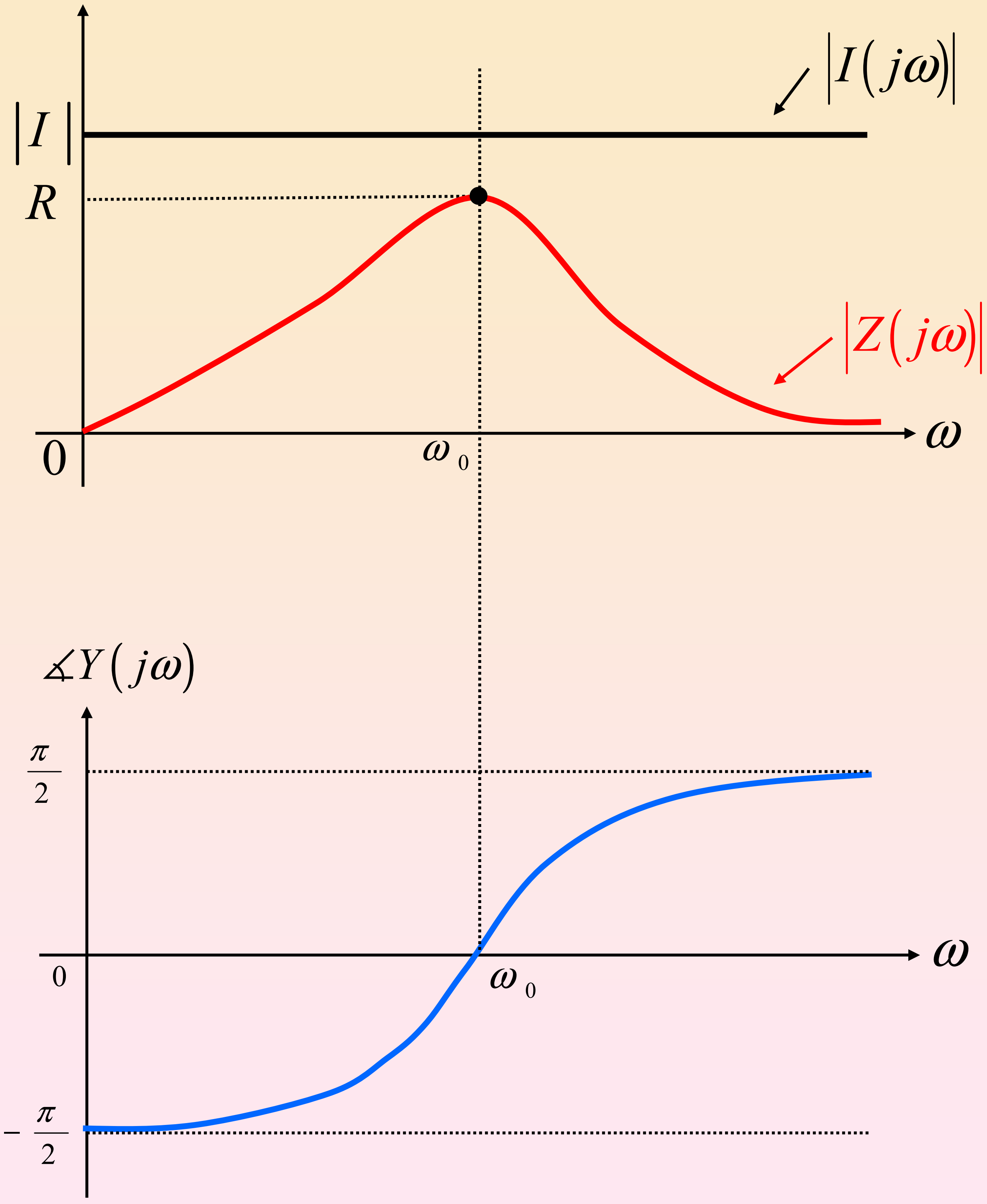
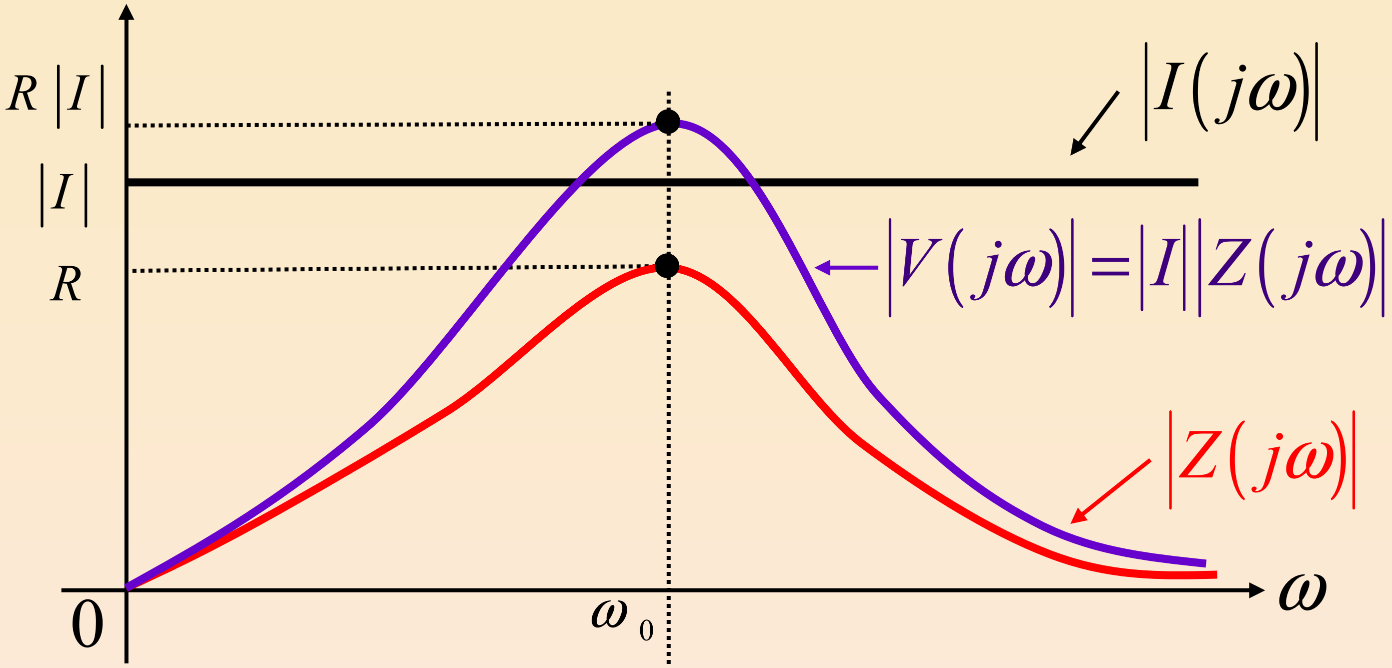
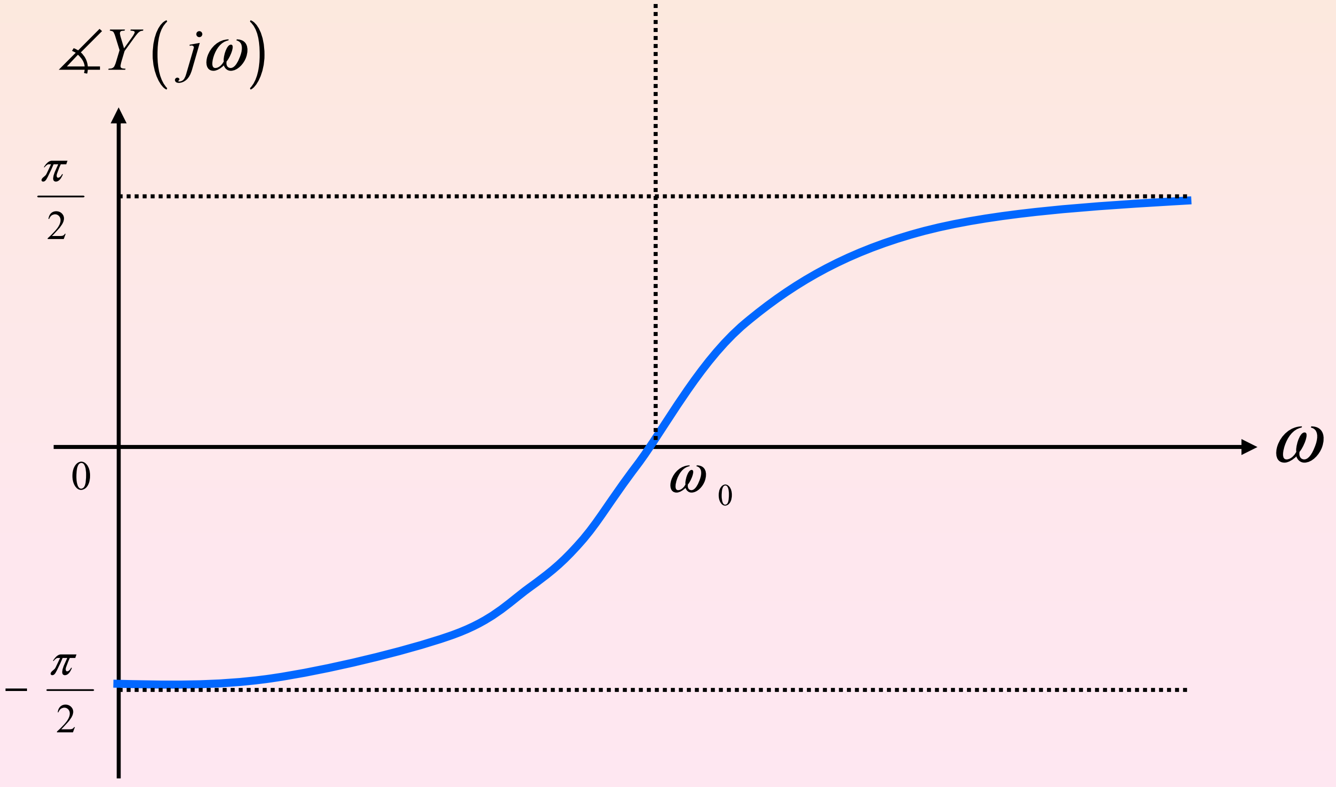


