SOCCER Ball Circuit

Consider a soccer ball circuit where each edge is a resistor.

number of nodes = n = 60

number of branches = b = 90

number of meshes = m = 32

node-voltage analysis $\Rightarrow \quad n-1=59$

equations involving $\{e_1, e_2, \cdots, e_{59}\}$ node-voltage variables.

mesh-current analysis $\Rightarrow \quad m-1=31$

equations involving $\{\hat{i}_1, \hat{i}_2, \cdots, \hat{i}_{31}\}$ mesh-current variables.

A **SOCCER** ball has :

n = 60 vertices

b = 90 edges

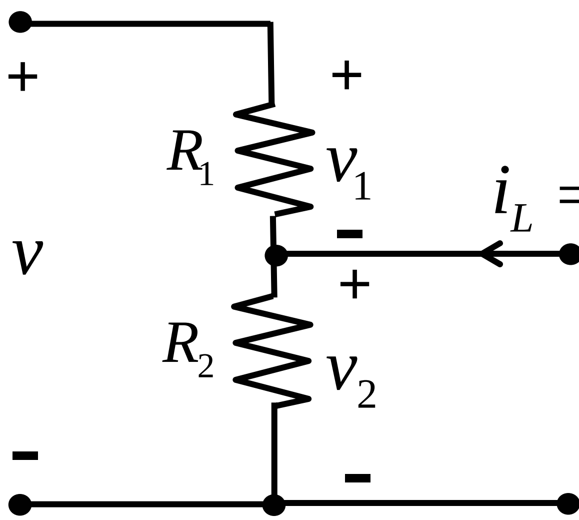
m = 32 faces

These numbers must satisfy Euler's formula

$$m + n = b + 2$$

***Check**: 32 + 60 = 90 + 2*

Voltage divider



$$i_L = 0$$

$$v_2 = \left(\frac{R_2}{R_1 + R_2} \right) v$$

$$v_2 = \left(\frac{R_2}{R_1 + R_2} \right) v$$

warning: This formula is valid only if

$i_L = 0$ (no loading!)

How to Construct Dual Circuits

Given a **planar** circuit N with n nodes and m-1 meshes, draw its digraph G. Label all branches, nodes, and meshes and let node $\textcircled{n}$ be the datum node. The dual circuit N' is constructed in 2 steps:

Step 1 draw the dual digraph G' of G.

(a) For each mesh m_j of G, draw a node **inside** mesh m_j and label it as node $\textcircled{j'}$, $j = 1, 2, \cdots, m-1$.

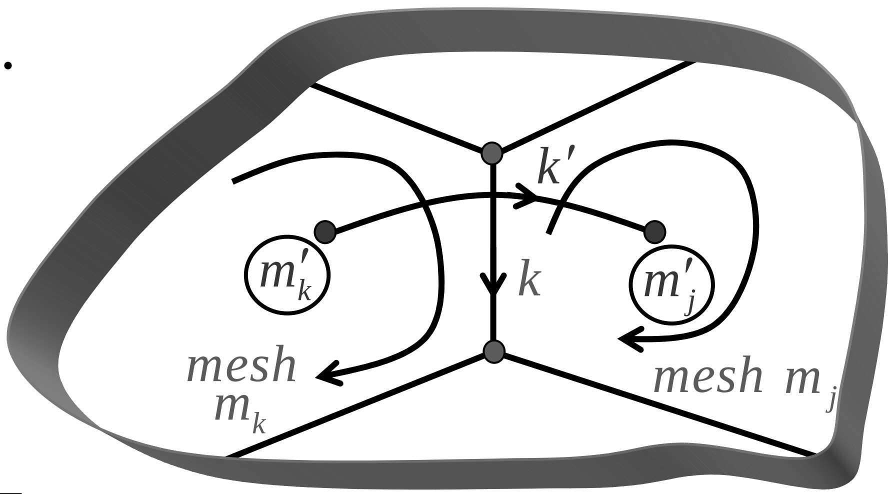
(b) Draw a node **outside** of G and label it as node $\textcircled{m'}$, and choose this ***exterior*** node $\textcircled{m'}$ as the datum node of G'.

(c) For each branch k of G belonging to 2 adjacent meshes m_j and m_k, draw a **dual** branch k' connecting node $\textcircled{m'_j}$ to node $\textcircled{m'_k}$.

