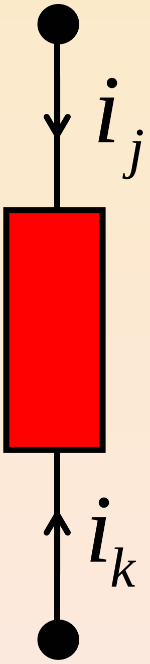
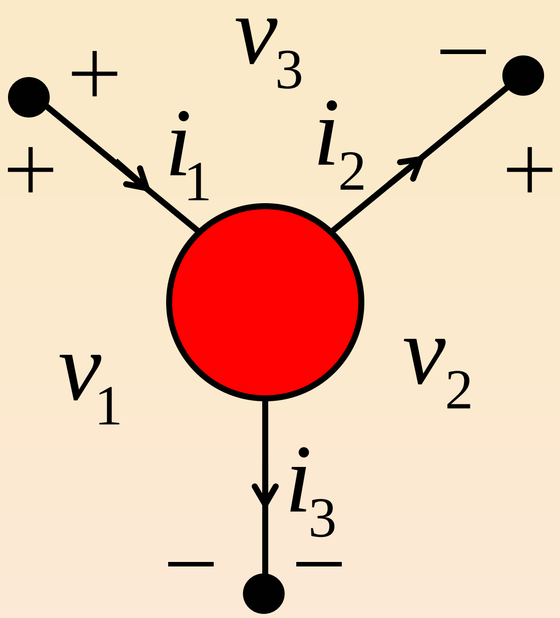
Independent Device Currents



Since **KCL** $\Rightarrow$

$$-i_j - i_k = 0$$

Only one independent current can be defined for each **2 - terminal device**.



Since **KCL** $\Rightarrow$

$$-i_1 + i_2 + i_3 = 0$$

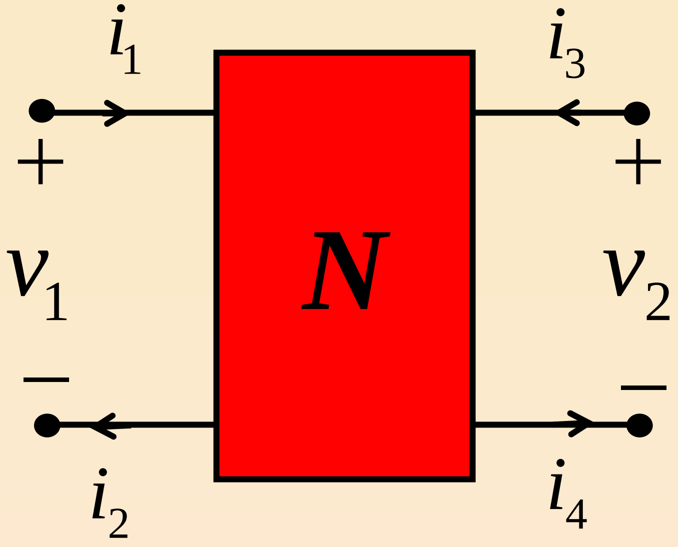
Only 2 independent currents can be defined for a 3 - **terminal device**.

Since **KVL** $\Rightarrow v_1 - v_2 - v_3 = 0$

Only 2 independent voltages can be defined for a 3 - **terminal device**.

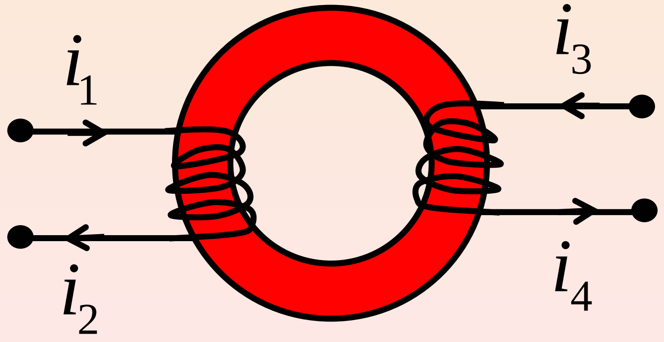
n-port device

There are electrical devices having an **even** number of terminals and constructed in such a way that the terminals can be grouped into pairs, called **ports**, such that the current entering one terminal of each pair is always equal to the current leaving the other terminal. A device with “*n*” pairs of such current-following terminals is called an ***n*-port**.



A 4-terminal device

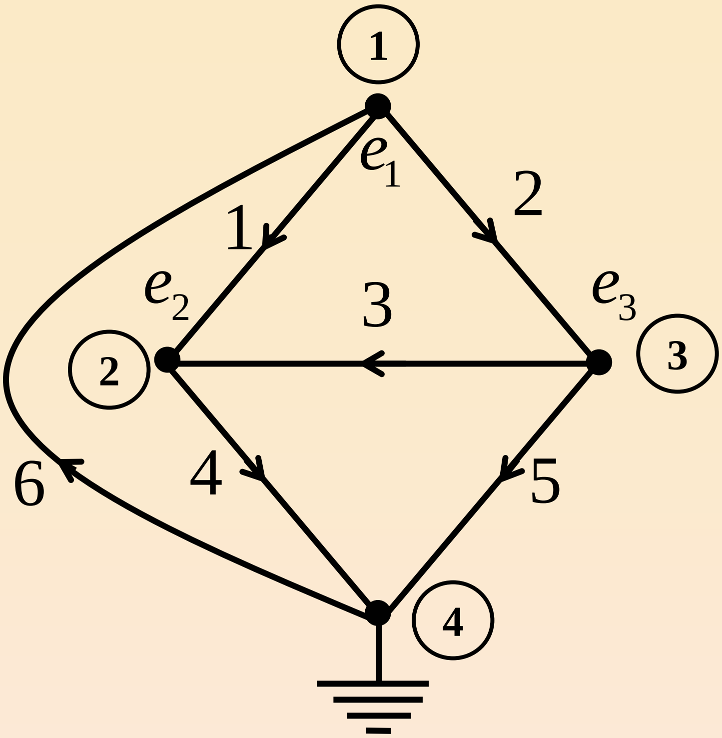
Example of a 2-port



In this example, the 2 separate physical windings guarantee that :

$$i_1 = i_2 \text{ and } i_3 = i_4$$

In this case, only 1 current and 1 voltage needs to be defined for each pair of terminals, henceforth called ports.



KCL Equations:

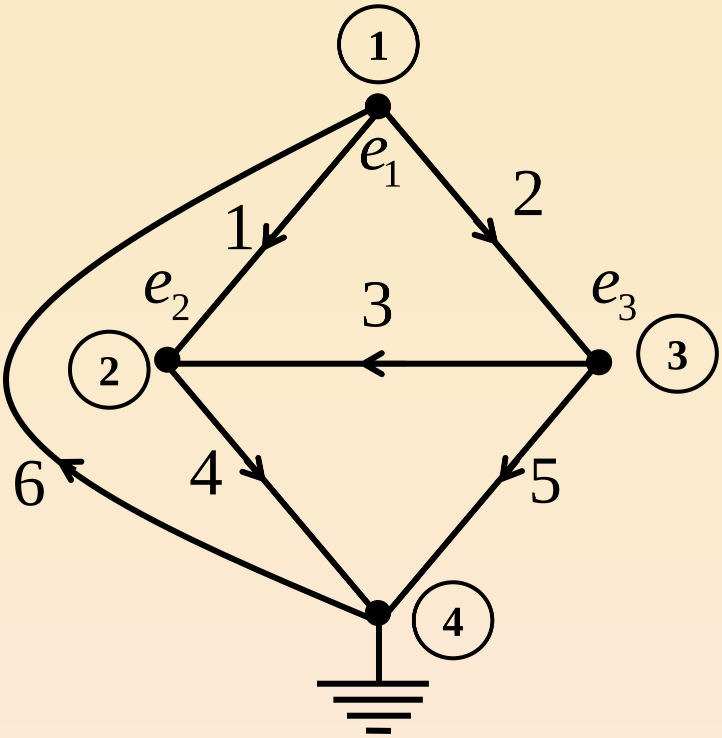
① $i_1 + i_2 - i_6 = 0$

② $-i_1 - i_3 + i_4 = 0$

③ $-i_2 + i_3 + i_5 = 0$

Branch no.

