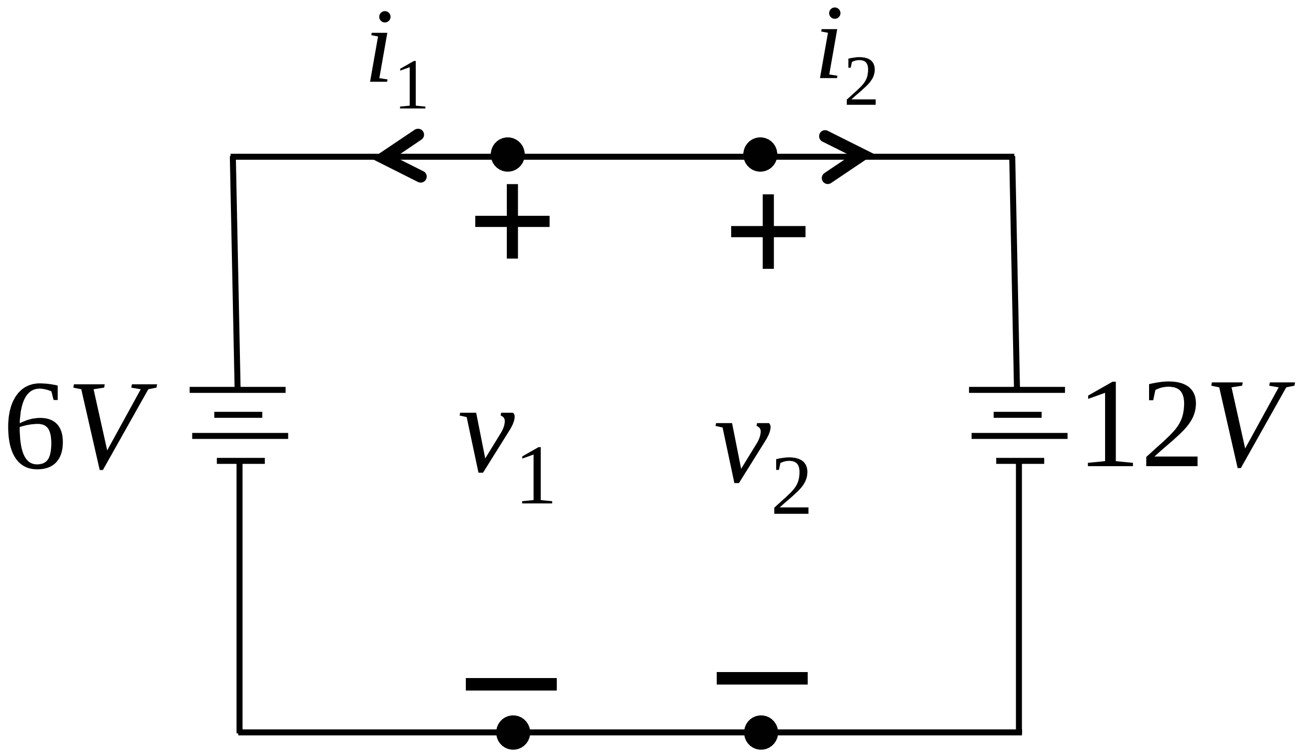


Problem: Find solutions for v_1 and v_2.

Solution:

Since it is impossible to satisfy KVL, KCL, and the element law governing the batteries, this circuit does **not** have a solution!

Tableau Equation

$$\underbrace{\left[\begin{array}{c|cc|cc} 0 & 0 & 0 & 1 & 1 \\ \hline -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]}_{\mathbf{T}} \left[\begin{array}{c} e_1 \\ \hline v_1 \\ v_2 \\ \hline i_1 \\ i_2 \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ \hline 6 \\ 12 \end{array}\right]$$

