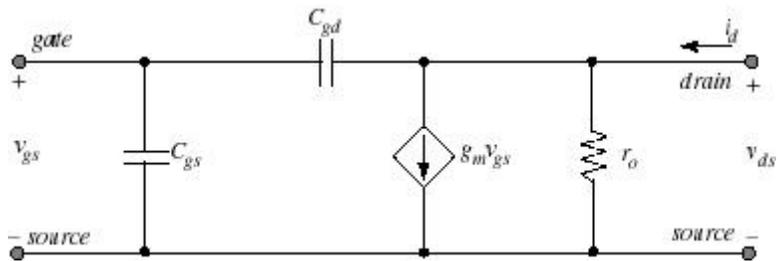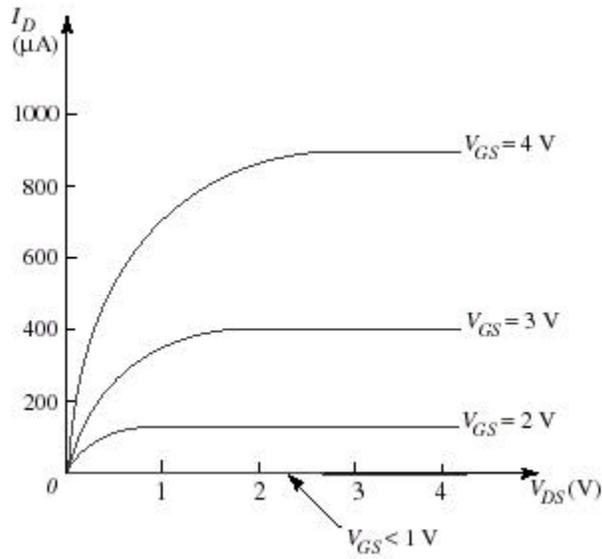
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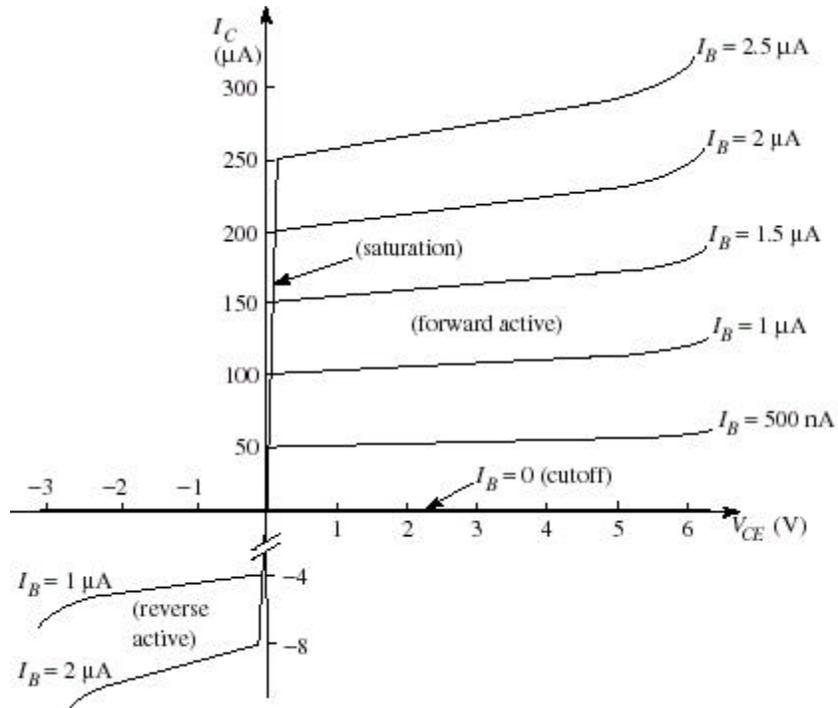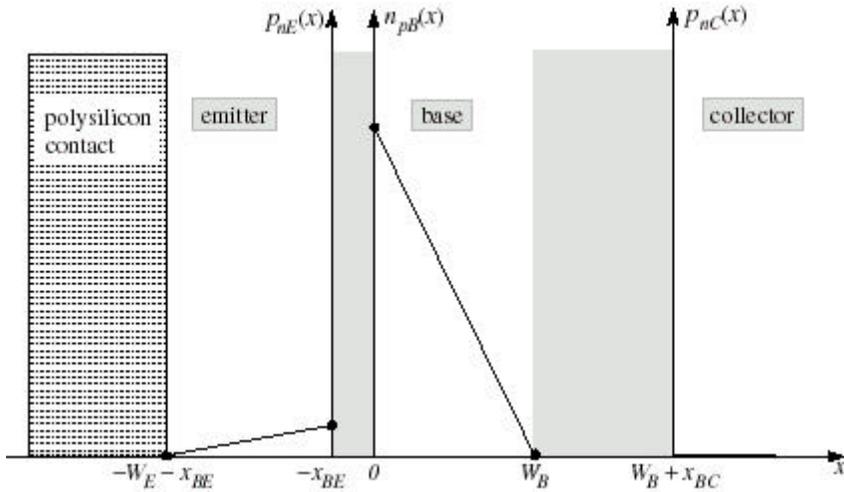
Course Overview