$$\mathbf{A}\,\mathbf{i} = \mathbf{0} \quad \Rightarrow \quad \begin{array}{c} \text{node no.} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \end{array}\right] \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



KCL Equations:

① $i_1 + i_2 - i_6 = 0$

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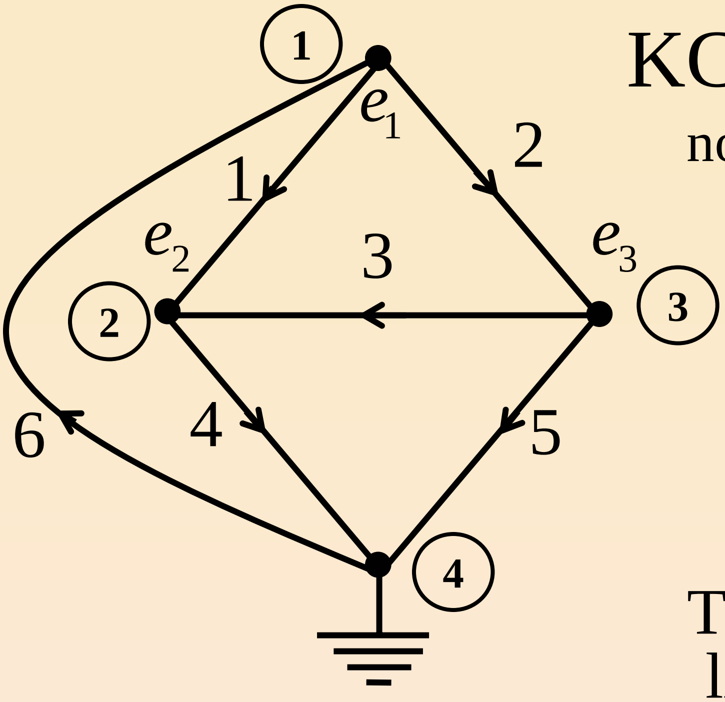
$$\mathbf{A\,i} = \mathbf{0} \quad \Rightarrow \quad \begin{array}{cc} \begin{array}{c}\text{node}\\ \text{no.}\end{array} & \begin{array}{cccccc} \multicolumn{6}{c}{\text{Branch no.}} \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{array} & \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{A}} \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A is called the **reduced Incidence Matrix** of the diagraph G relative to datum node ④.



KCL Node Equations:

node No.

① $i_1 + i_2 - i_6 = 0$

② $-i_1 - i_3 + i_4 = 0$

③ $-i_2 + i_3 + i_5 = 0$

④ $-i_4 - i_5 + i_6 = 0$

These 4 equations are linearly-dependent.

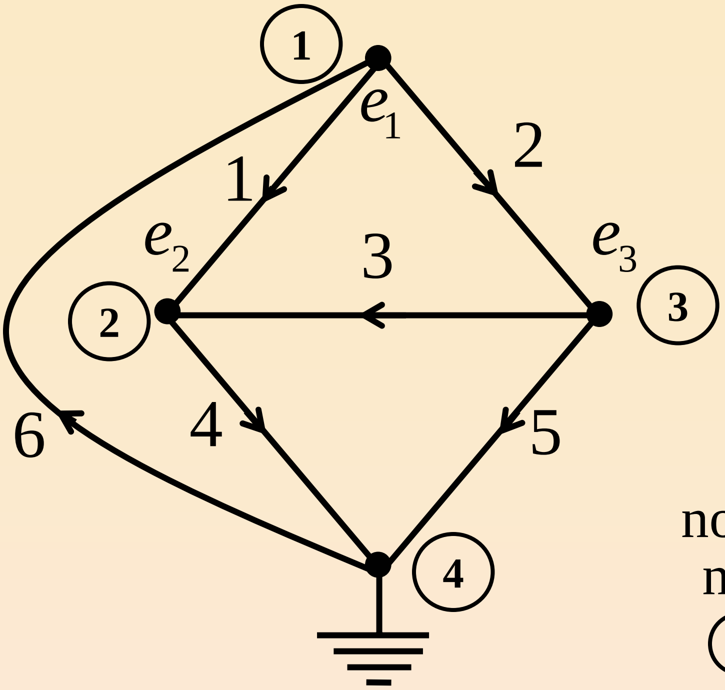
Matrix Formulation:

Branch no.

$$\begin{array}{cc} \text{node no.} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{array} & \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix}}_{\mathbf{A}_a} \end{array} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}}_{\mathbf{i}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{0}}$$

INCIDENCE MATRIX →

$$a_{jk} = \begin{cases} 1 & \text{if branch } k \text{ leaves node } \textcircled{j} \\ -1 & \text{if branch } k \text{ enters node } \textcircled{j} \\ 0 & \text{if branch } k \text{ is not connected to node } \textcircled{j} \end{cases}$$



KCL Equations:

$$\textcircled{1}\quad i_1 + i_2 - i_6 = 0$$
$$\textcircled{2}\quad -i_1 - i_3 + i_4 = 0$$
$$\textcircled{3}\quad -i_2 + i_3 + i_5 = 0$$

$$\mathbf{A\,i} = \mathbf{0} \quad \Rightarrow \quad \begin{array}{c|cccccc} \text{node no.} & \multicolumn{6}{c}{\text{Branch no.}} \\ \text{no.} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline \textcircled{1} & 1 & 1 & 0 & 0 & 0 & -1 \\ \textcircled{2} & -1 & 0 & -1 & 1 & 0 & 0 \\ \textcircled{3} & 0 & -1 & 1 & 0 & 1 & 0 \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

KVL Equations:

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}}_{\mathbf{v}} = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}}_{\mathbf{A}^T} \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}}_{\mathbf{e}} \quad \Leftarrow \quad \begin{cases} v_1 = e_1 - e_2 \\ v_2 = e_1 - e_3 \\ v_3 = e_3 - e_2 \\ v_4 = e_2 \\ v_5 = e_3 \\ v_6 = -e_1 \end{cases}$$

$$\text{KVL:} \quad \boxed{\mathbf{v} = \mathbf{A}^T\,\mathbf{e}}$$

Since v_j is present only in the jth equation, these k equations are **linearly - independent**.

Theorem

$$\boxed{\mathbf{A}\,\mathbf{i} = \mathbf{0}}$$

gives the **maximum possible** number of **linearly-independent KCL equations** for a connected circuit.

Reduced Incidence Matrix

Let G be a connected **digraph** with “*n*” nodes and “*b*” branches. Let $\mathbf{A}_a$ be the **Incidence Matrix** of G. The (n-1) x b matrix **A** obtained by deleting any one row of $\mathbf{A}_a$ is called a **Reduced-Incidence Matrix** of G.

Observation : The 4 KCL node equations are ***not*** linearly independent.

Adding the left side of the 4 KCL node equations, we obtain:

$$(\underbrace{i_1+i_2-i_6}_{\textcircled{1}})+(\underbrace{-i_1-i_3+i_4}_{\textcircled{2}})+(\underbrace{-i_2+i_3+i_5}_{\textcircled{3}})$$

$$+(\underbrace{-i_4-i_5+i_6}_{\textcircled{4}})\equiv 0$$

This means we can derive any one of these 4 equations from the other 3.