(d) For each branch k of G belonging to only 1 mesh m_j, draw a **dual** branch k' from node $\textcircled{m_j}$ to the ***exterior*** node $\textcircled{m'}$.

(e) Draw an **arrowhead** for each branch k' of G' as follow:

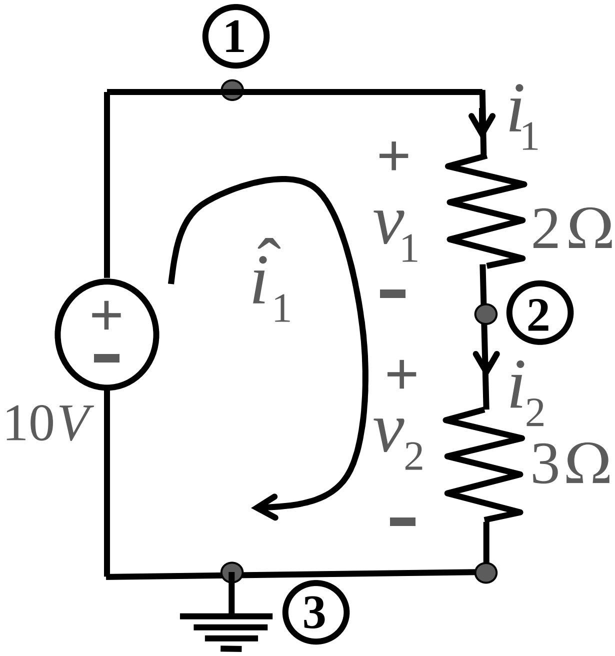
The **arrowhead** of each branch k' connected to nodes (m'_j) and (m'_k) is drawn **toward** node (m'_j) if the direction of branch k of G is in a **counterclockwise** direction relative to node (m'_j).



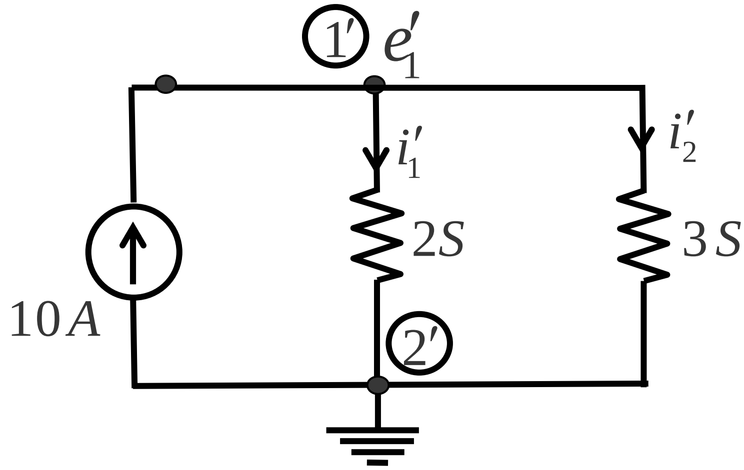
Step 2 For each branch k' in the dual digraph G', draw a circuit element which is ***dual*** of the corresponding circuit element from N.

Examples.

1. If element k is a 5 Ω linear resistor in N, then its dual is 1/5 Ω (or 5 Siemen) linear resistor in N'.
2. If element k is a 2 V voltage source in N, then its dual is a 2 A current source in N'.
3. If element k is a 5 A current source in N, then its dual is a 5 V voltage source in N'.



Circuit N



Circuit N'