Weeks ~8-10 small-signal amplifiers
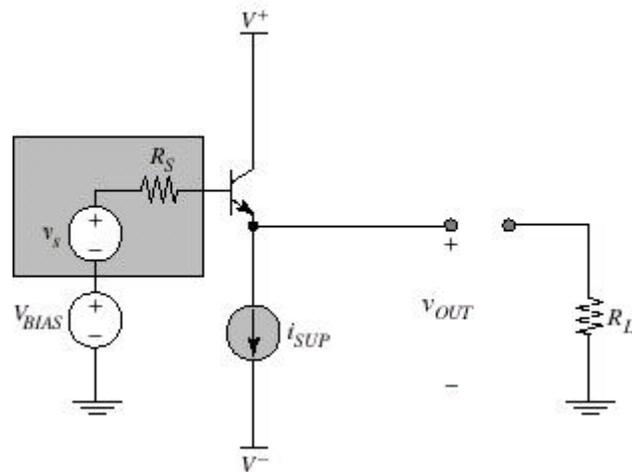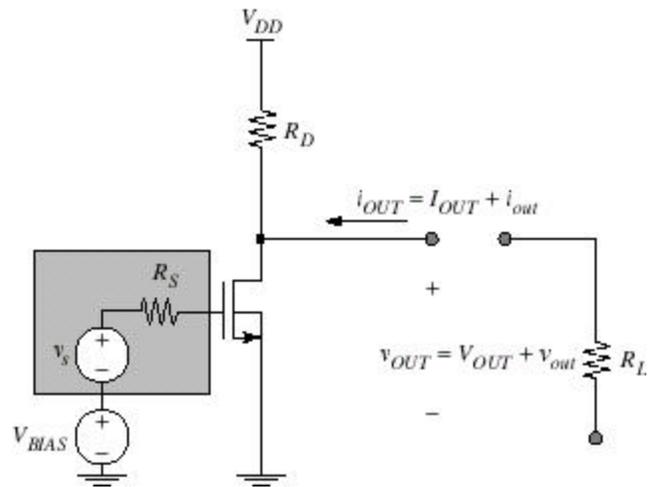
Course Overview

Weeks ~10-11, phasor analysis and frequency response
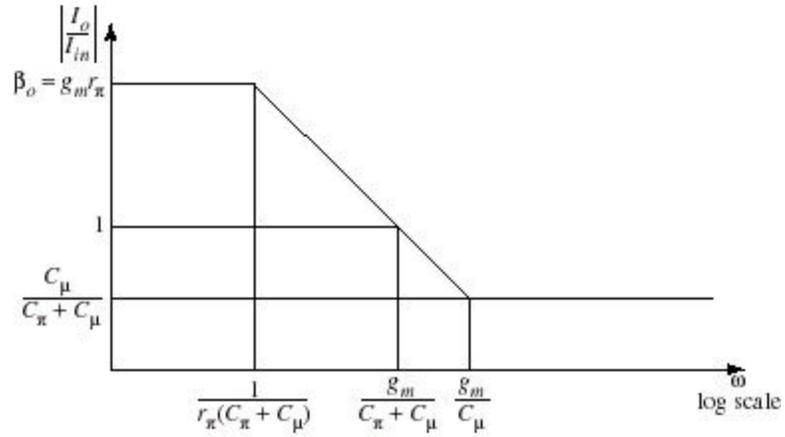
Weeks ~11-15, multistage amplifiers

Course Overview
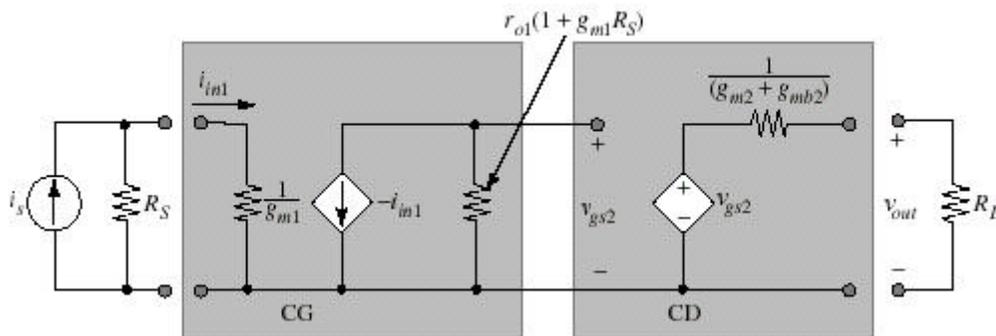
Silicon: *The* Element for Microelectronics

∗ Group IV of the periodic table:

IV

C
Si
Ge
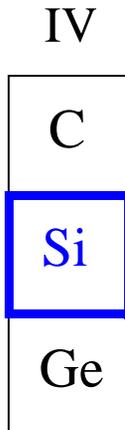
very desirable properties:

1.
2.
3.

∗ electronic structure: four bonding (valence) electrons

$(1s)^2$

$(2s)^2$

$(2p)^6$

$(3sp)^4$
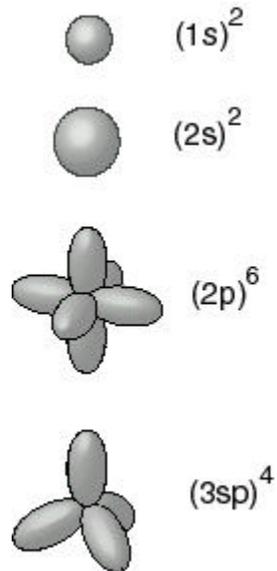
Silicon Crystal Structure

* Diamond lattice: defined by covalent bonds between tetrahedral hybrid orbitals

5.43 Å

2.35Å

3sp tetrahedral bond
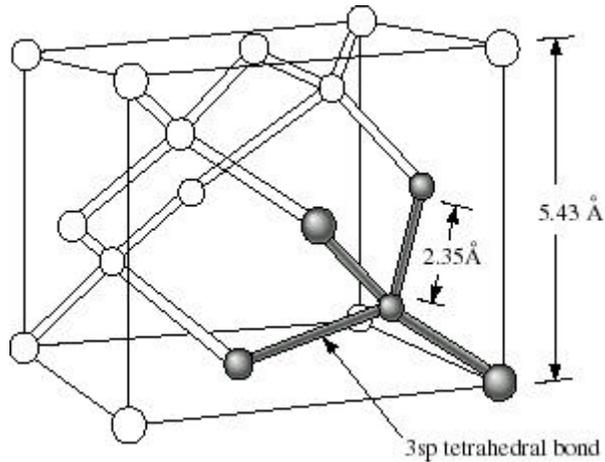
How many silicon atoms cm^{-3}?