Example: Derive KCL equations at node ④:

Adding the first 3 node equations gives:

$$(\underbrace{i_1+i_2-i_6}_{\textcircled{1}})+(\underbrace{-i_1-i_3+i_4}_{\textcircled{2}})+(\underbrace{-i_2+i_3+i_5}_{\textcircled{3}})$$

$$=\underbrace{i_4+i_5-i_6}_{\textcircled{4}}$$

Reduced Incidence Matrix **A**

Let G be a connected **digraph** with "n" nodes and "b" branches, the **reduced incidence matrix** **A** relative to **datum node** ⓝ is an $(n$-1) x b matrix whose coefficients a_{jk} are obtained from the (n-1) KCL equations written at the n-1 non-datum nodes:

$$a_{jk} = \begin{cases} 1 & \text{if branch } k \text{ leaves node } ⓙ \\ -1 & \text{if branch } k \text{ enters node } ⓙ \\ 0 & \text{if branch } k \text{ is not connected to node } ⓙ \end{cases}$$

By applying the various versions of KCL, we can write many different KCL equations for each circuit. However, these equations are usually **not** linearly independent in the sense that each equation can be derived by a linear combination of the others.

How can we write a maximum set of **linearly-independent** KCL equations?

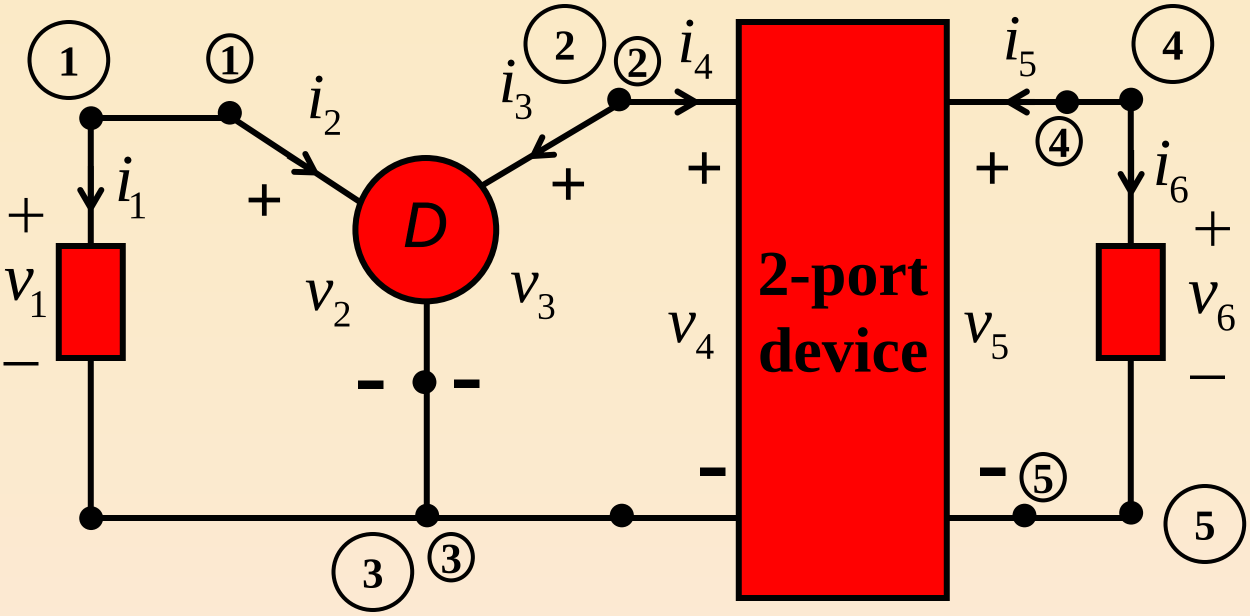
Simplest Method to write linearly-Independent KCL Equations.

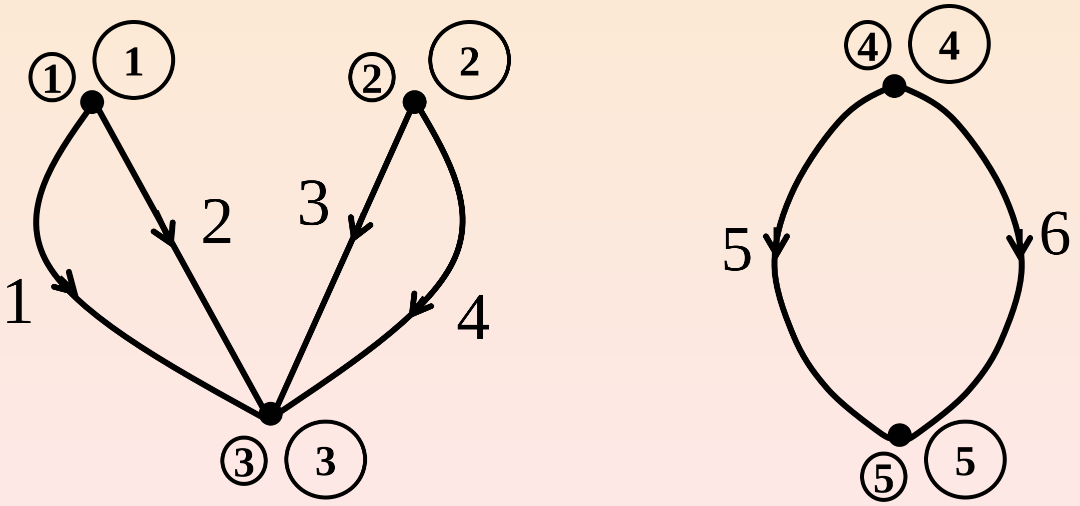
Given a connected circuit with "*n*" nodes, choose an arbitrary node as **datum**. Write a KCL equation at each of the remaining (*n*-1) nodes.

Relationship between **A** and $\mathbf{A}_a$

Let $\mathbf{A}_a$ be the n x b **Incidence matrix** of a connected digraph G with "n" nodes and "b" branches.

By deleting any row corresponding to node ⓜ from $\mathbf{A}_a$, we obtain the **reduced incidence matrix** **A** of G relative to the datum node ⓜ.