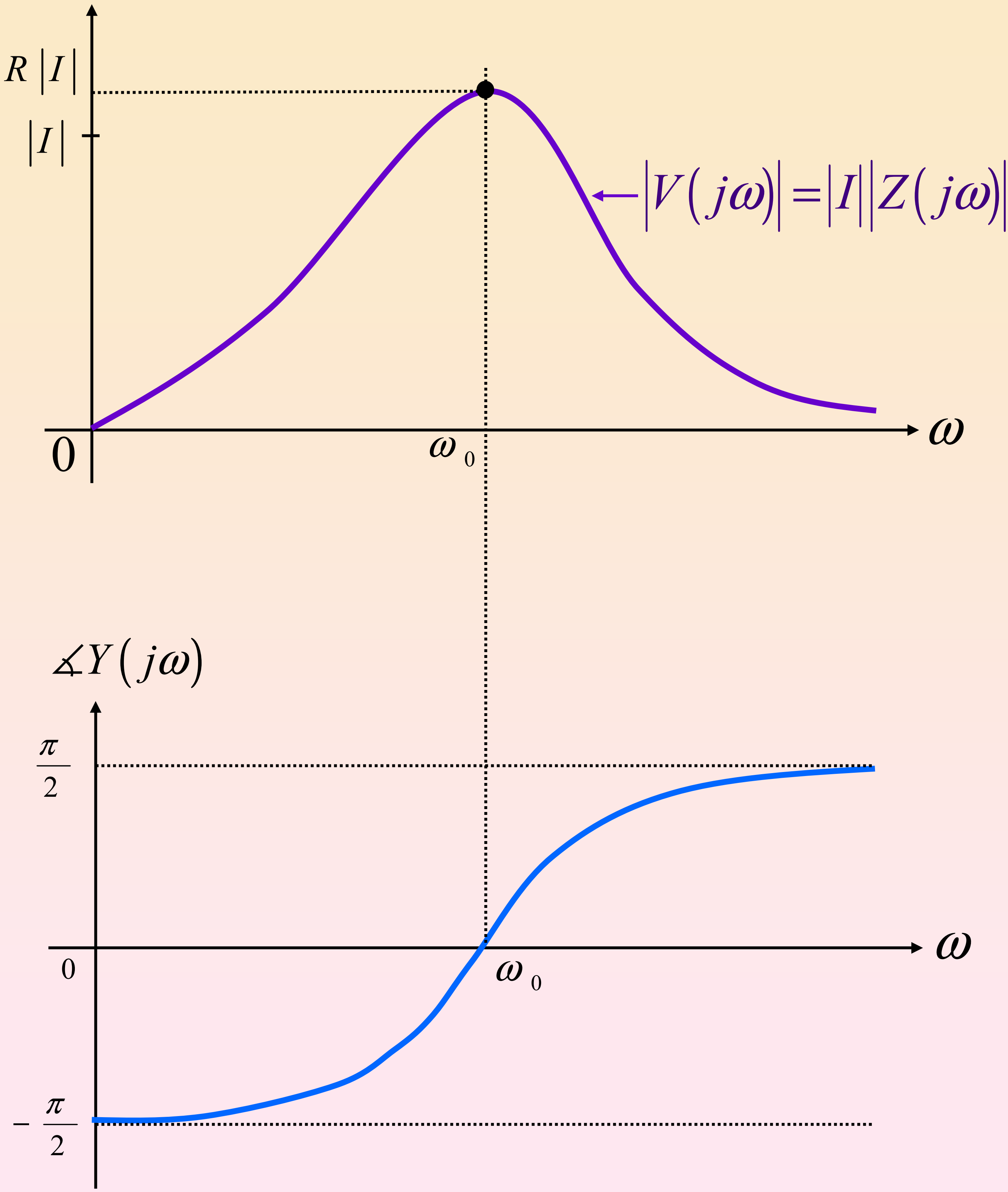
$\left|I(j\omega)\right|$
$\left|I\right|$
R
$\left|Z(j\omega)\right|$
0
ω_0
ω
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
0
ω_0
ω
$-\frac{\pi}{2}$



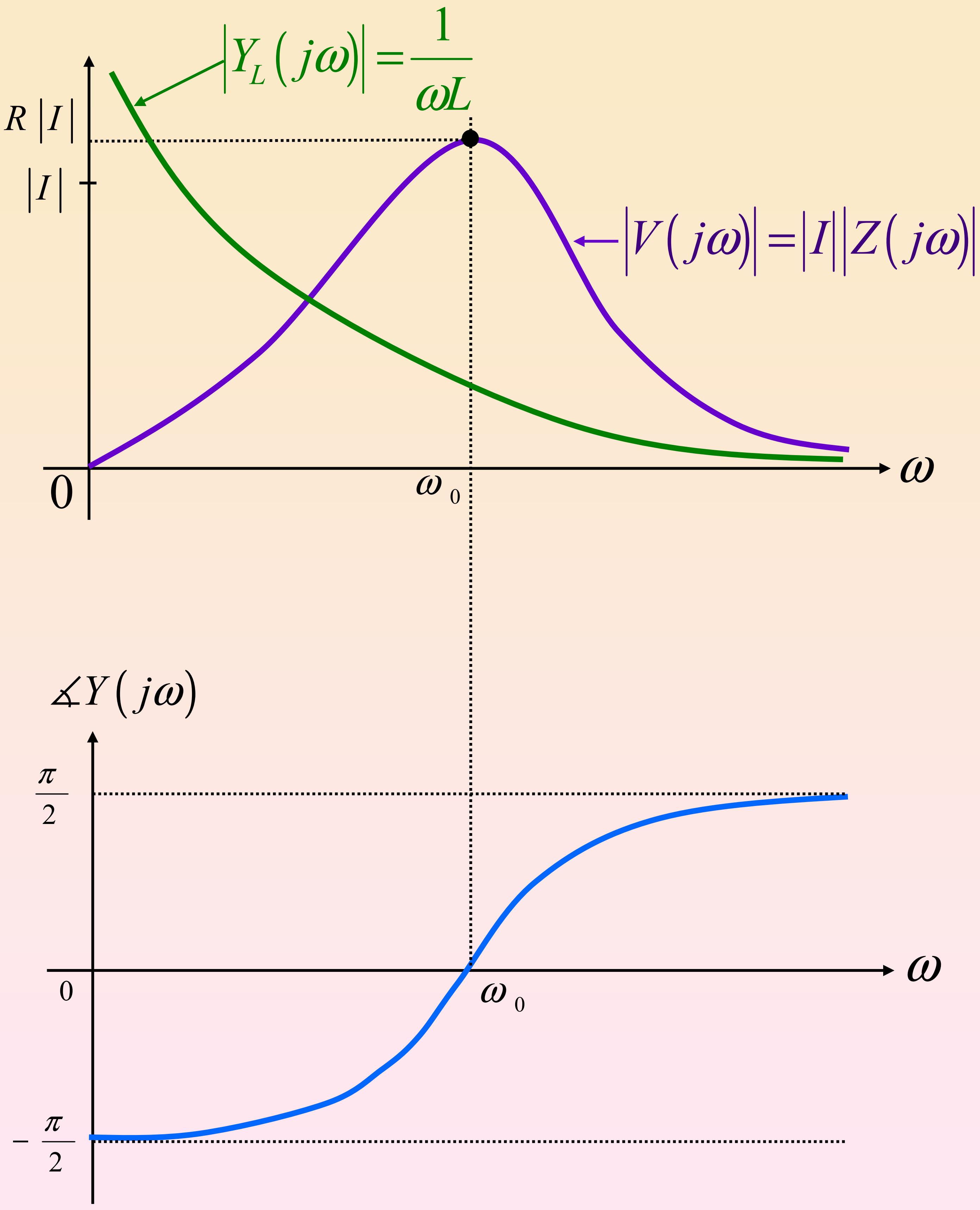
$R\left|I\right|$
$\left|I\right|$
R
$\left|I(j\omega)\right|$
$\left|V(j\omega)\right| = \left|I\right|\left|Z(j\omega)\right|$
$\left|Z(j\omega)\right|$
0
ω_0
ω