Mesh-current Equation

$$5\ \hat{i}_1 = 10$$

$\Downarrow$

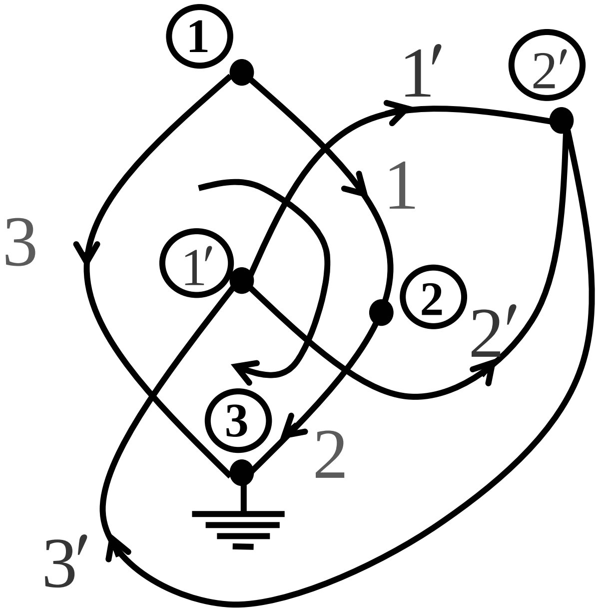
Digraph G

Node-voltage Equation

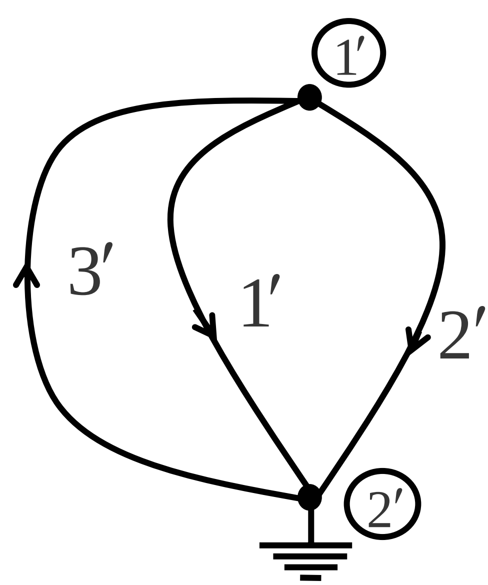
$$5\ e_1' = 10$$

$\Uparrow$

Digraph G'



$\Rightarrow$



Theorem.

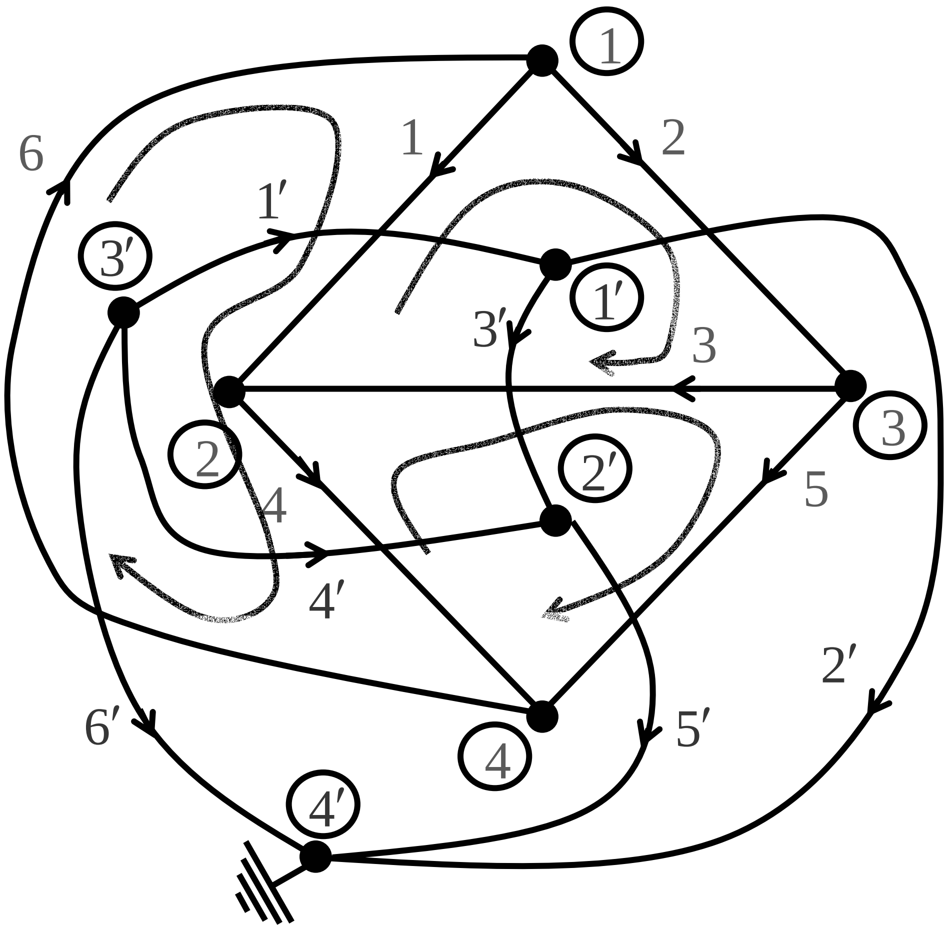
The **mesh-current equations** $\mathbf{Z}_m \hat{\mathbf{i}} = \mathbf{v}_s$ for a planar circuit N constitute an **independent** system of equations whose solution can be used to find ***all*** branch voltages and currents of N trivially via Ohm's law.