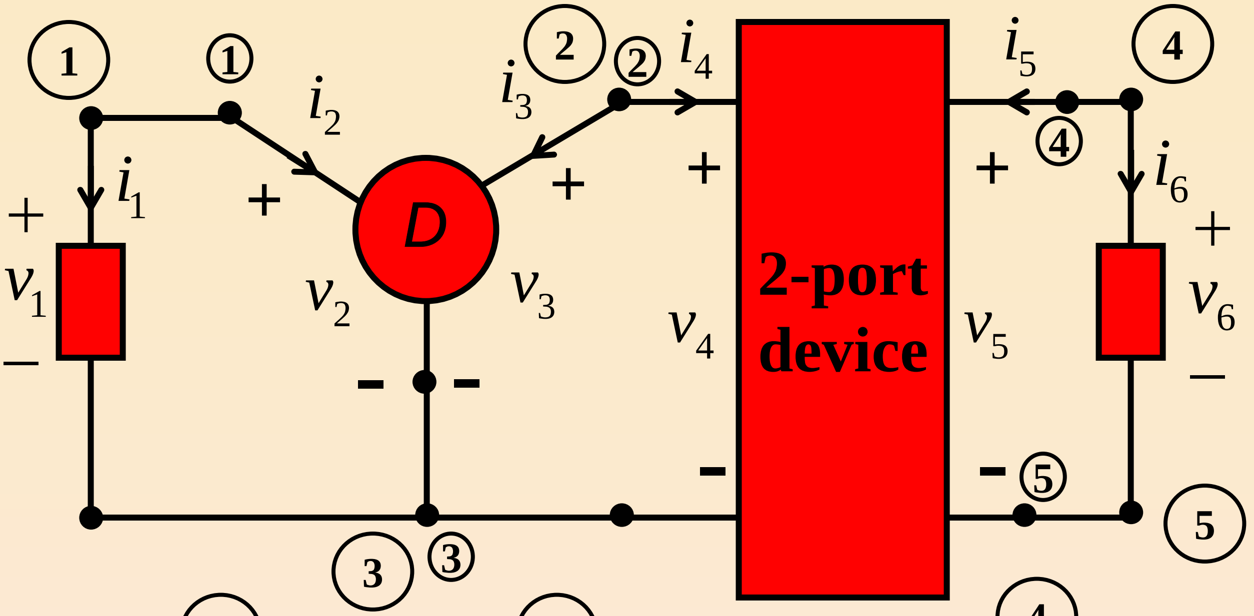
disconnected diagraph

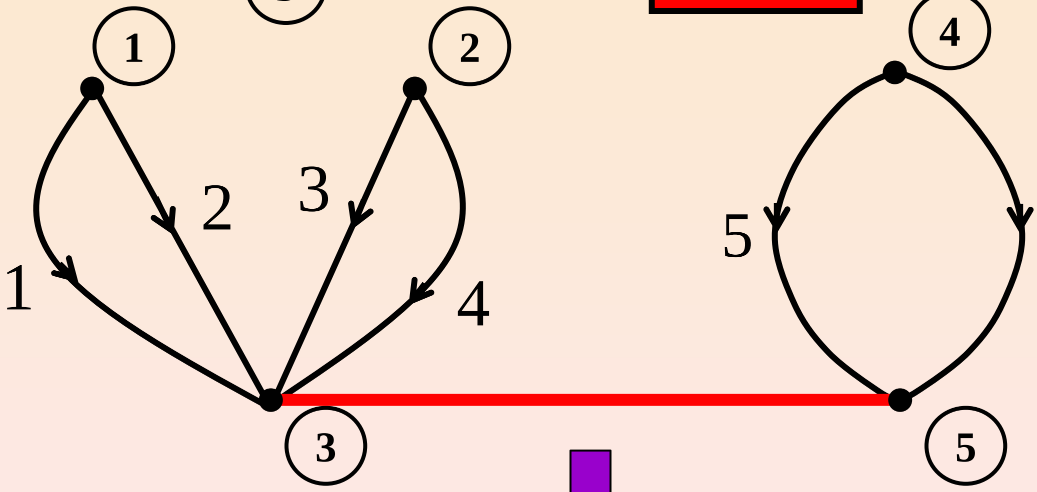
KCL at ② : $i_3 + i_4 = 0$

KCL at ④ : $i_5 + i_6 = 0$

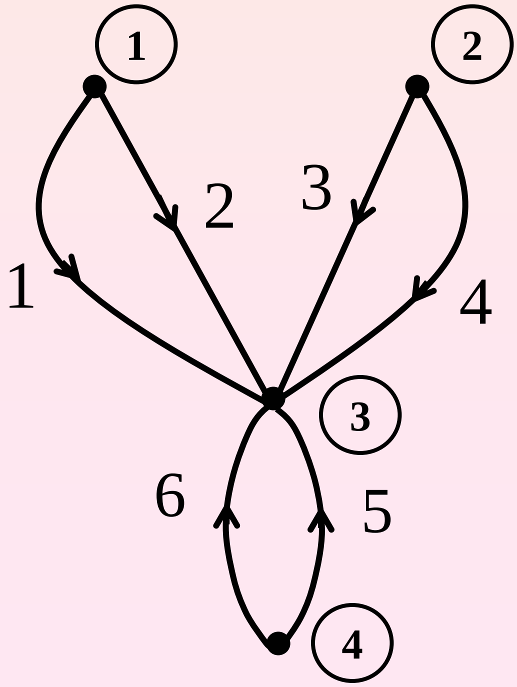
KVL around ②–③–② : $v_4 - v_3 = 0$

KVL around ④–⑤–④ : $v_6 - v_5 = 0$

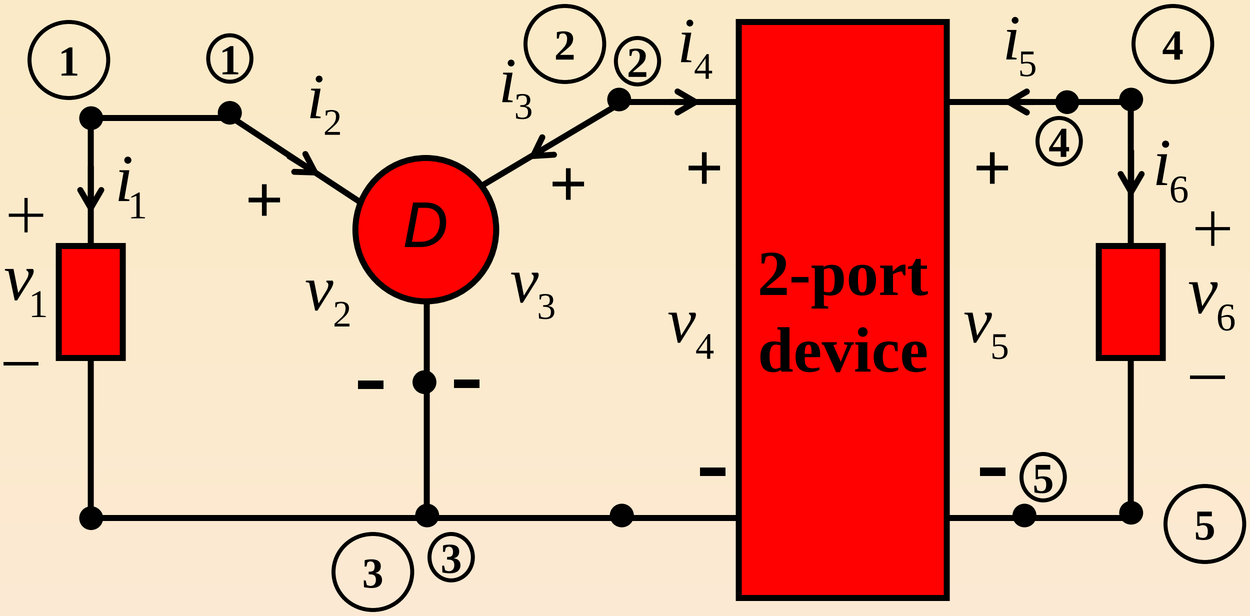