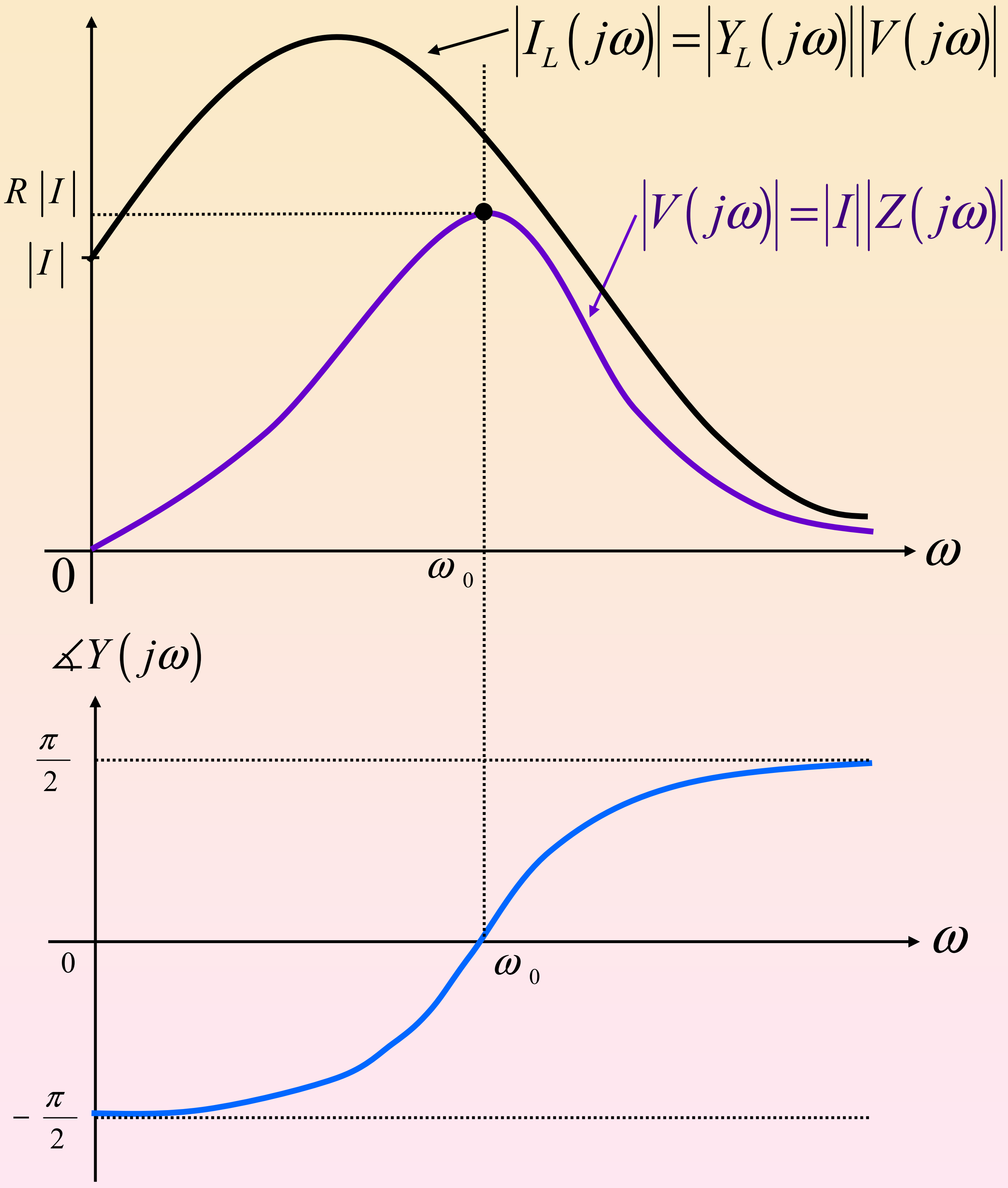
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
0
ω_0
ω
$-\frac{\pi}{2}$



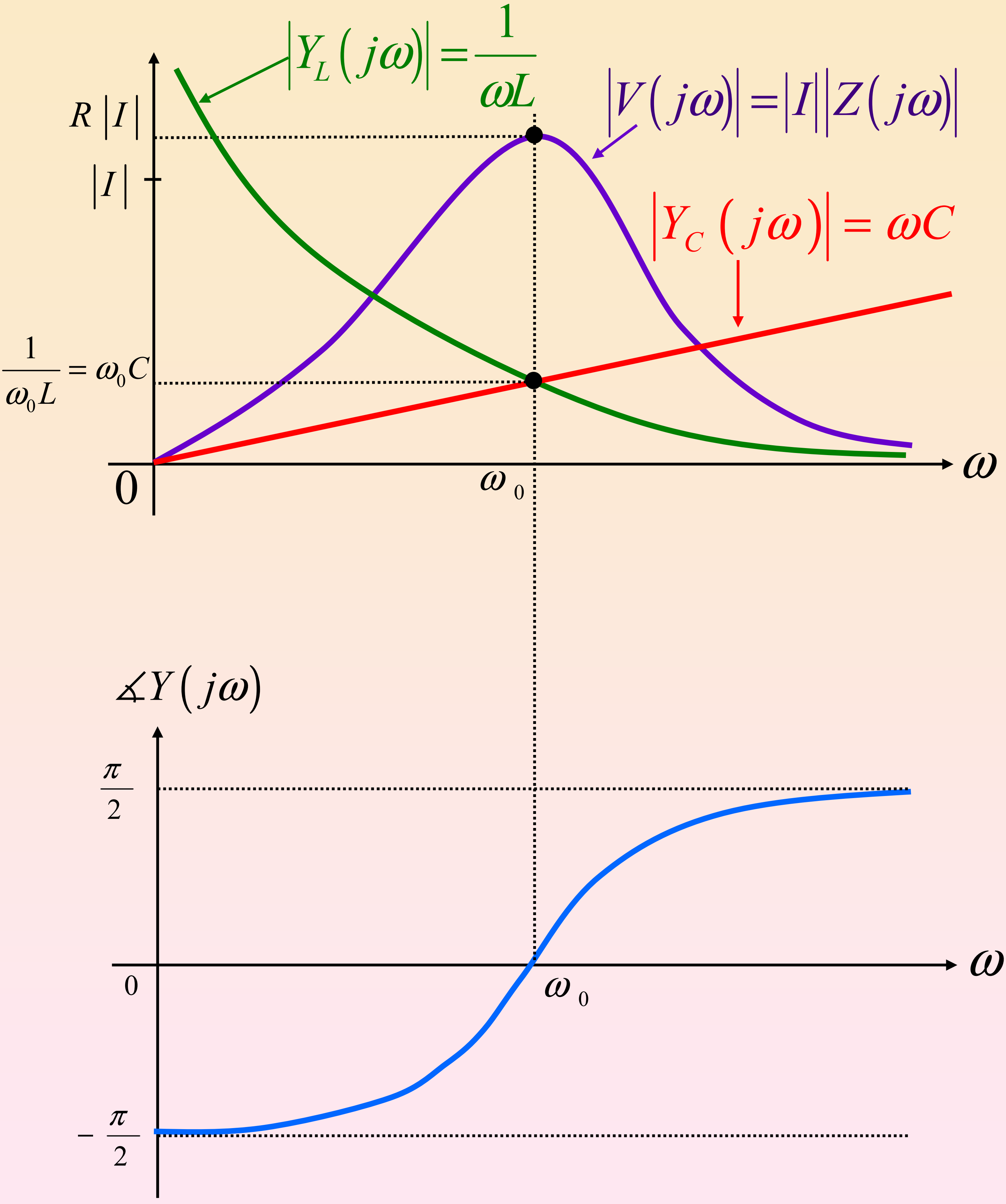
$R|I|$
$|I|$
$|V(j\omega)| = |I||Z(j\omega)|$
0
ω_0
ω
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
0
ω_0
ω
$-\frac{\pi}{2}$



