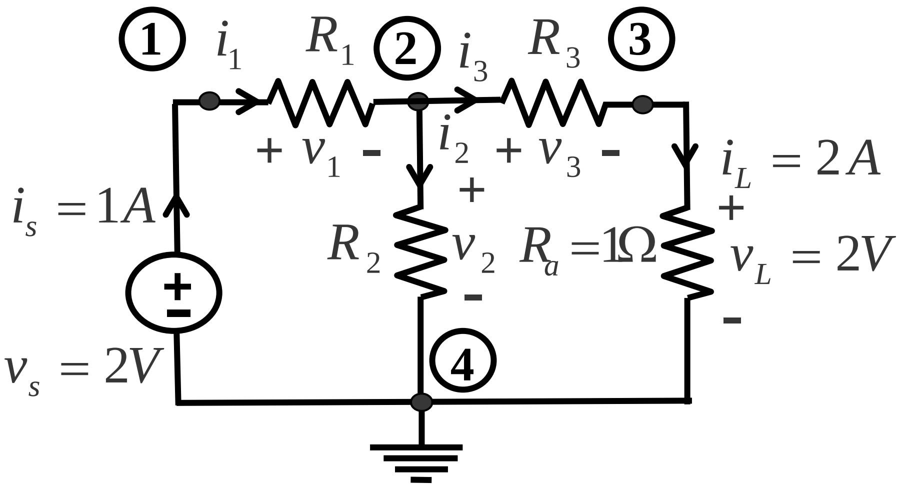
An Example Illustrating A Non-trivial Application of Tellegen's Theorem

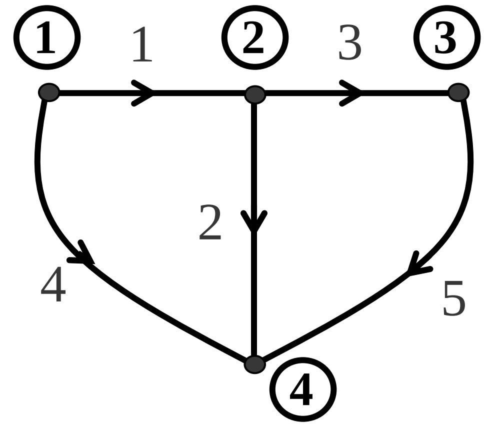
Consider the following 2 circuits N and $\hat{N}$.
Let v_j and i_j denote the voltage and current of branch j of N. Let $\hat{v}_j$ and $\hat{i}_j$ denote the voltage and current of $\hat{N}$. The values of R_1, R_2 and R_3 are not known in both circuits. But instead, $i_s = 1A$ and $v_L = 2V$ are given for N and $\hat{v}_s = 3V$ is given for $\hat{N}$. The problem is to find the voltage $\hat{v}_L$ of $\hat{N}$.

Note : Although the 2 circuits are different (N is driven by a voltage source, but $\hat{N}$ is driven by a current source; the values of R_a and R_b are also different), they have the same digraph.