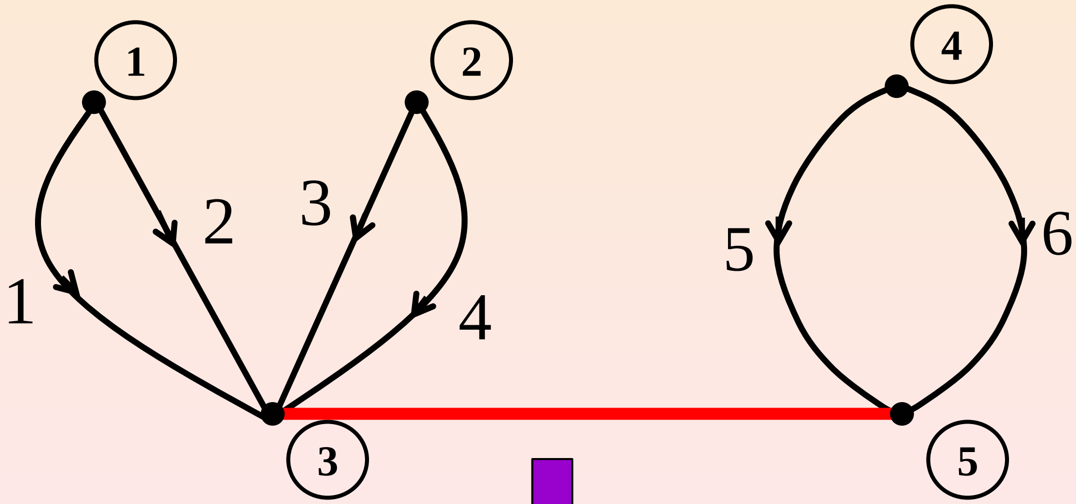


HINGED DIAGRAPH

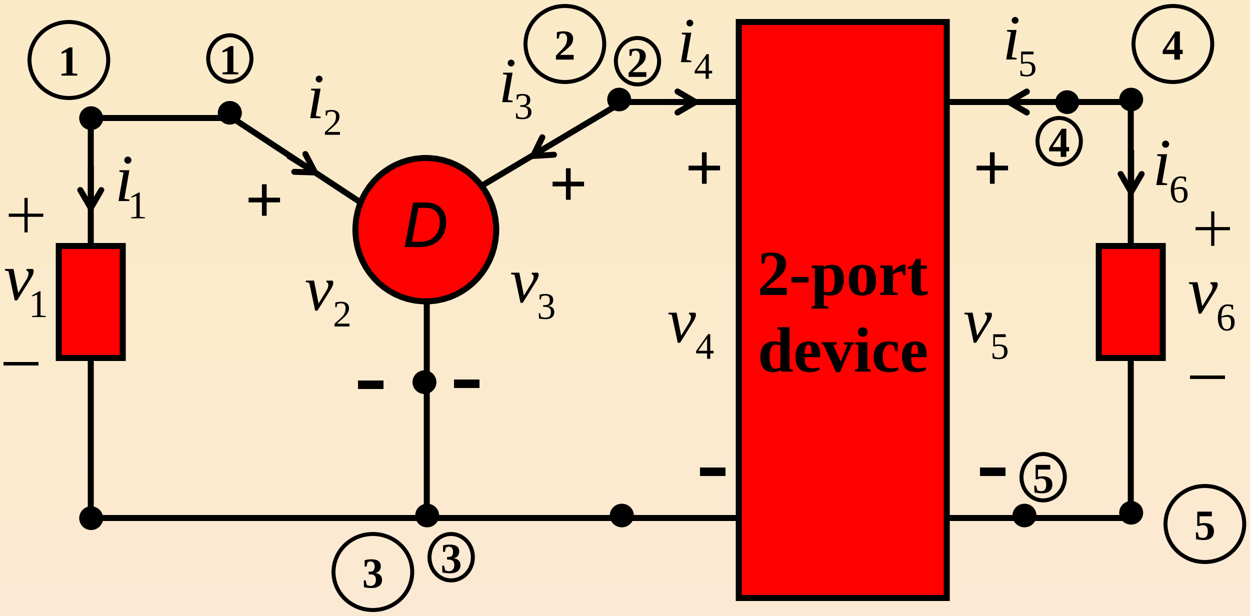


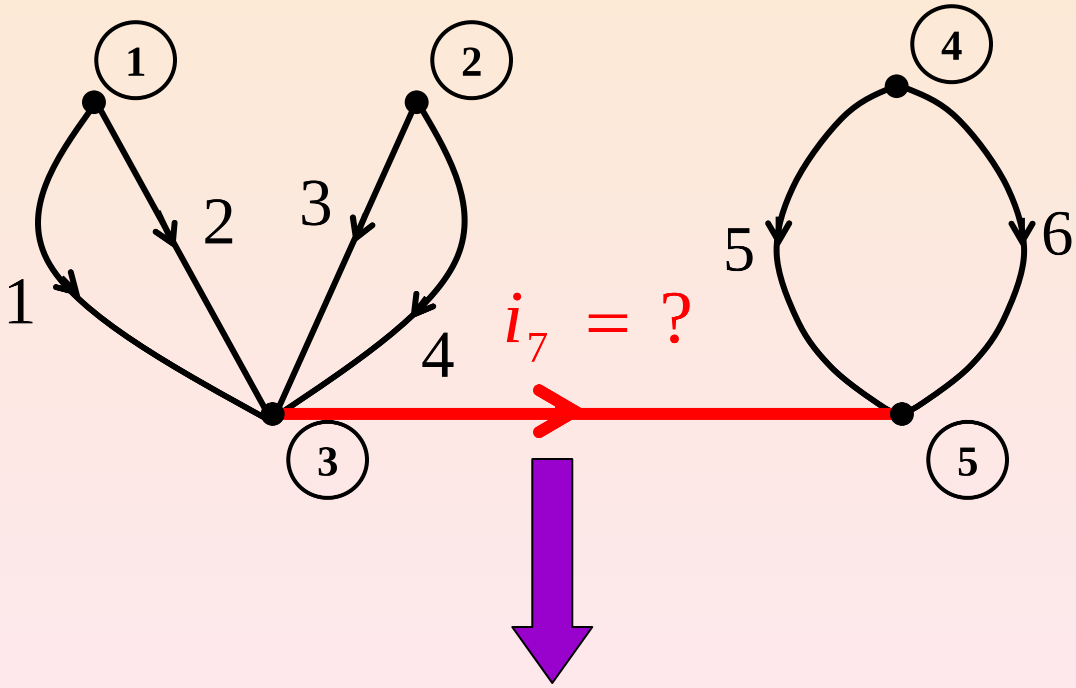
Since nodes ③ and ⑤ are now the same node, they can be combined into one node, and the redrawn diagraph is called a **hinged** graph.