atoms per unit cell =

unit cells cm^{-3} = 1 / (5.43 x 10^{-8} cm)^3

Picturing the Valence Electrons: The Bond Model

* A silicon crystal has a network of single bonds (one electron from each Si atom)

* How to depict the electrons in a silicon crystal?

border of "bulk" silicon region   two electrons in each bond
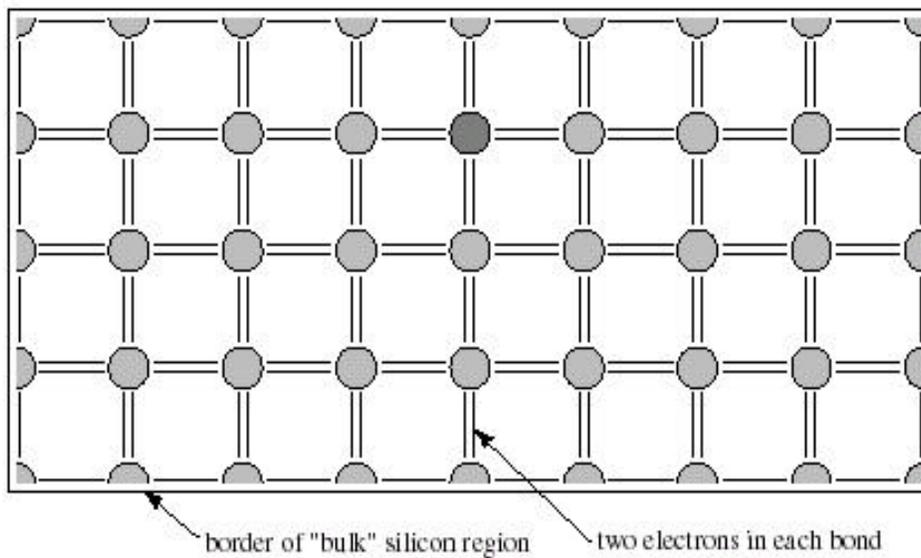
"Intrinsic" Silicon ($T > 0$ K)

A few broken bonds due to thermal agitation

border of bulk silicon region
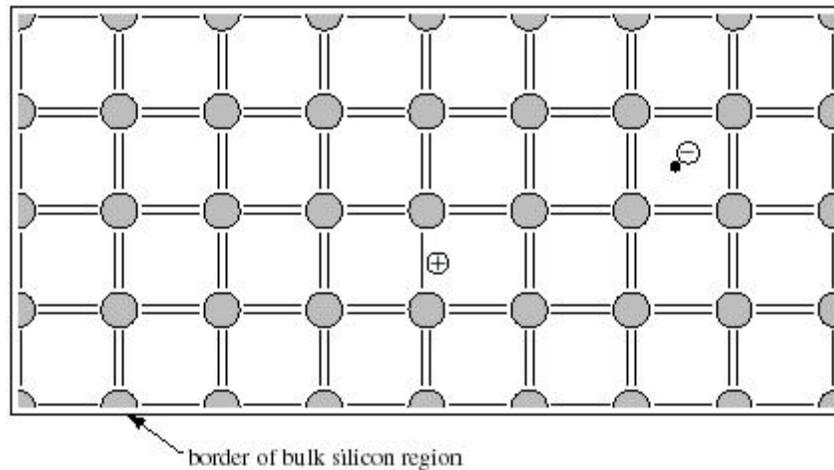
Unit of charge: $q = 1.6$ x 10^{-19} Coulombs [C]

* *electron*: mobile negative unit charge, concentration n (cm^{-3})

   no surprise ... but crystal causes a change in effective mass

* *hole*: mobile positive unit charge, concentration p (cm^{-3})

   surprise: holes move through silicon crystal almost as well as electrons

Thermal velocity

Thermal Equilibrium

Concept: balance between internal processes with no external stimulus

Electron and hole *generation rate*: G units: $\text{cm}^{-3}\,\text{s}^{-1}$ (thermal, optical processes)

Electron and hole *recombination rate*: $R \propto n \cdot p$

    n = electron concentration cm^{-3} p = hole concentration cm^{-3}

In thermal equilibrium, we add "*o*" as a subscript and we must have a balance

    $G_o = R_o$

    $n_o\,p_o = \text{constant} = n_i^2 = 10^{20}\ \text{cm}^{-3}$ at room temperature (approx.)

                           (more exactly, $\sim 2 \times 10^{20}\ \text{cm}^{-3}$)

Since holes and electrons are created *together* in intrinsic silicon,

    $n_o = p_o$

Doping

Donors (group V) *donate* their 5th valence electron and become fixed positive charges in the lattice. Examples: Arsenic, Phosphorus.
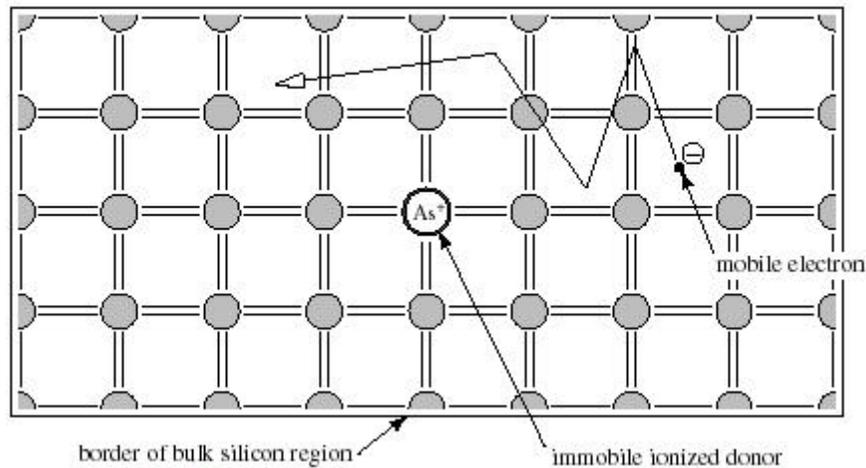
At room temperature, the very weak bond between the 5th valence electron and the arsenic breaks and the electron can "roam around"