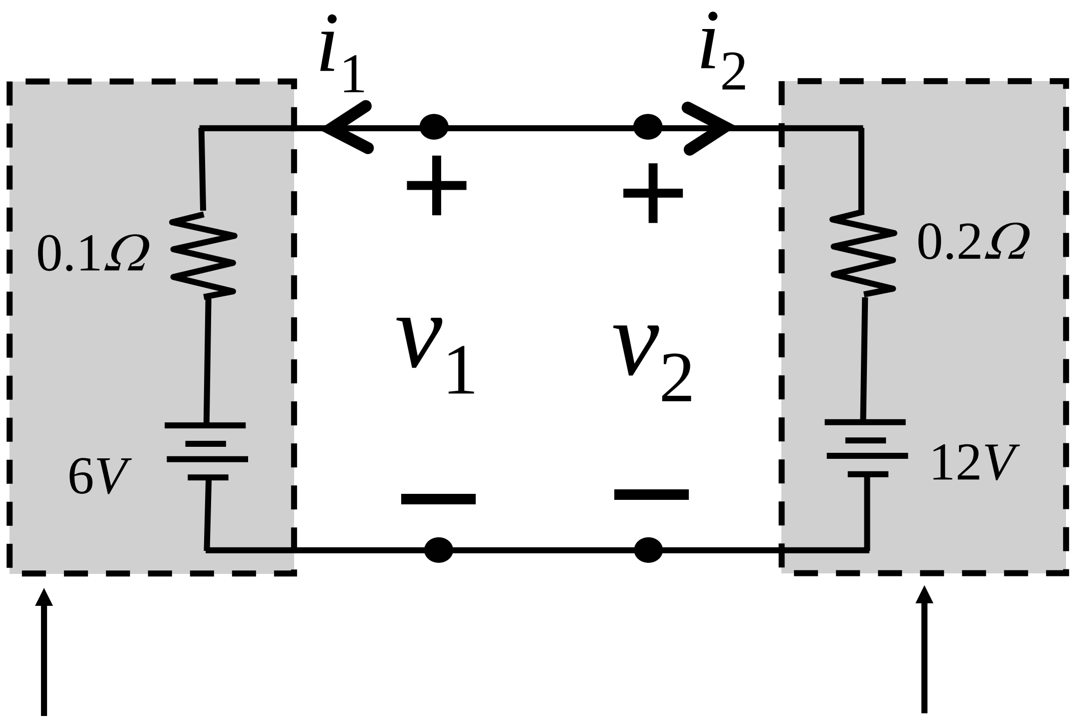
Exercise:

Verify

$$\det \mathbf{T} = 0$$

$\therefore$ This circuit does **not** have a solution when the batteries have **unequal** voltages, or it has an **infinite** number of solutions when the batteries have equal voltages.

How do we resolve the Paradox?



More-realistic model of 6-volt battery

More-realistic model of 12-volt battery

Solution:

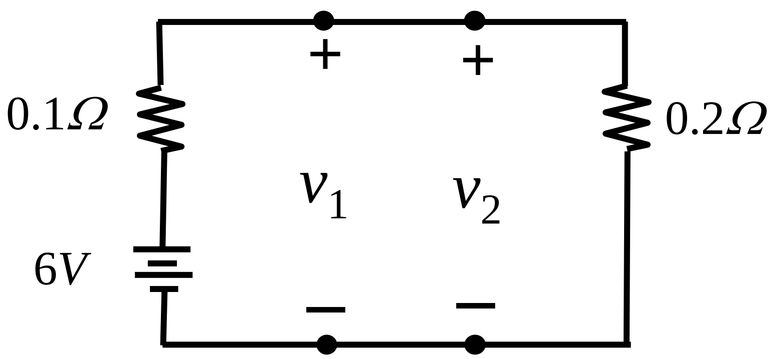
$$i_1 = \frac{12-6}{0.1+0.2} = \frac{6}{0.3} = 20\ A$$