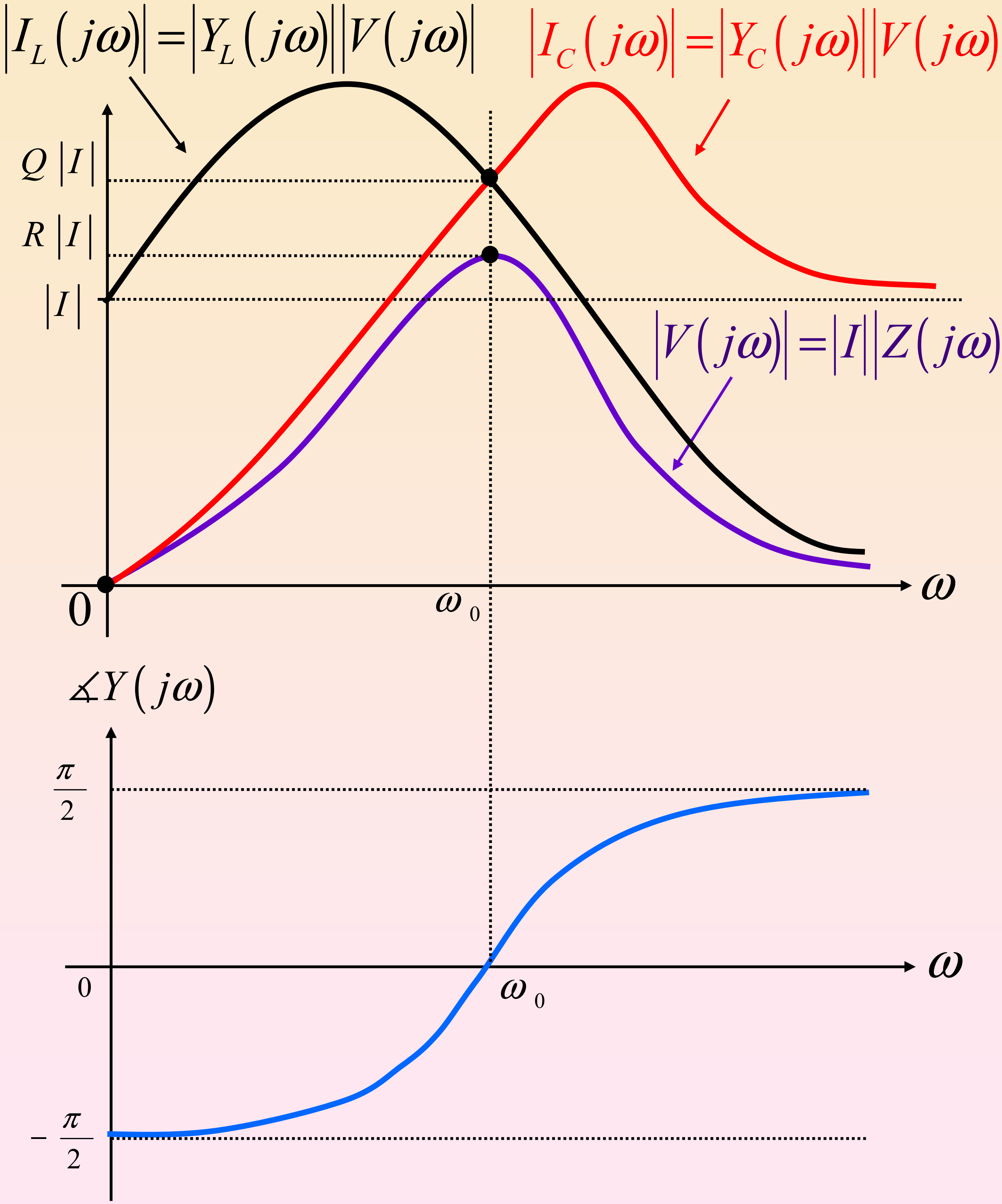
$\left|Y_L(j\omega)\right| = \frac{1}{\omega L}$
$R\left|I\right|$
$\left|I\right|$
$\left|V(j\omega)\right| = \left|I\right|\left|Z(j\omega)\right|$
0
ω_0
ω
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
0
ω_0
ω
$-\frac{\pi}{2}$



$\left|I_L(j\omega)\right| = \left|Y_L(j\omega)\right|\left|V(j\omega)\right|$
$\left|V(j\omega)\right| = \left|I\right|\left|Z(j\omega)\right|$
$R\left|I\right|$
$\left|I\right|$
0
ω_0
ω
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
ω_0
ω



$\left|Y_L(j\omega)\right| = \frac{1}{\omega L}$
$\left|V(j\omega)\right| = \left|I\right|\left|Z(j\omega)\right|$
$\left|Y_C(j\omega)\right| = \omega C$
$R\left|I\right|$
$\left|I\right|$
$\frac{1}{\omega_0 L} = \omega_0 C$
0
ω_0
ω
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
ω_0
ω



$|I_L(j\omega)| = |Y_L(j\omega)||V(j\omega)|$
$|I_C(j\omega)| = |Y_C(j\omega)||V(j\omega)|$
$Q|I|$
$R|I|$
$|I|$
$|V(j\omega)| = |I||Z(j\omega)|$
0
ω_0
ω
$\measuredangle Y(j\omega)$
$\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
ω_0
ω

$$\omega = \omega_0 \quad \Rightarrow \left|V(j\omega_0)\right| = \left|Z(j\omega_0)I(j\omega_0)\right| = R\left|I\right|$$

Definitions :

Damping Constant : $\alpha \triangleq \dfrac{1}{2RC}$

Quality Factor : $Q \triangleq \dfrac{R}{\omega_0 L} = \dfrac{\omega_0}{2\alpha}$