Key idea: we can introduce electrons *selectively* and change n_o

Electron Concentration in Donor-Doped Silicon

How are the equilibrium electron and hole concentrations changed by donor doping?

Key idea:

interior region of the silicon crystal was electrically neutral before arsenic donated its electron and must be neutral afterwards

charge density ρ [C/cm^3] in the region is *zero*, before and after doping:

three components to charge density:

1. ionized donors (positive charge, immobile) $\rho_1 = + q\, N_d$

(donor concentration is N_d [cm^{-3}])

2. electrons (negative charge, mobile) $\rho_2 = - q\, n_o$

3. holes (positive charge, mobile) $\rho_3 = + q\, p_o$

$$\rho = 0 = +qN_d - qn_o + qp_o$$

Substitute the hole concentration p_o in terms of the electron concentration n_o

$$p_o = n_i^2 / n_o$$

$$0 = +qN_d - qn_o + q(n_i^2 / n_o)$$

Electron Concentration (cont.)

Quadratic formula

$$n_o = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2} = \frac{N_d}{2} + \frac{N_d}{2}\sqrt{1 + \frac{4n_i^2}{N_d^2}}$$

We *always* dope the crystal some, so that $N_d \gg n_i$ ... ($N_d = 10^{13}$ - 10^{19} cm^{-3}), so the square root reduces to 1:

$$\boxed{n_o = N_d}$$

"one electron per donor" is a way to remember the electron concentration in silicon doped with donors.

Since we are in thermal equilibrium, $n_o\, p_o = n_i^2$ (not changed by doping)

The equilibrium hole concentration is:

$$\boxed{p_o = n_i^2 / N_d}$$

Numerical Example

Donor concentration: $N_d = 10^{15}$ cm^{-3}

Thermal equilibrium electron concentration:

$$n_o \approx N_d = 10^{15} \text{ cm}^{-3}$$

Thermal equilibrium hole concentration:

$$p_o = n_i^2 / n_o \approx n_i^2 / N_d = (10^{10} \text{ cm}^{-3})^2 / 10^{15} \text{ cm}^{-3} = 10^5 \text{ cm}^{-3}$$

Silicon doped with donors is called **n-type** and electrons are the **majority carriers**.

Holes are the (nearly negligible) **minority carriers**.

An Analogy from Chemistry 1A

H_2O is the product of a reaction of hydrogen ions (H^+) and hydroxyl ions (OH^-)

In thermal equilibrium, the product of the concentration of hydrogen ions [H^+] and the concentration of hydroxyl ions [OH^-] is a constant at a given temperature

$$[H^+][OH^-] = K_{eq}$$

In an acid, the hydrogen ion concentration dominates, implying that [OH^-] is much lower

$$[OH^-] = \frac{K_{eq}}{[H^+]}$$

The chemists (for reasons known only to themselves) call this type of expression

"the mass action law"

Doping with Acceptors

Acceptors (group III) are deficient one electron compared to Si, but they can *accept* an electron from the lattice to fill the incomplete fourth covalent bond and thereby create a mobile hole and become fixed negative charges. Example: Boron.

mobile hole and later trajectory   immobile negatively ionized acceptor
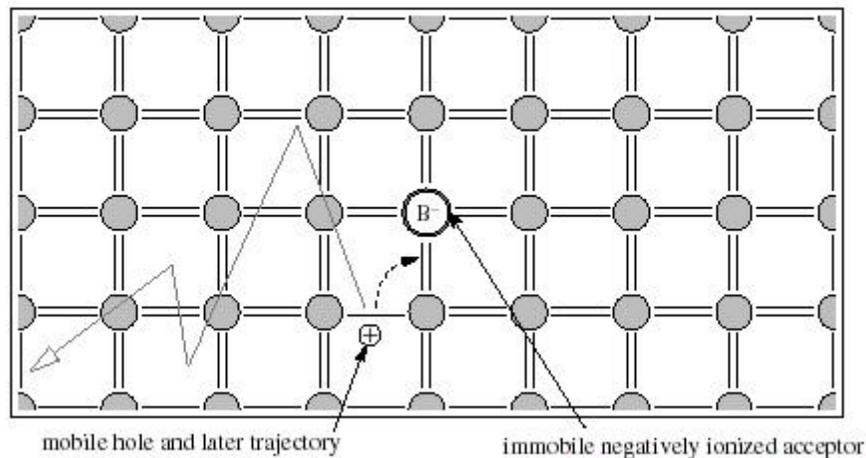
Acceptor concentration is N_a (cm^{-3}), we have $N_a \gg n_i$ typically and so:

one hole is added per acceptor:

$$p_o = N_a$$

equilibrium electron concentration is::

$$n_o = n_i^2 / N_a$$

Example: $N_a = (10^{16}/\text{cm}^3) \rightarrow p_o \simeq 10^{16}/\text{cm}^3, n_o \simeq 10^4/\text{cm}^3$

Doping with both Donors and Acceptors: Compensation

What happens if both donors and acceptors are present?

whichever is more numerous determines the type

Example: twice as many donors (As) as acceptors (B)