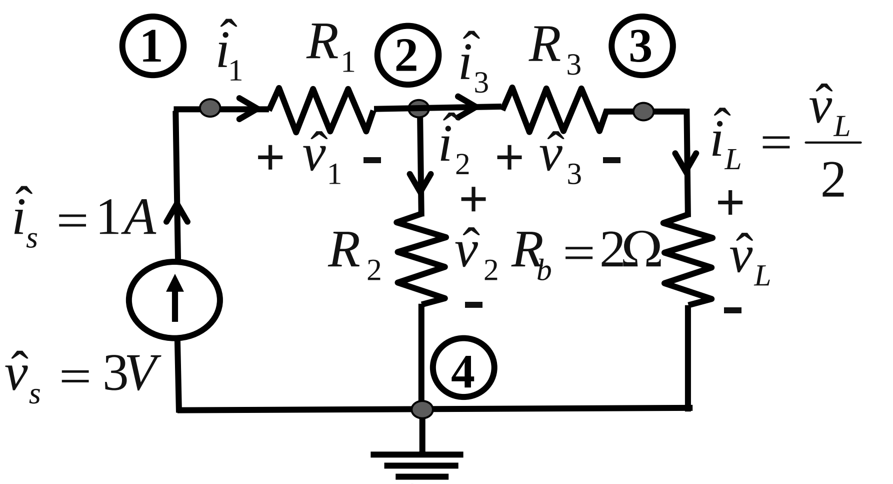
Circuit *N*



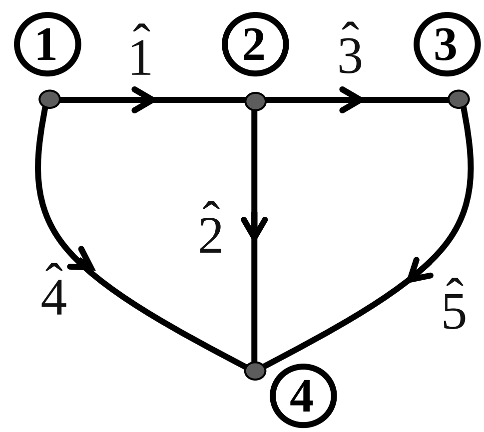
Graph *G*



Circuit $\hat{N}$



Graph $\hat{G}$



Since the 2 digraph G and $\hat{G}$ are identical, we can apply Tellegen's theorem to either digraph using **any set** of voltages which satisfy KVL for N, and **any set** of currents which satisfy KCL for $\hat{N}$, and vice versa, paying attention that we must use Associated Reference Convention :

$$\text{For } N \; : v_4 = 2V \; , \quad i_4 = -1A$$

$$\text{For } \hat{N} \; : \hat{v}_4 = 3V \; , \quad \hat{i}_4 = -1A$$

(a) Applying Tellegen's Theorem using the voltage solutions v_j for N (which must satisfy KVL) and the current solutions $\hat{i}_j$ (which must satisfy KCL) for $\hat{N}$:

$$\underbrace{(v_1)(\hat{i}_1) + (v_2)(\hat{i}_2) + (v_3)(\hat{i}_3)} + (v_4)(\hat{i}_4) + (v_5)(\hat{i}_5) = 0$$

$$\Rightarrow \quad I = \sum_{j=1}^{3} (v_j)(\hat{i}_j) \qquad + (2)(-1) + (2)\left(\frac{\hat{v}_L}{2}\right) = 0 \qquad (1)$$

(b) Applying Tellegen's Theorem using the voltage solutions $\hat{v}_j$ for $\hat{N}$ (which must satisfy KVL) and the current solutions i_j (which must satisfy KCL) for N :

$$\underbrace{(\hat{v}_1)(i_1) + (\hat{v}_2)(i_2) + (\hat{v}_3)(i_3)} + (\hat{v}_4)(i_4) + (\hat{v}_5)(i_5) = 0$$

$$\Rightarrow \quad \hat{I} = \sum_{j=1}^{3} (\hat{v}_j)(i_j) \qquad + (3)(-1) + (\hat{v}_L)(2) = 0 \qquad (2)$$

Observe

$$I = \sum_{j=1}^{3} (v_j)(\hat{i}_j) = (R_1 i_1)(\hat{i}_1) + (R_2 i_2)(\hat{i}_2) + (R_3 i_3)(\hat{i}_3) \qquad (3)$$

$$I = \sum_{j=1}^{3} (\hat{v}_j)(i_j) = (R_1 \hat{i}_1)(i_1) + (R_2 \hat{i}_2)(i_2) + (R_3 \hat{i}_3)(i_3) \qquad (4)$$

$$\Rightarrow \quad I = \hat{I}$$

Substracting (1) - (2) :

$$\left[(2)(-1) + (2)\left(\frac{\hat{v}_L}{2}\right)\right] - \left[(3)(-1) + (\hat{v}_L)(2)\right] = 0$$

$$-2 + \hat{v}_L + 3 - 2\hat{v}_L = 0$$

$$\Rightarrow \boxed{\hat{v}_L = 1V}$$