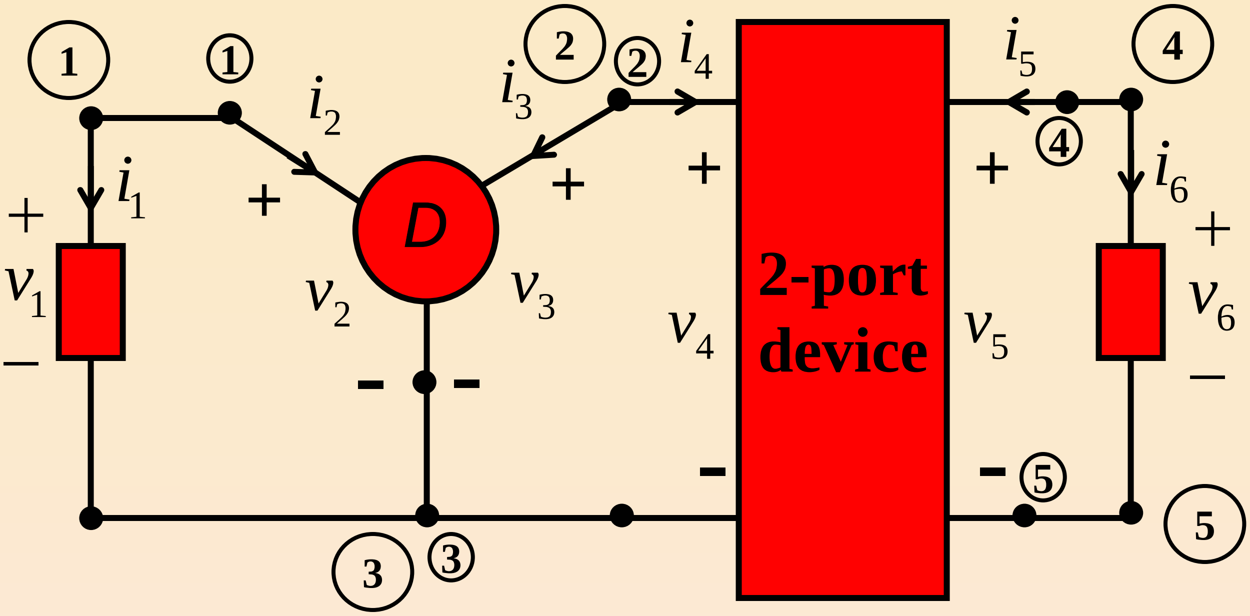


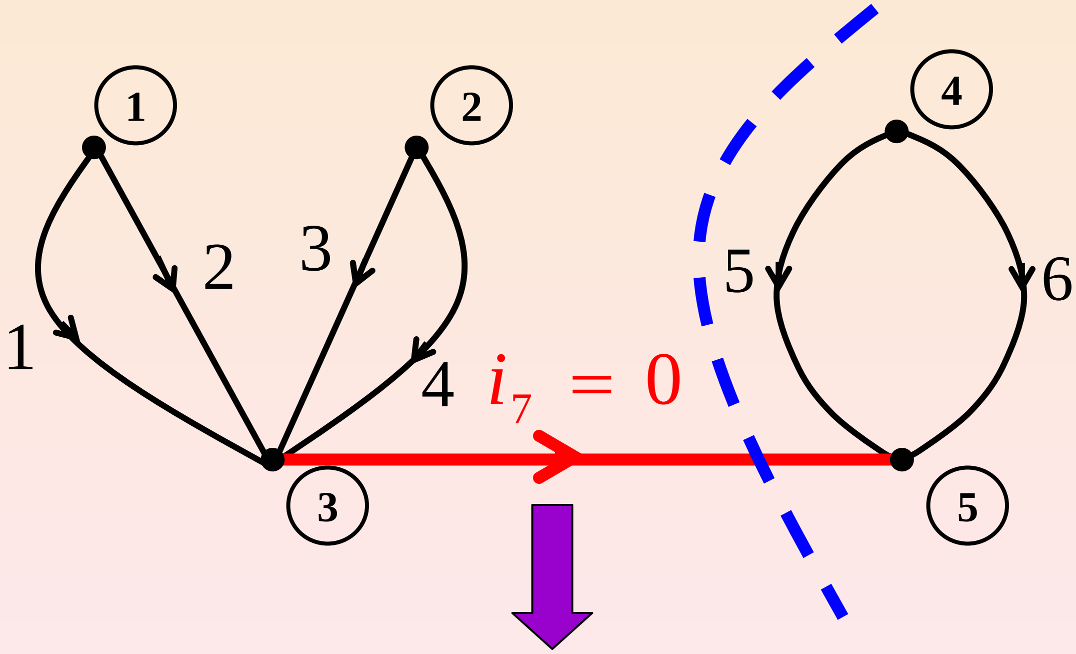
Adding a wire connecting one node from each separate component does **not** change KVL or KCL equations.





Adding a wire connecting one node from each separate component does **not** change KVL or KCL equations.

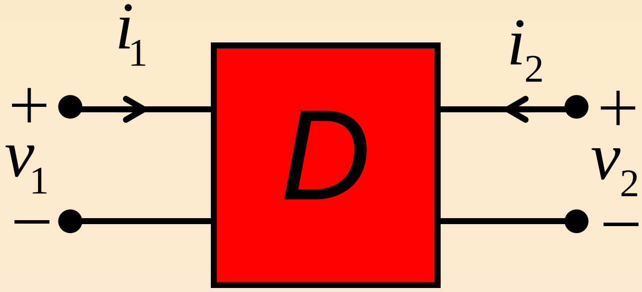




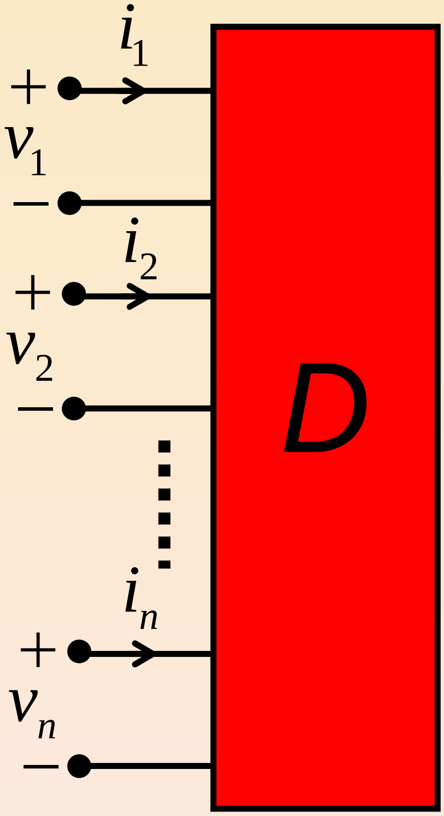
Adding a wire connecting one node from each separate component does **not** change KVL or KCL equations.

$\{7\}$ is a **cut set** $\Rightarrow$ $i_7 = 0$

Associated Reference Convention :