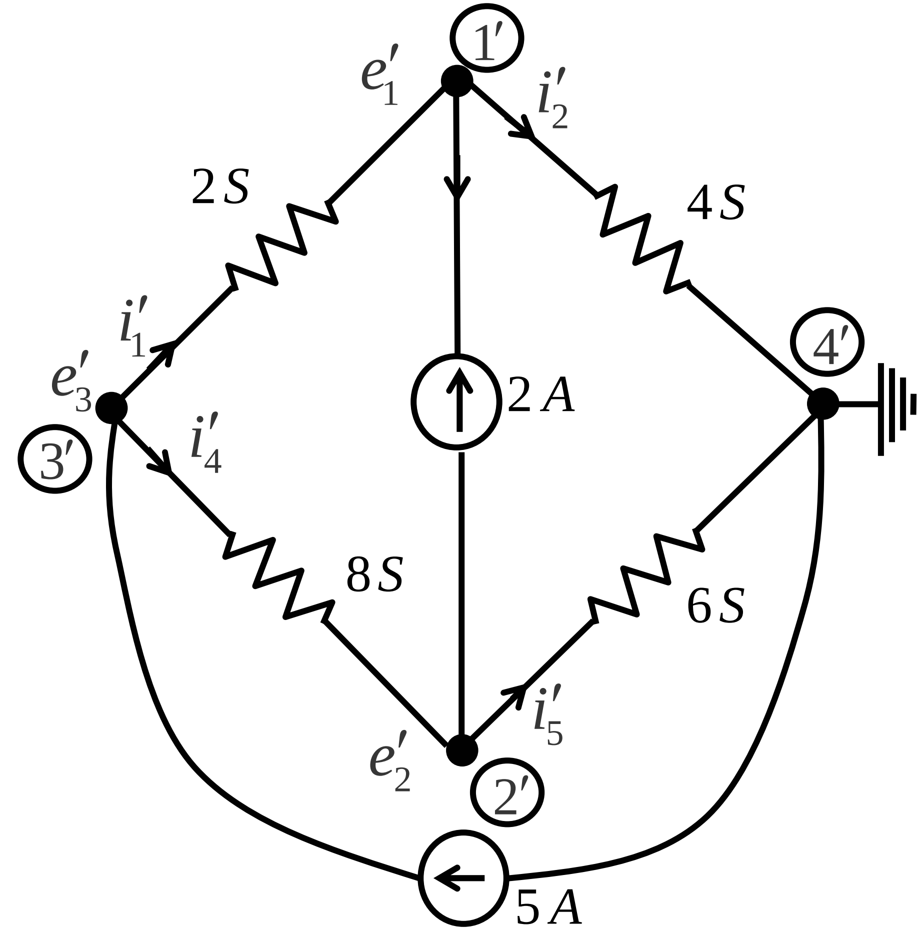
Proof.

For each planar circuit N, we can always construct its **dual** circuit N'. By the duality principle, the **node-voltage equations** $\mathbf{Y}_n \mathbf{e}' = \mathbf{i}'_s$ of N' are identical to the **mesh-current equations** of N, except for a trivial change of dual symbols

mesh-current $\hat{i}_j$ $\rightarrow$ node-voltage e'_j

But we have already proved the **node-voltage equations** constitute an **independent** system of equations whose solution can be used to find all branch voltages and currents of N' trivially via Ohm's law. ■





Mesh-current Equations

$$\begin{bmatrix} 6 & 0 & -2 \\ 0 & 14 & -8 \\ -2 & -8 & 10 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \\ \hat{i}_2 \\ \hat{i}_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix}$$

Node-voltage Equations

$$\begin{bmatrix} 6 & 0 & -2 \\ 0 & 14 & -8 \\ -2 & -8 & 10 \end{bmatrix} \begin{bmatrix} e_1' \\ e_2' \\ e_3' \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix}$$