1. the first N_a of donated electrons get accepted by the boron ions, leaving

2. $N_d - N_a$ leftover as mobile electrons
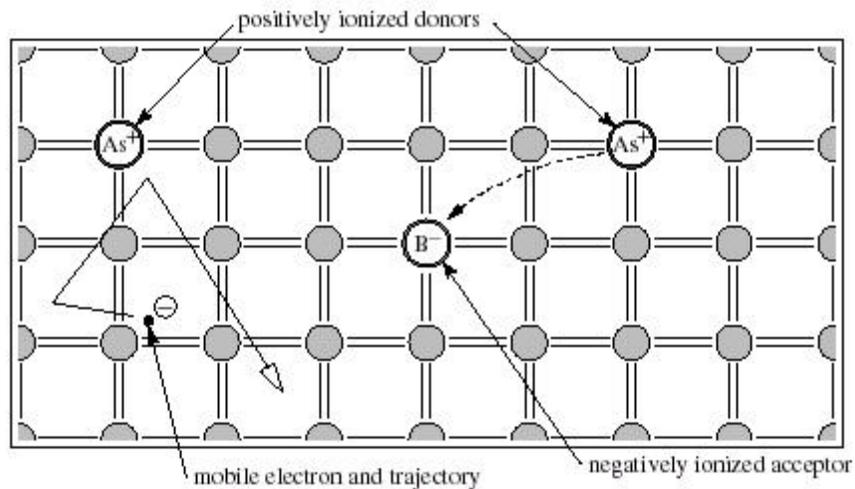
Typically, $| N_d - N_a | \gg n_i$

Carrier Concentrations in Compensated Silicon

For the case where $N_d > N_a$, the electron and hole concentrations are:

$$n_o \cong N_d - N_a \quad \text{and} \quad p_o \cong \frac{n_i^2}{N_d - N_a}$$

For the case where $N_a > N_d$, the hole and electron concentrations are:

$$p_o \cong N_a - N_d \quad \text{and} \quad n_o \cong \frac{n_i^2}{N_a - N_d}$$

Note that these approximations assume that $|N_d - N_a| \gg n_i$, which is nearly always true.

Carrier Transport: Drift

* If an electric field is applied to silicon, the holes and the electrons "feel" an electrostatic force $F_e = (+q$ or $-q)E$.

* Picture of effect of electric field on representative electrons: moving at the thermal velocity $= 10^7$ cm/s ... *very fast*, but colliding every 0.1 ps $= 10^{-13}$ s. Distance between collisions $= 10^7$ cm/s x 10^{-13} cm $= 0.01$ μm

(a) Thermal Equilibrium, $E = 0$ (b) Electric Field $E > 0$

Electron # 1

Electron # 1

E

x_i $x_{f,1}$ x

x_i $x_{f,1}$ x

Electron # 2

Electron # 2

E

$x_{f,2}$ x_i x

$x_{f,2}$ x_i x

Electron # 3

Electron # 3

E

$x_{f,3}$ x_i x

$x_{f,3}$ x_i x
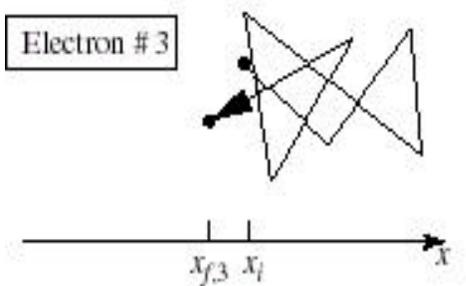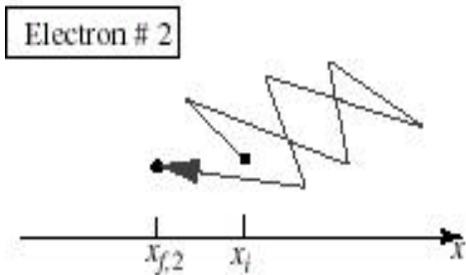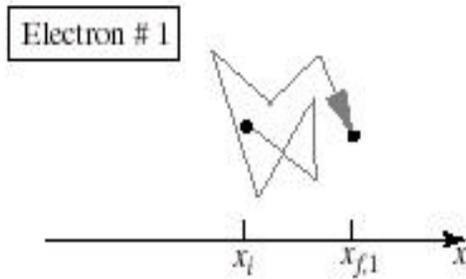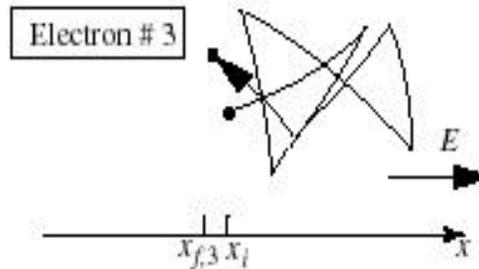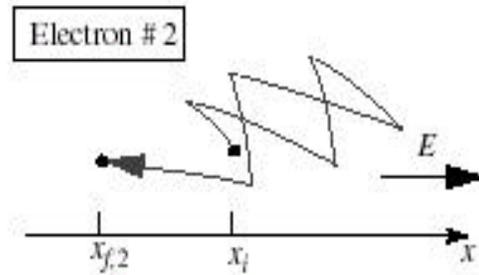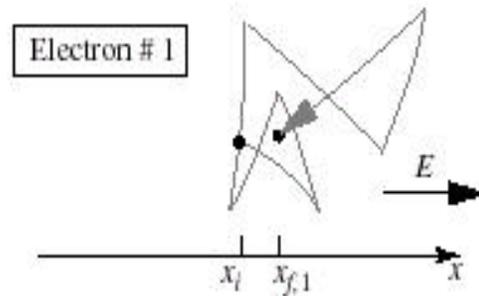
* x_i = initial position   * $x_{f,n}$ = final position of electron n after 7 collisions

Drift Velocity and Mobility

* Experiment shows that the drift velocity is proportional to the electric field for electrons (but they move, of course, in the opposite direction)

$$v_{dn} = -\mu_n E,$$

with the constant μ_n defined as the *electron mobility*.