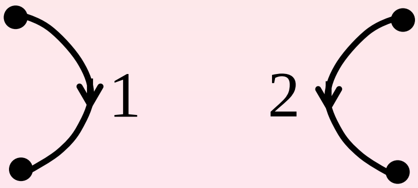


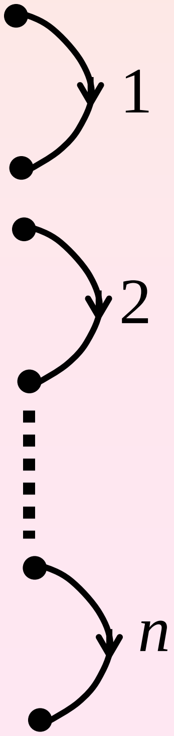
2-port Device

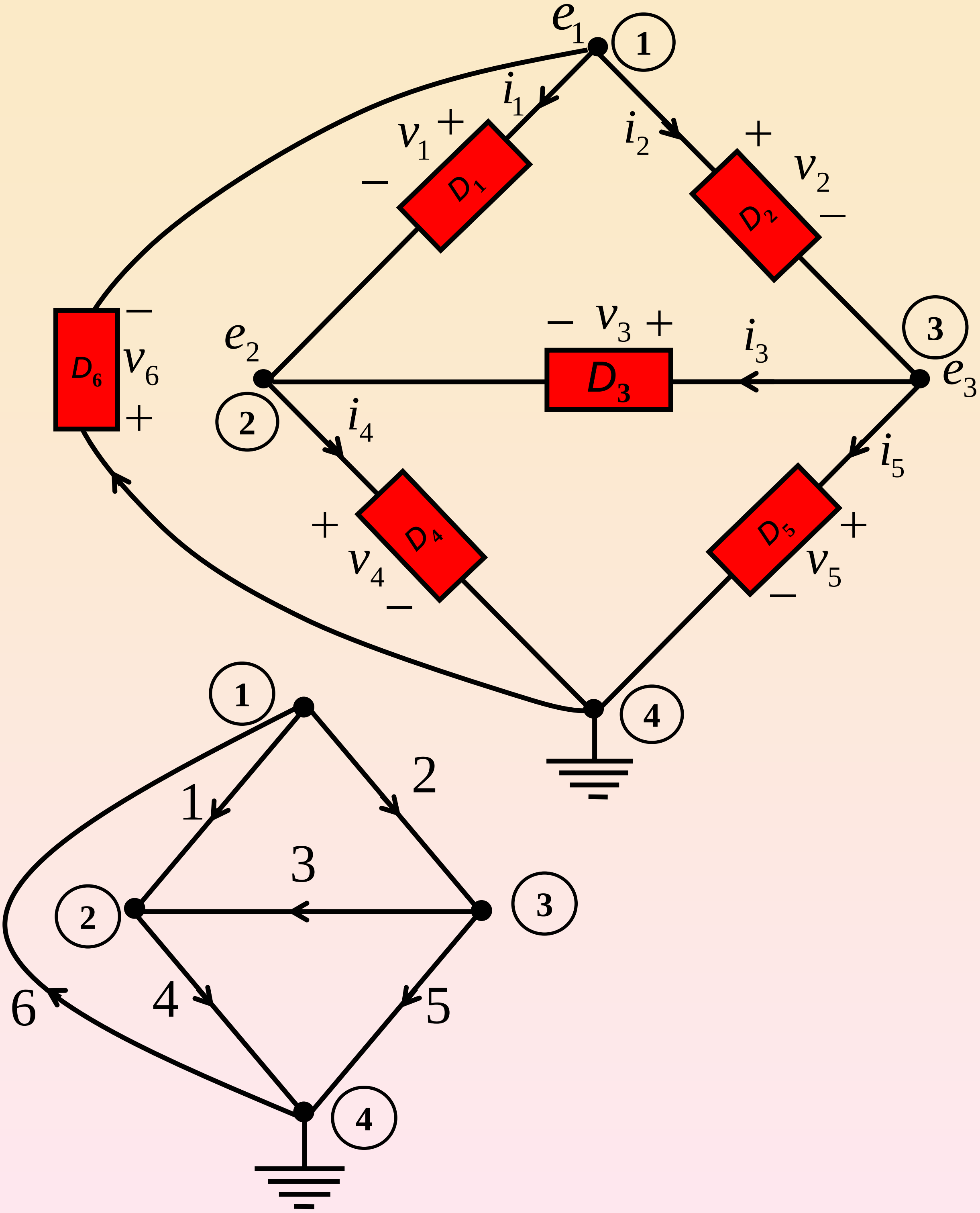


n-port Device

Device Graph







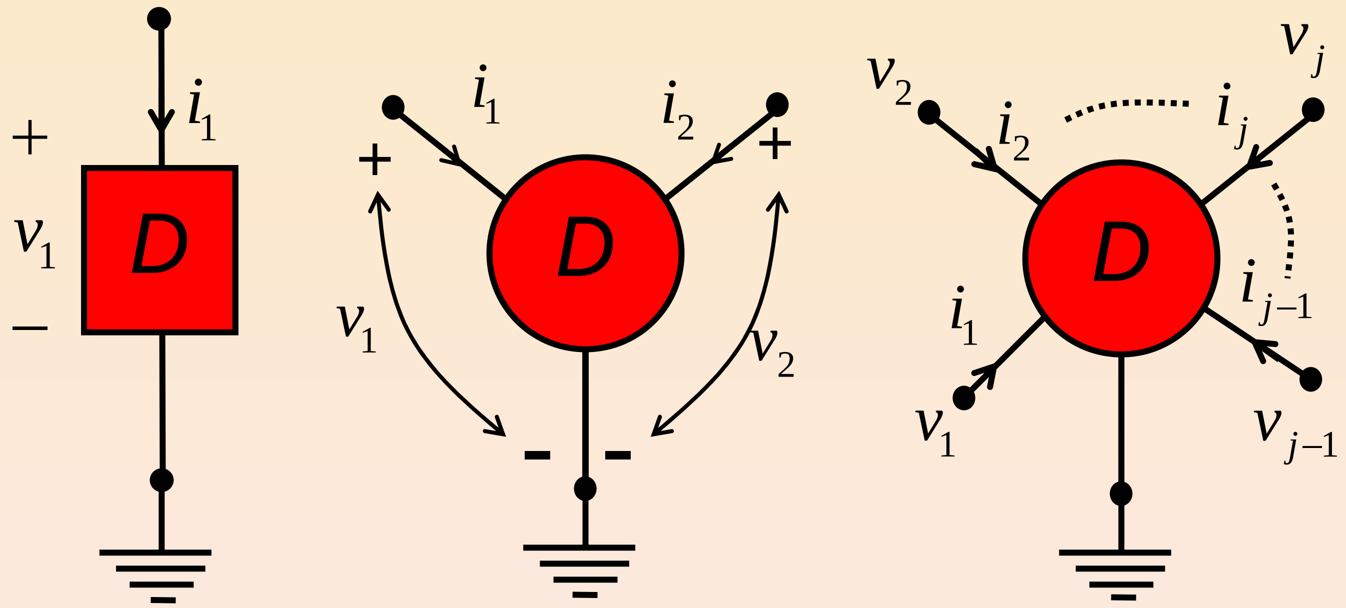
KCL at ① : $i_1 + i_2 - i_6 = 0$

KVL around ①–③–④–②–① :