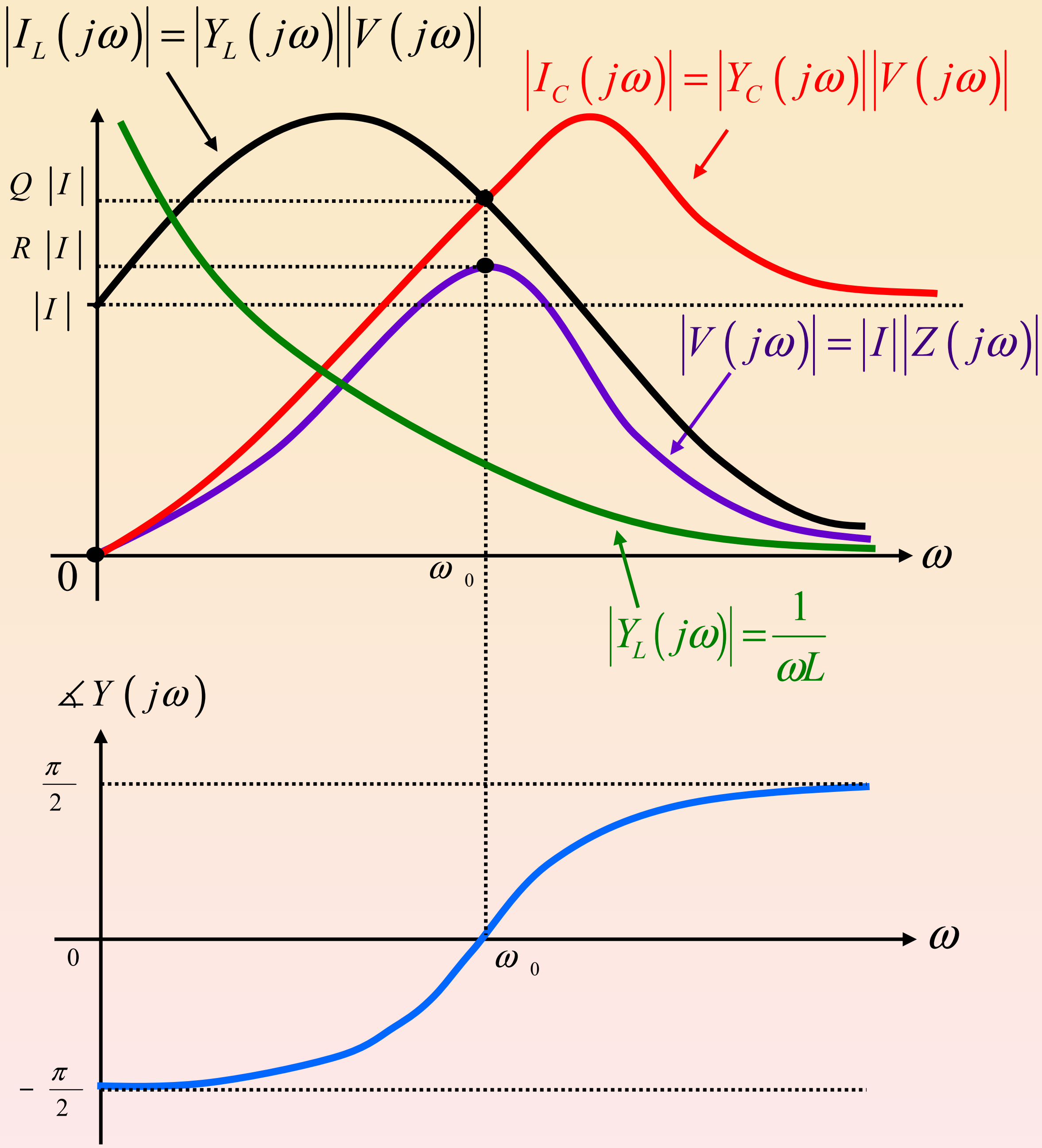
$$\frac{\left|I_L(j\omega_0)\right|}{\left|I(j\omega_0)\right|} = \frac{\left|\left(1/j\omega_0 L\right)V(j\omega_0)\right|}{|I|} = \frac{\left|R/j\omega_0 L\right||I|}{|I|} = \frac{R/L}{\omega_0} = \frac{\omega_0}{2\alpha} \triangleq Q$$

$$\frac{\left|I_C(j\omega_0)\right|}{\left|I(j\omega_0)\right|} = \frac{\left|(j\omega_0 C)V(j\omega_0)\right|}{|I|} = \frac{\left|j\omega_0 C\ R\right||I|}{|I|} = \frac{\omega_0^2 RC}{\omega_0} = \frac{R/L}{\omega_0} \triangleq Q$$

$$\therefore \quad \left|I_L(j\omega_0)\right| = \left|I_C(j\omega_0)\right| = Q\left|I\right|$$

Note: $\max\left|I_L(j\omega)\right| > Q\left|I\right|$

$\max\left|I_C(j\omega)\right| > Q\left|I\right|$



at $\omega = 0$

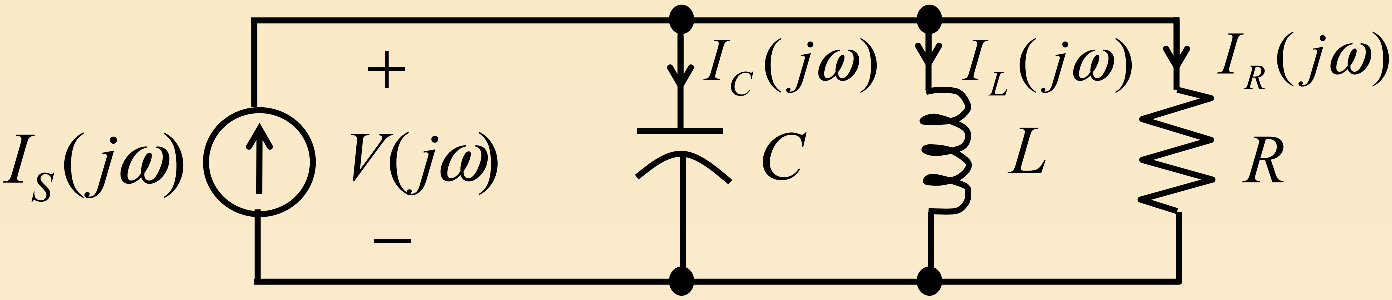
$$|I_L(j0)| = |I| \quad , \quad |I_C(j0)| = 0$$

at $\omega = \infty$

$$|I_L(j\infty)| = 0 \quad , \quad |I_C(j\infty)| = |I|$$

at $\omega = \omega_0$

$$|I_L(j\omega_0)| = |I_C(j\omega_0)| = Q\,|I|$$



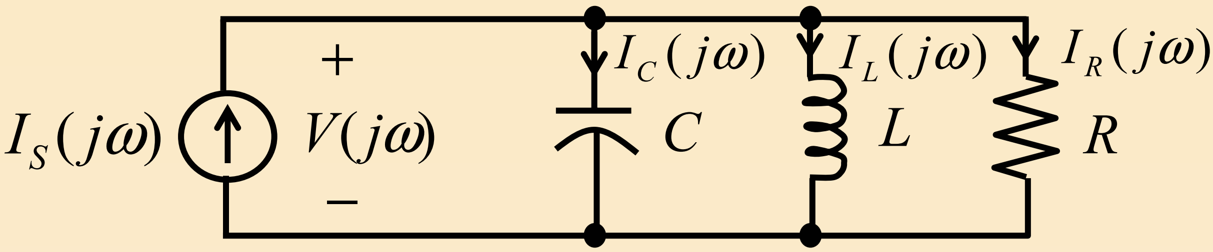
Find $H(j\omega) \triangleq \dfrac{I_R(j\omega)}{I_S(j\omega)}$ $\qquad \omega_0 \triangleq \dfrac{1}{\sqrt{LC}}$

$$H(j\omega) \triangleq \frac{I_R(j\omega)}{I_S(j\omega)} = \frac{V(j\omega)/R}{I_S(j\omega)} = \frac{Z(j\omega)I_S(j\omega)}{R\,I_S(j\omega)}$$

$$= \frac{1}{R\,Y(j\omega)} = \frac{1}{R\left[1/R + j\left(\omega C - 1/\omega L\right)\right]}$$

$$= \frac{1}{1 + j\left[\omega_0 RC\left(\omega/\omega_0 - \omega_0/\omega\right)\right]}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1 + Q^2\left(\omega/\omega_0 - \omega_0/\omega\right)^2}}$$



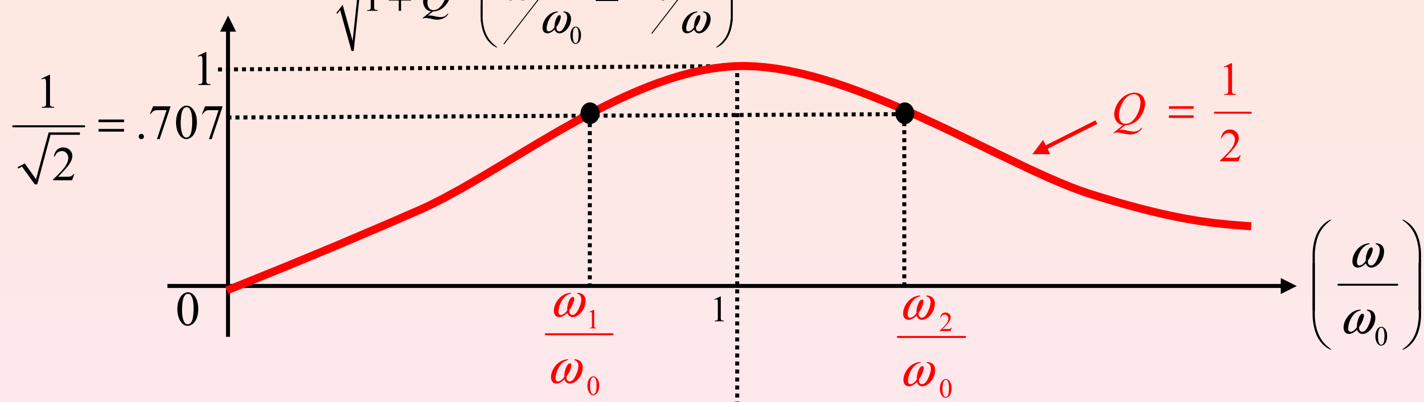
Find $H(j\omega) \triangleq \dfrac{I_R(j\omega)}{I_S(j\omega)}$ $\qquad \omega_0 \triangleq \dfrac{1}{\sqrt{LC}}$

$$H(j\omega) \triangleq \frac{I_R(j\omega)}{I_S(j\omega)} = \frac{1}{1 + j\left[\underbrace{\omega_0 RC}\left(\omega/\omega_0 - \omega_0/\omega\right)\right]}$$