* Holes drift in the direction of the applied electric field, with the constant μ_p defined as the *hole mobility*.

$$v_{dp} = \mu_p E$$

How do we know what's positive and what's negative?

positive: **negative:**
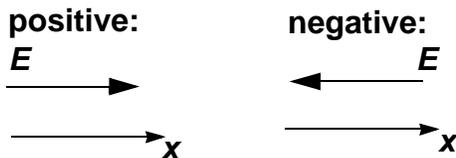
Electron and Hole Mobilities

Mobilities vary with doping level: plot is for 300 K
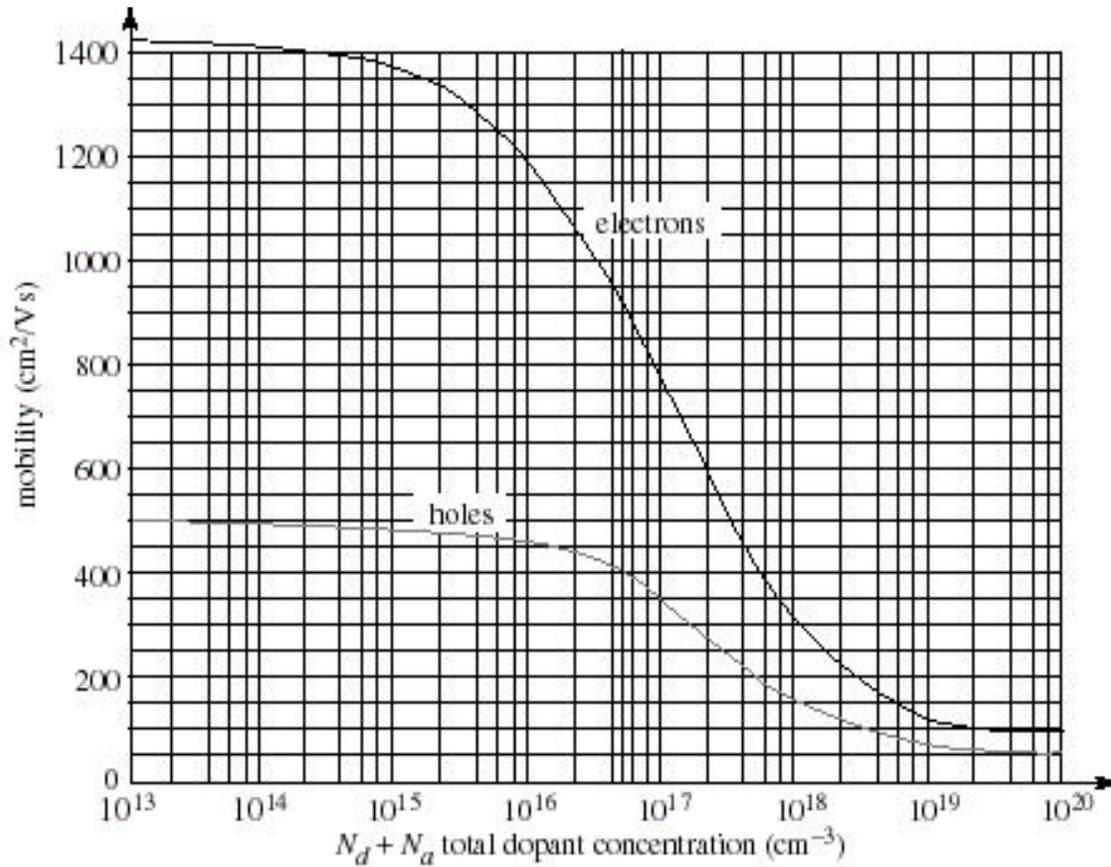
* "typical values" for bulk silicon - assuming around 5 x 10^{16} cm^{-3} doping

$$\mu_n = 1000 \text{ cm}^2/(\text{Vs}) \qquad\qquad \mu_p = 400 \text{ cm}^2/(\text{Vs})$$

Mobility and Drift Velocity

Carrier is accelerated by electric field in between collisions

zero E field

v_{th}

positive E

$a\boldsymbol{t}_c$

v_{th}
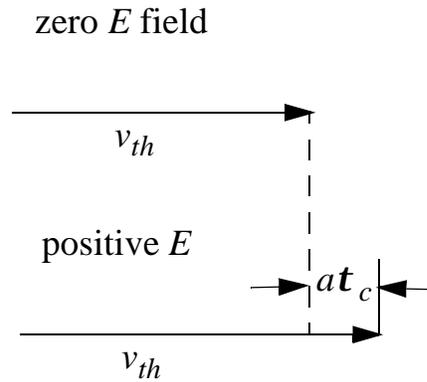
$$v_{dr} = a \cdot \tau_c = \left(\frac{F_e}{m_n}\right)\tau_c = \left(\frac{qE}{m_n}\right)\tau_c = \left(\frac{q\tau_c}{m_n}\right)E$$

Velocity Saturation

∗ "linear relationship must end at some point"