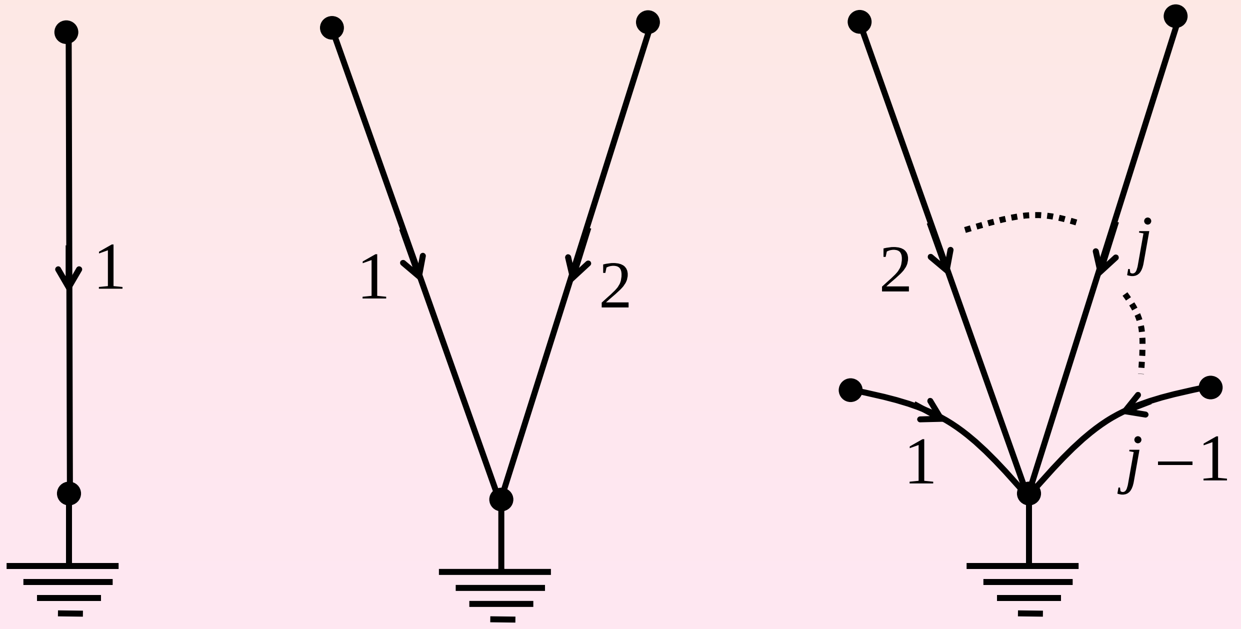
$$v_2 + v_5 - v_4 - v_1 = 0$$

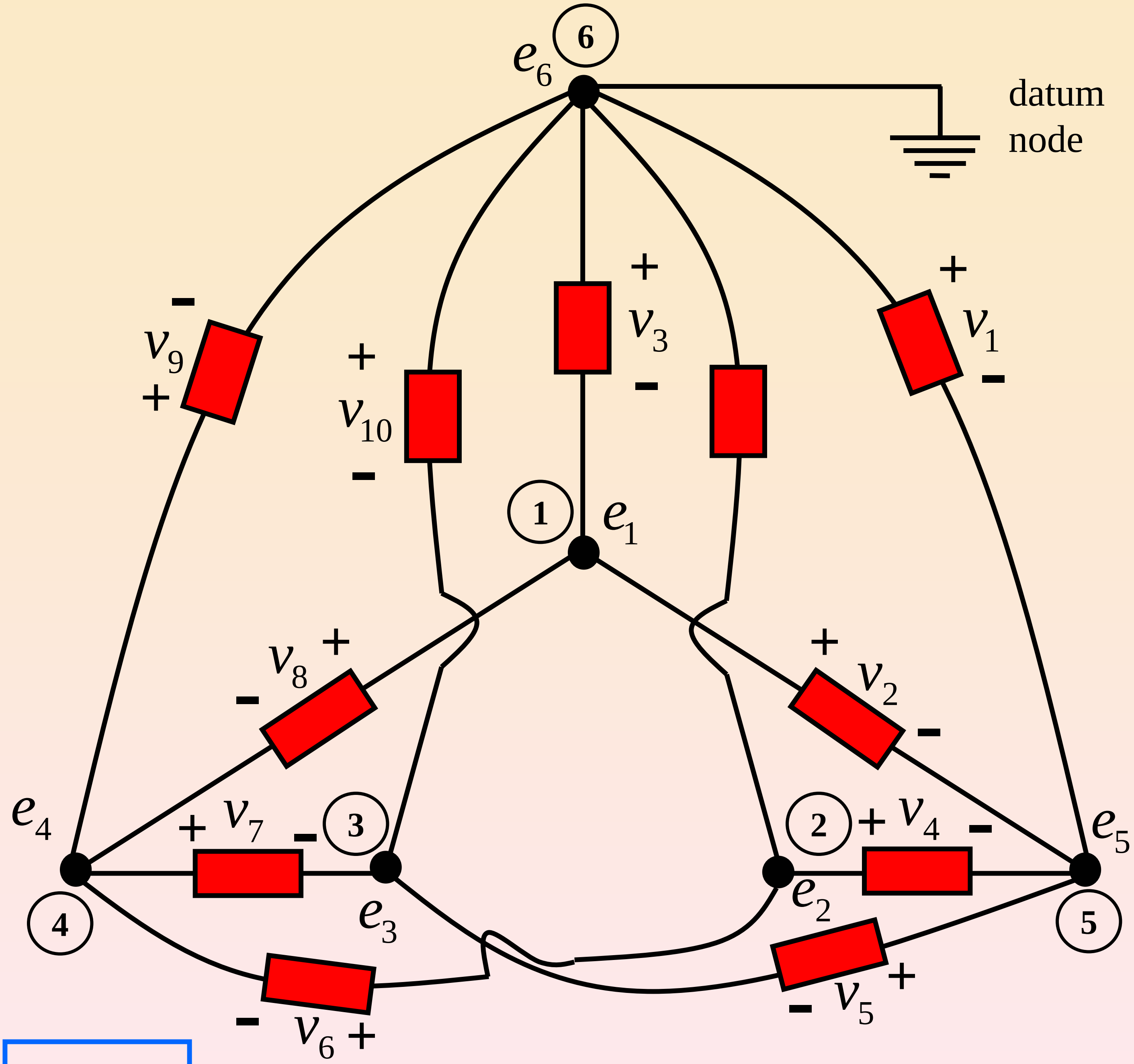
Associated Reference Convention :

A current direction is chosen entering each positively-referenced terminal.



Device Graph : DIGRAPH (Directed Graph)





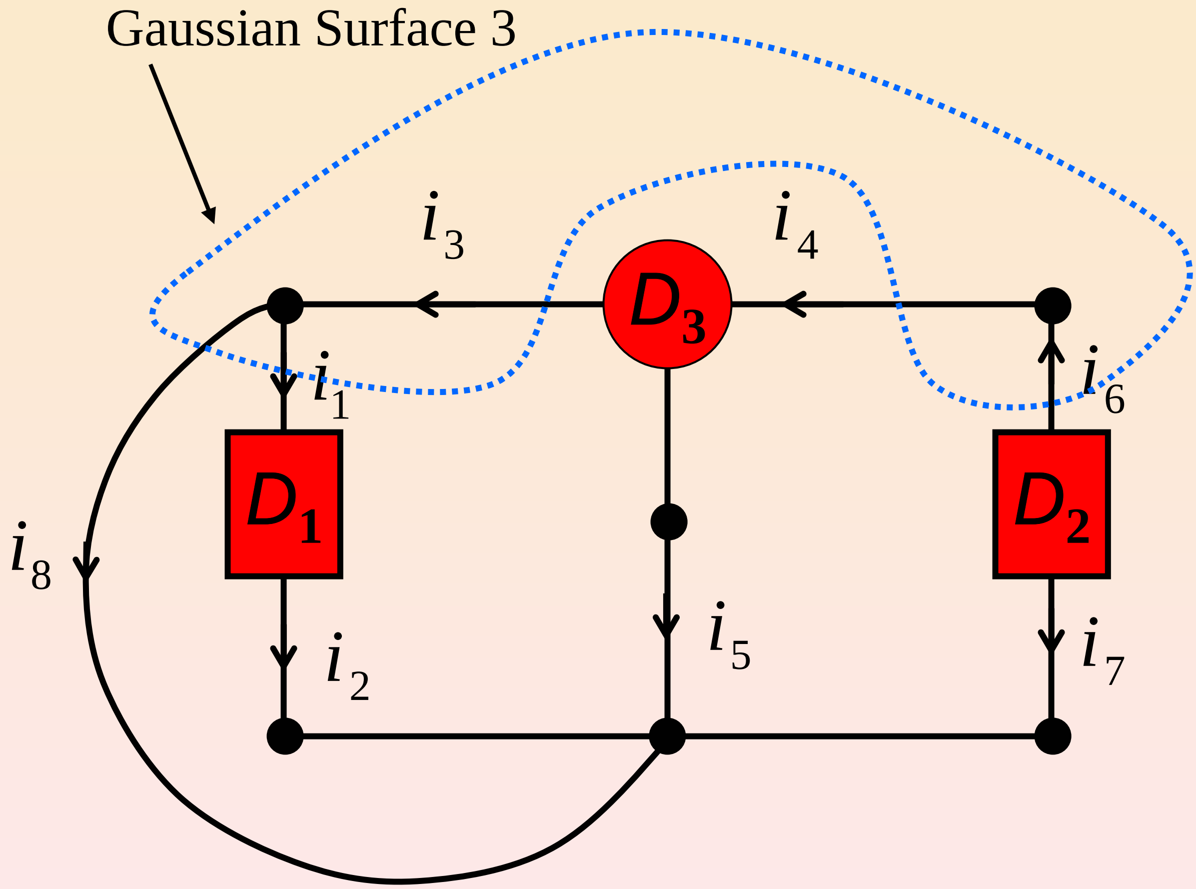
KVL

$$v_1 = e_6 - e_5 = -e_5, \quad v_4 = e_2 - e_5$$

$$v_2 = e_1 - e_5 \qquad , \quad v_5 = e_5 - e_3$$

$$v_3 = e_6 - e_1 = -e_1, \quad v_6 = e_2 - e_4$$

KCL



Gaussian Surface 3:

$$i_1 - i_3 + i_4 - i_6 + i_8 = 0$$

This KCL equation can be decomposed into the sum of two KCL cut set equations:

$$\text{cut set } \{4,6\} \quad \Rightarrow \quad i_4 - i_6 = 0$$

$$\text{cut set } \{1,3,8\} \quad \Rightarrow \quad i_1 - i_3 + i_8 = 0$$