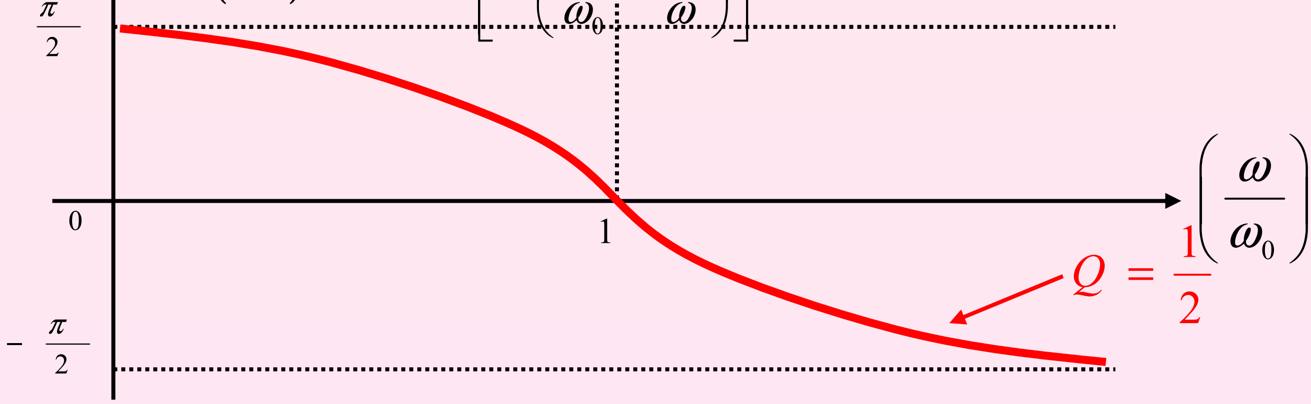
$$\frac{\omega_0^2 RC}{\omega_0} = \frac{R/L}{\omega_0} \triangleq Q$$

$$= \frac{1}{1 + jQ\left(\omega/\omega_0 - \omega_0/\omega\right)}$$

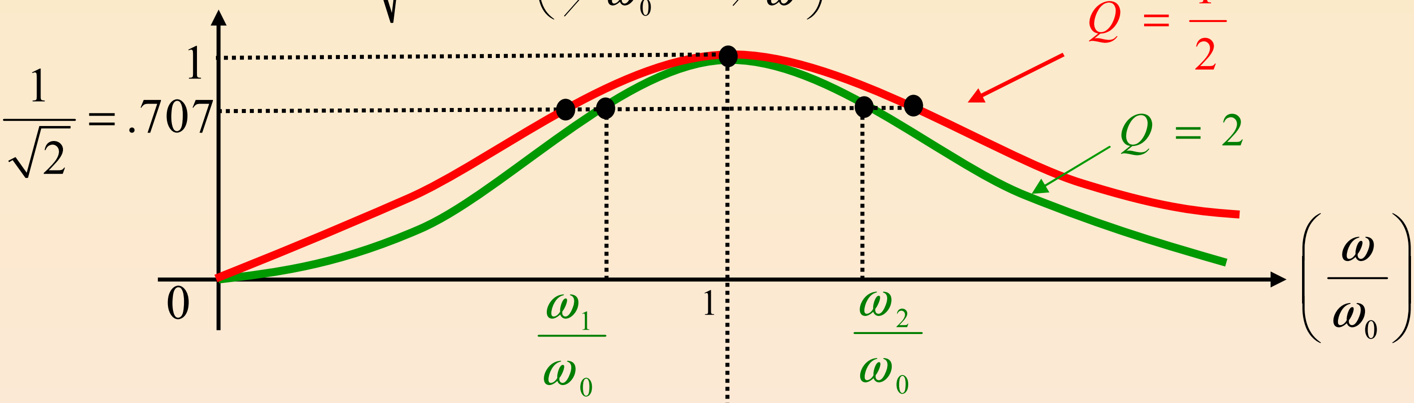
$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1 + Q^2\left(\omega/\omega_0 - \omega_0/\omega\right)^2}}$$