∗ at electric fields greater than around 10^4 V/cm, the drift velocities max. out at around 10^7 cm/s.
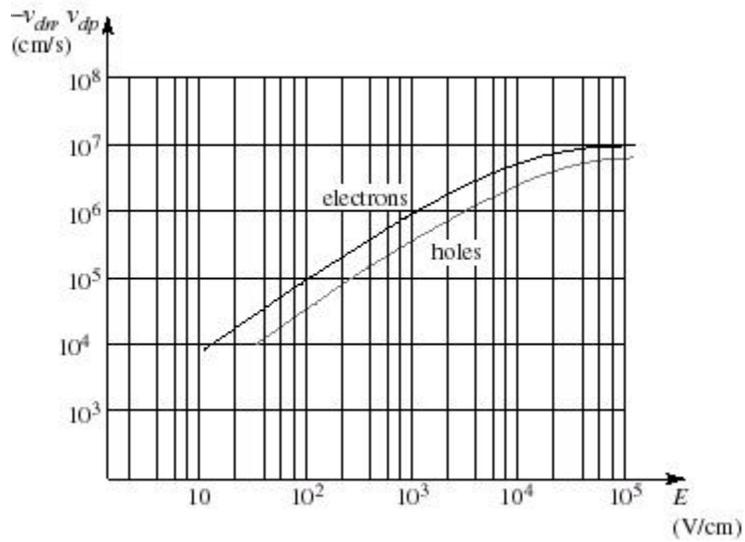
Electric fields can be huge with small voltage drops:

Carrier Transport: Drift Current Density

Electrons drifting opposite to the electric field are carrying negative charge; therefore, the *drift current density* is:

$$J_n^{dr} = (-q)\, n\, v_{dn} \qquad \text{units: } Ccm^{-2}\, s^{-1} = Acm^{-2}$$

$$\boxed{J_n^{dr} = (-q)\, n\, (-\mu_n\, E) = q\, n\, \mu_n\, E}$$

Note that J_n^{dr} is in the *same* direction as the electric field.

For holes, the mobility is μ_p and the drift velocity is in the same direction as the electric field: $v_{dp} = \mu_p\, E$

The hole drift current density is:

$$J_p^{dr} = (+q)\, p\, v_{dp}$$

$$\boxed{J_p^{dr} = q\, p\, \mu_p\, E}$$

Drift Current Directions and Signs

* For electrons, an electric field in the +x direction will lead to a drift velocity in the -x direction ($v_{dn} < 0$) and a drift current density in the +x direction ($J_n^{dr} > 0$).

**electron drift
current density**

$$\longrightarrow J_n^{dr}$$

$$\longleftarrow v_{dn}$$

$$\longrightarrow E$$

$$\longrightarrow$$
$$x$$
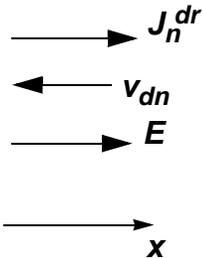
* For holes, an electric field in the +x direction will lead to a drift velocity in the +x direction ($v_{dp} > 0$) and a drift current density in the +x direction ($J_n^{dr} > 0$).

**hole drift
current density**

$$\longrightarrow J_p^{dr}$$

$$\longrightarrow v_{dp}$$

$$\longrightarrow E$$

$$\longrightarrow$$
$$x$$
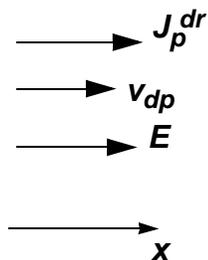
Ohm's Law for Silicon

*Bulk silicon*: uniform doping concentration, away from surfaces

   n-type example: in equilibrium, $n_o = N_d$.

   When we apply an electric field, $n = N_d$.

$$J_n = \qquad\qquad\qquad \equiv \sigma_n E$$

where σ_n is the *conductivity*   [Units: S/cm = 1 / (Ω cm)]

note: holes contribute almost nothing to the conductivity of n-type silicon.
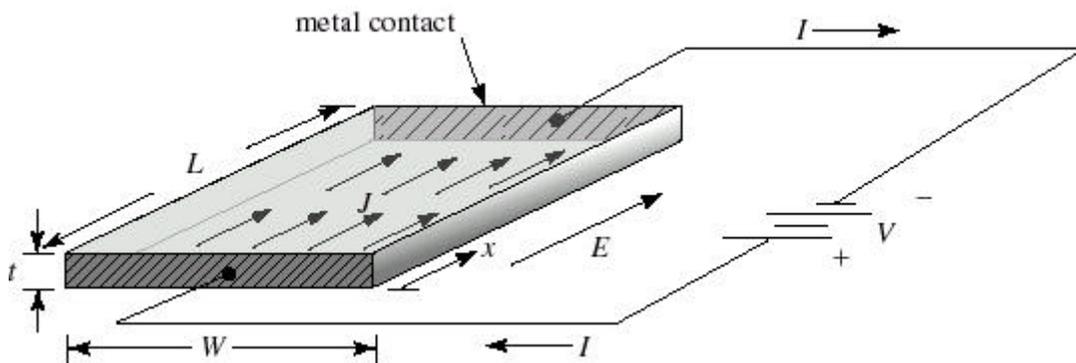
J_n is the current density, uniform over the area $t \times W$

E is electric field, here V/L

Carrier Transport: Diffusion

Diffusion is a transport process driven by *gradients* in the concentration of particles in random motion and undergoing frequent collisions -- such as ink molecules in water ... or holes and electrons in silicon.
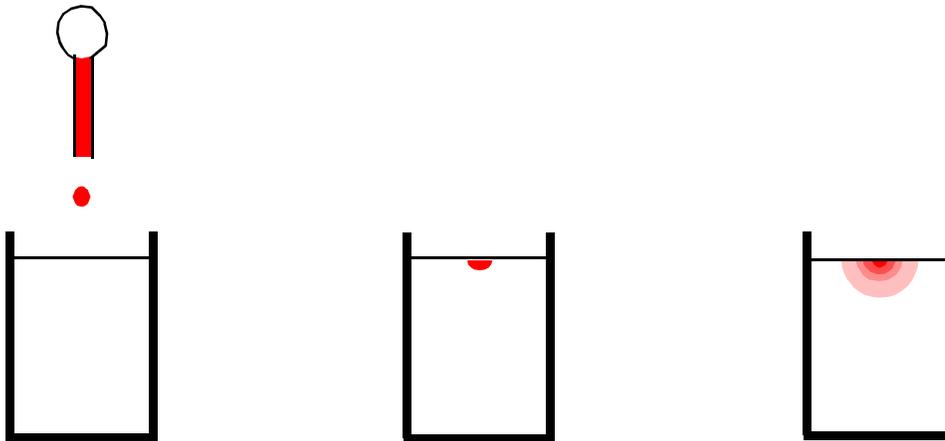
Mathematical analysis:

Some numbers: average carrier velocity = $v_{th} = 10^7$ cm/s

average interval between collisions = $\tau_c = 10^{-13}$ s = 0.1 picoseconds

define λ: the *mean free path* $= v_{th}\, t_c$

mean free path = $\lambda = v_{th}\, \tau_c = 10^{-6}$ cm = 0.01 μm ... not that small

Diffusion of Holes in One Dimension
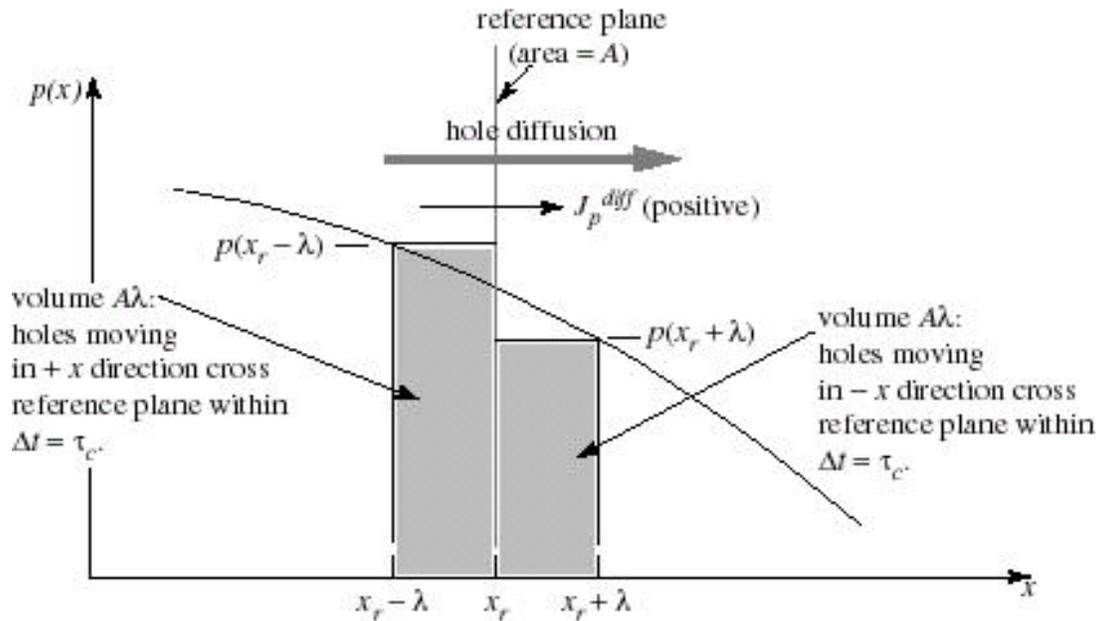
* half of the carriers in each volume will pass through the plane before their next collision, since their 1-D motion is random

* The *flux F* of holes is the number passing the unit-area reference plane per collision time

* The concentrations at $x_r \pm \lambda$ can be found by a first-order Taylor expansion, if λ is small

Hole Diffusion Current Density

Current density J = (charge) x (Flux) = (charge) x (# carriers per area per second)

$$J_p^{diff} = -qD_p\frac{dp}{dx}, \quad \text{where } D_p \text{ is the diffusion coefficient}$$

The diffusion coefficient is related to the mean free path and the collision time by

$$D_p = \frac{\lambda^2}{\tau_c} = v_{th}\lambda$$

Mobility and the Diffusion Coefficient

The ratio of mobility to the diffusion coefficient is:

$$\frac{D}{\mu} = \frac{(\lambda^2/\tau_c)}{(q\tau_c)/m} = \left(\frac{\lambda^2}{\tau_c^2}\right)\left(\frac{m}{q}\right) = v_{th}^2\left(\frac{m}{q}\right)$$

The thermal velocity is a function of the temperature and the mass (Lecture 2)

$$\frac{1}{2}mv_{th}^2 = \frac{1}{2}kT \qquad \text{so} \qquad v_{th}^2 = (kT)/m$$

$$\boxed{\frac{D}{\mu} = \left(\frac{kT}{m}\right)\left(\frac{m}{q}\right) = \frac{kT}{q}}$$

∗ The quantity kT/q has units of volts and is called the *thermal voltage*, V_{th}:

$$V_{th} = \frac{kT}{q} = 25 - 26 \text{ mV},$$

at "room temperature," with 25 mV for a cool room (62 $^\circ$F) and 26 mV for a warm room (83 $^\circ$F).

Total Current Densities

Electron diffusion current density: same approach, sign is different

$$J_n^{diff} = qD_n\frac{dn}{dx},$$

where D_n is the electron diffusion coefficient (units: cm^2/s)

* Add drift and diffusion components for electrons and for holes --

$$J_n = J_n^{dr} + J_n^{diff} = qn\mu_n E + qD_n\frac{dn}{dx}$$

$$J_p = J_p^{dr} + J_p^{diff} = qp\mu_p E - qD_p\frac{dp}{dx}$$

* Fortunately, we will be able to eliminate one or the other component of the electron or the hole current in our analysis of semiconductor devices.