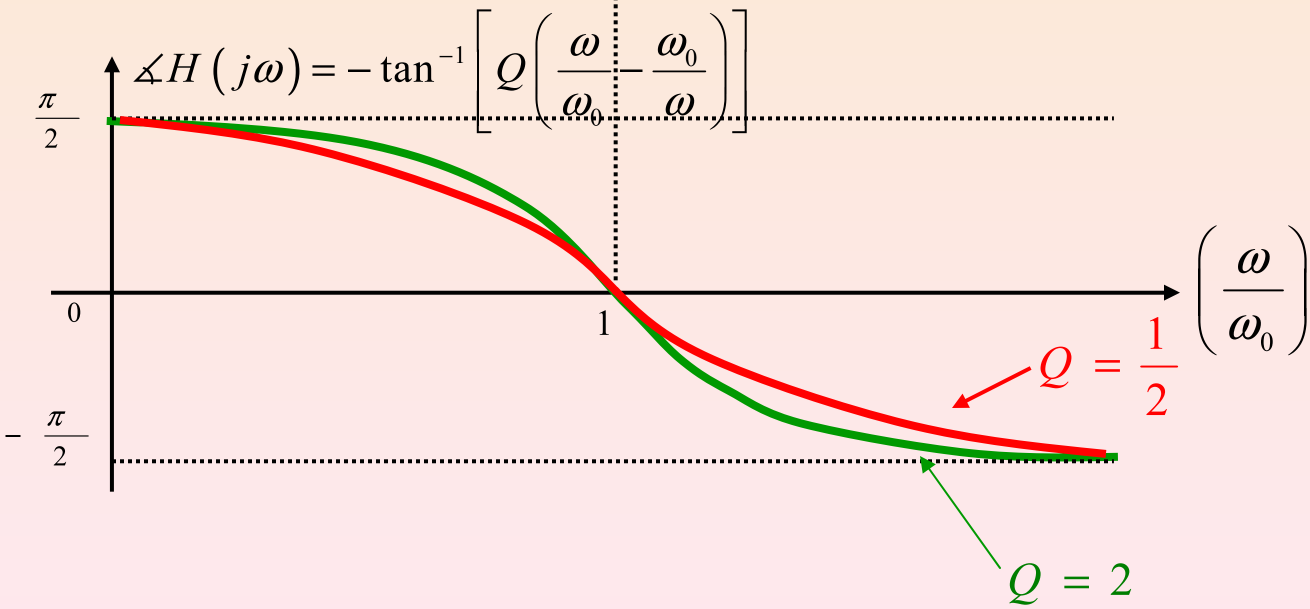


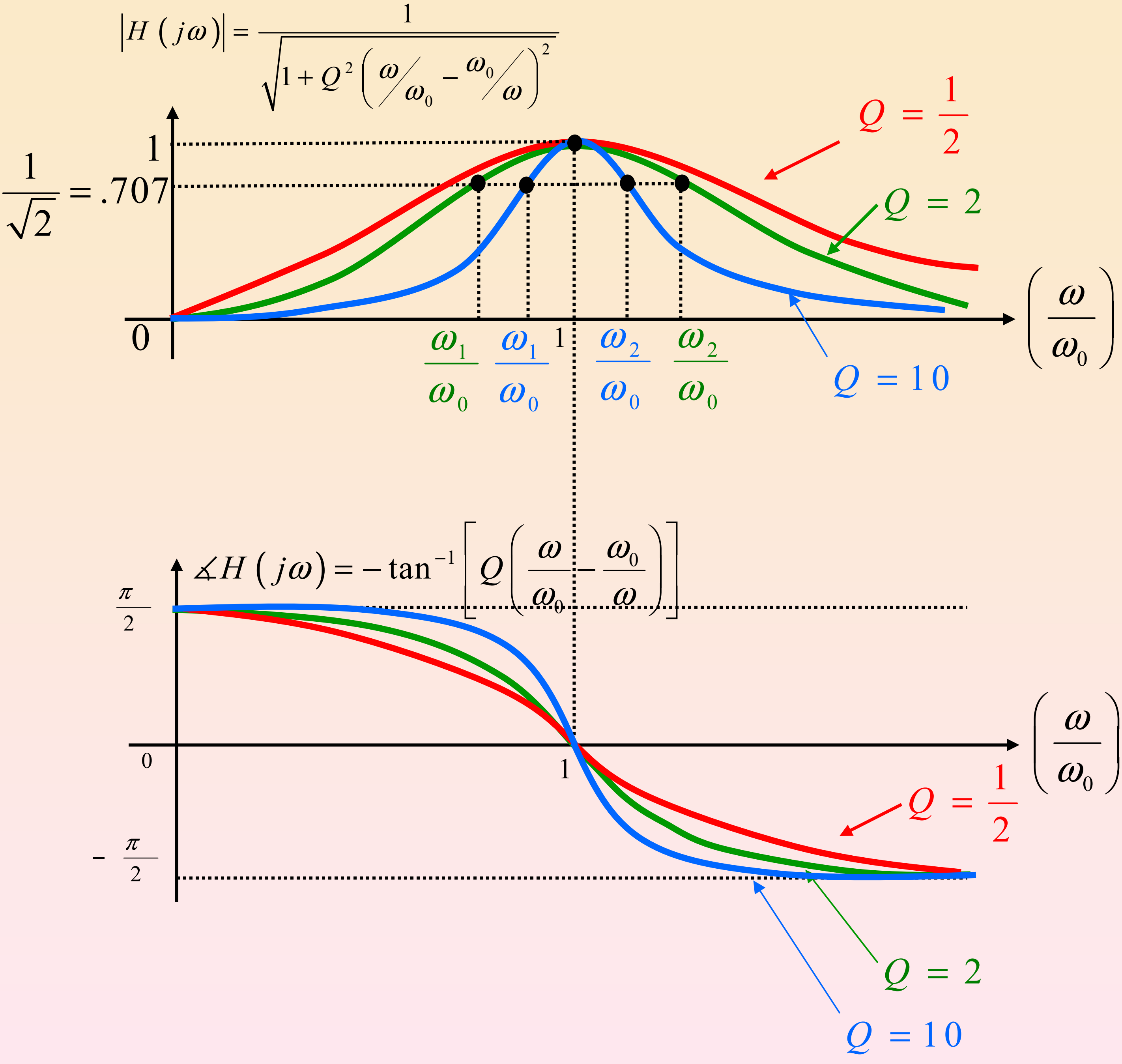
$$\measuredangle H(j\omega) = -\tan^{-1}\left[Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$



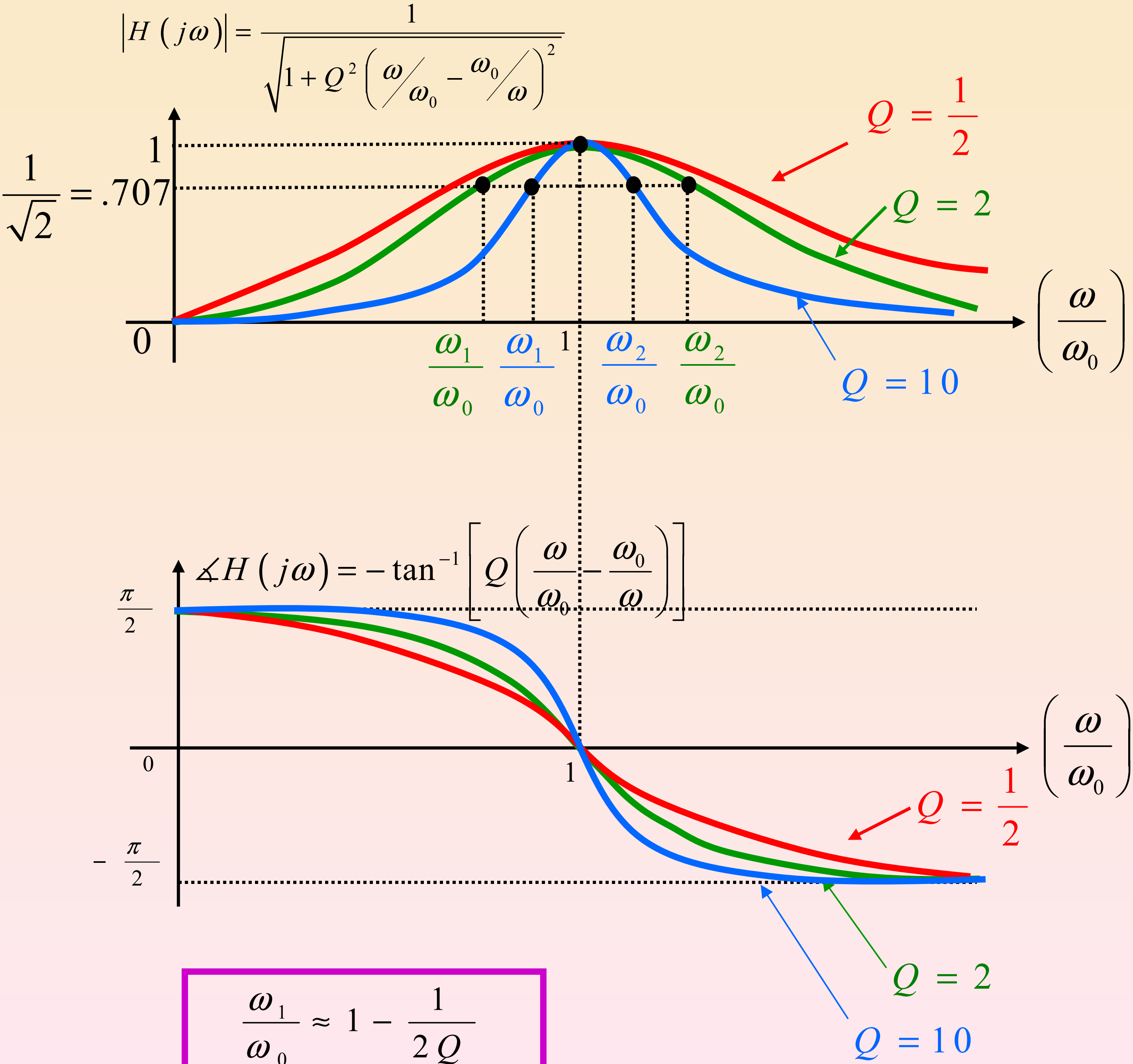
$$|H(j\omega)| = \frac{1}{\sqrt{1 + Q^2\left(\omega/\omega_0 - \omega_0/\omega\right)^2}}$$







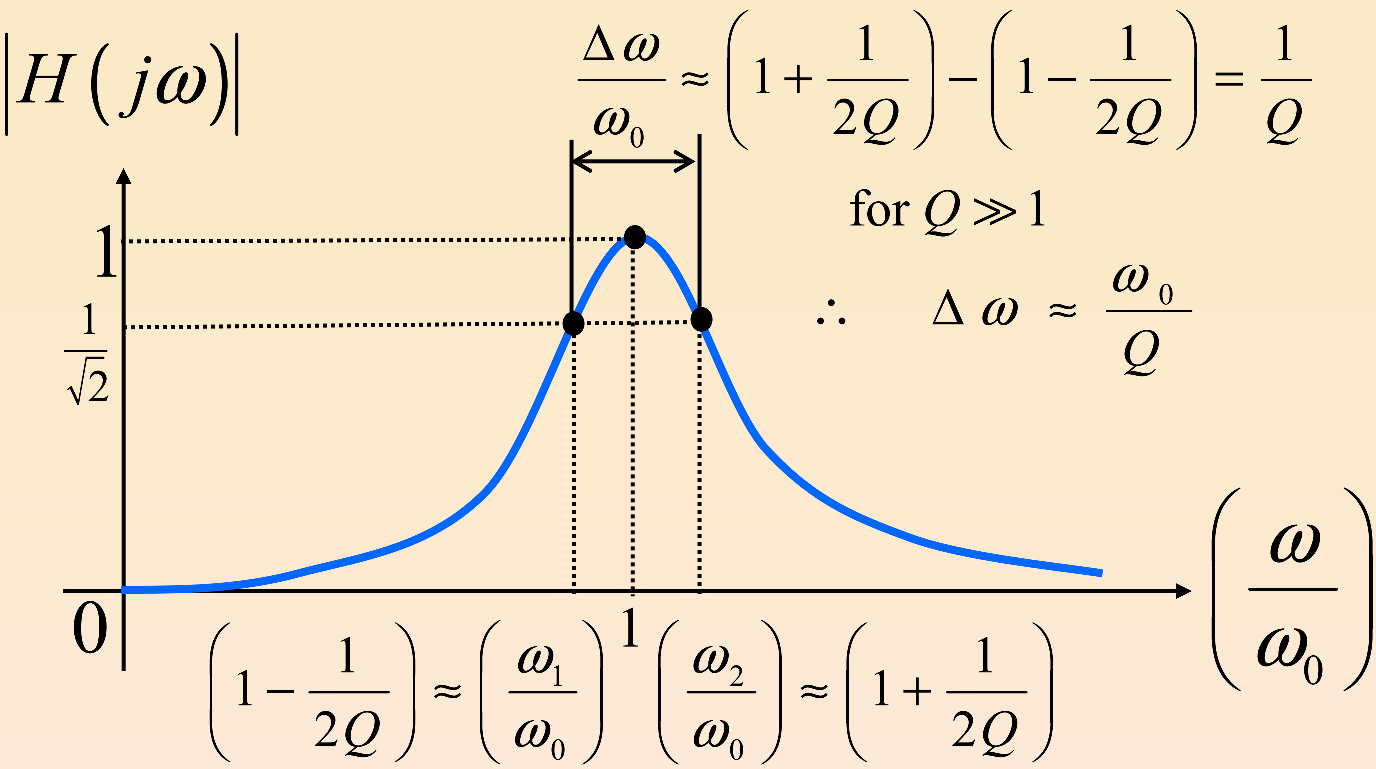
$|H(j\omega)| = \frac{1}{\sqrt{1+Q^2\left(\omega/\omega_0 - \omega_0/\omega\right)^2}}$
$\frac{1}{\sqrt{2}} = .707$
1
0
$Q = \frac{1}{2}$
$Q = 2$
$Q = 10$
$\frac{\omega_1}{\omega_0}$
$\frac{\omega_1}{\omega_0}$
1
$\frac{\omega_2}{\omega_0}$
$\frac{\omega_2}{\omega_0}$
$\left(\frac{\omega}{\omega_0}\right)$
$\measuredangle H(j\omega) = -\tan^{-1}\left[Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$
$\frac{\pi}{2}$
0
1
$-\frac{\pi}{2}$
$\left(\frac{\omega}{\omega_0}\right)$
$Q = \frac{1}{2}$
$Q = 2$
$Q = 10$



$$\frac{\omega_1}{\omega_0} \approx 1 - \frac{1}{2Q}$$

$$\frac{\omega_2}{\omega_0} \approx 1 + \frac{1}{2Q}$$

for $Q \gg 1$



$$|H(j\omega)|_{dB} \triangleq 20\log|H(j\omega)|$$

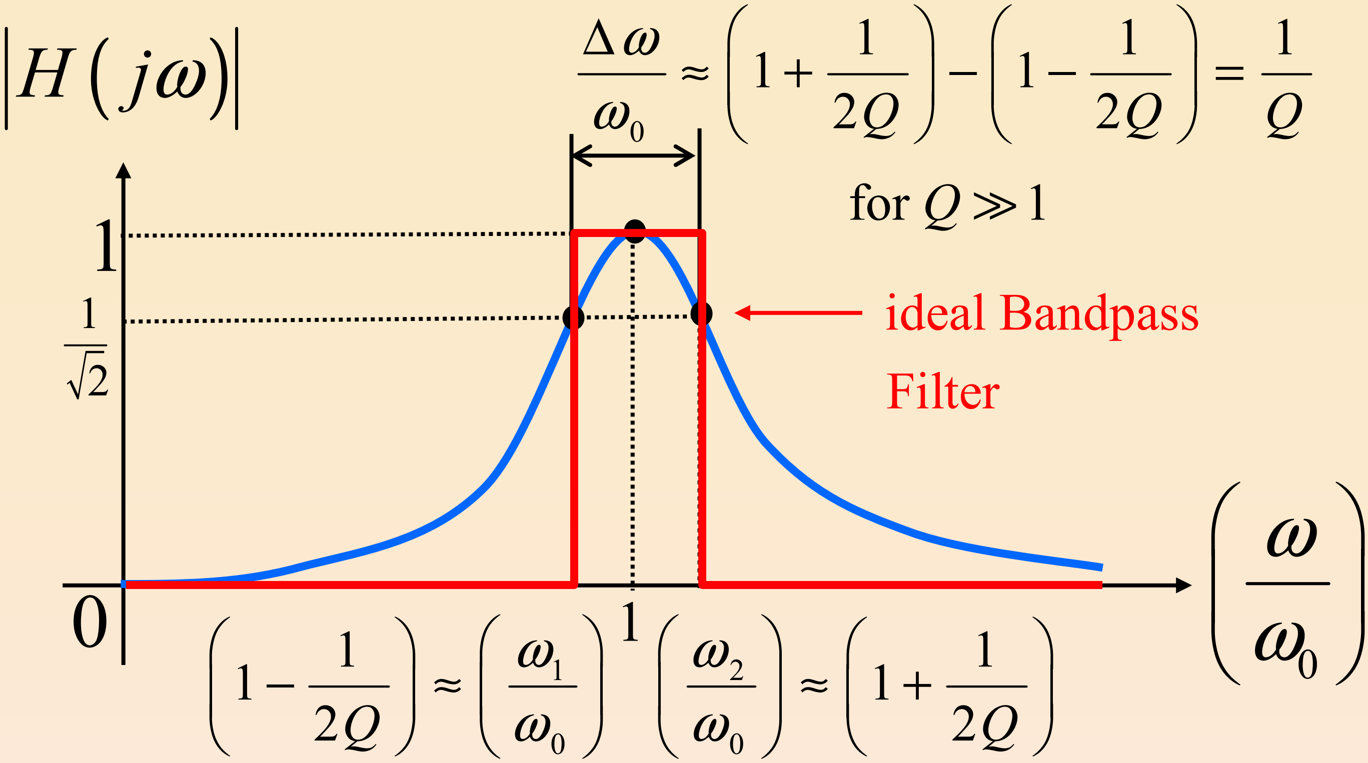
At $\frac{\omega_1}{\omega_0}$ and $\frac{\omega_2}{\omega_0}$, we have

$$20\log|H(j\omega)| \approx -3$$

$$\left.\begin{aligned} f_1 &\triangleq \frac{\omega_1}{2\pi} \\ f_2 &\triangleq \frac{\omega_2}{2\pi} \end{aligned}\right\} \quad \text{3dB frequencies}$$

$$f = f_2 - f_1 = \text{3dB Bandwidth}$$

$$\Delta\omega = \frac{\omega_0}{Q} = 2\pi\Delta f \qquad \therefore \Delta f = \frac{\omega_0}{2\pi Q} = \frac{\omega_0}{2\pi\left(\omega_0 / 2\alpha\right)} = \frac{\alpha}{\pi}$$



$|H(j\omega)|$
$\frac{\Delta\omega}{\omega_0} \approx \left(1+\frac{1}{2Q}\right)-\left(1-\frac{1}{2Q}\right)=\frac{1}{Q}$
for $Q \gg 1$
1
$\frac{1}{\sqrt{2}}$
ideal Bandpass
Filter
0
1
$\left(\frac{\omega}{\omega_0}\right)$
$\left(1-\frac{1}{2Q}\right) \approx \left(\frac{\omega_1}{\omega_0}\right)$
$\left(\frac{\omega_2}{\omega_0}\right) \approx \left(1+\frac{1}{2Q}\right)$