$$i_2 = -20\ A$$

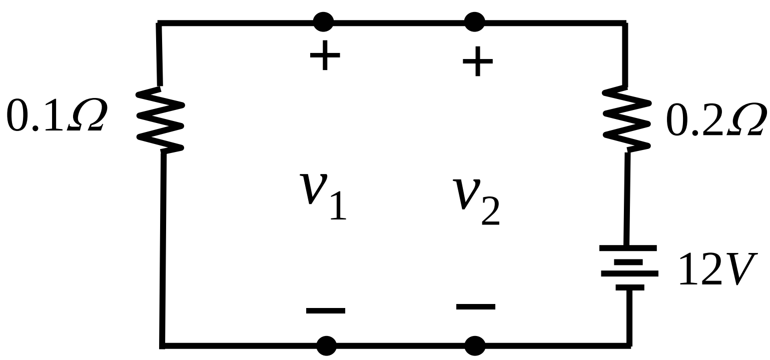
$$v_1 = 20(0.1) + 6 = 8\,V$$

$$v_2 = (-20)(0.2) + 12 = 8\,V$$

Solution by Superposition



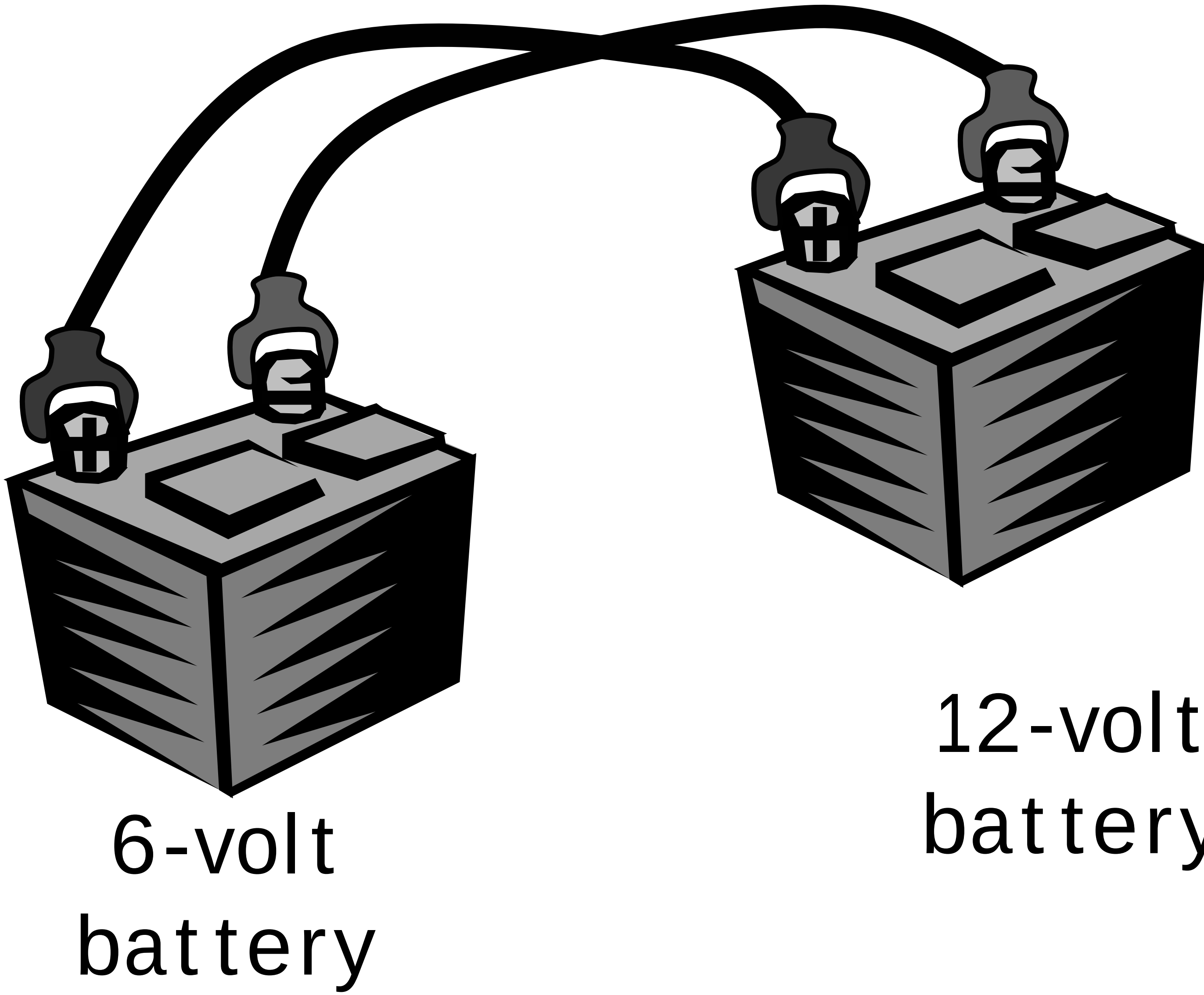
$$v_1' = v_2' = \frac{0.2}{0.3}(6) = 4V$$

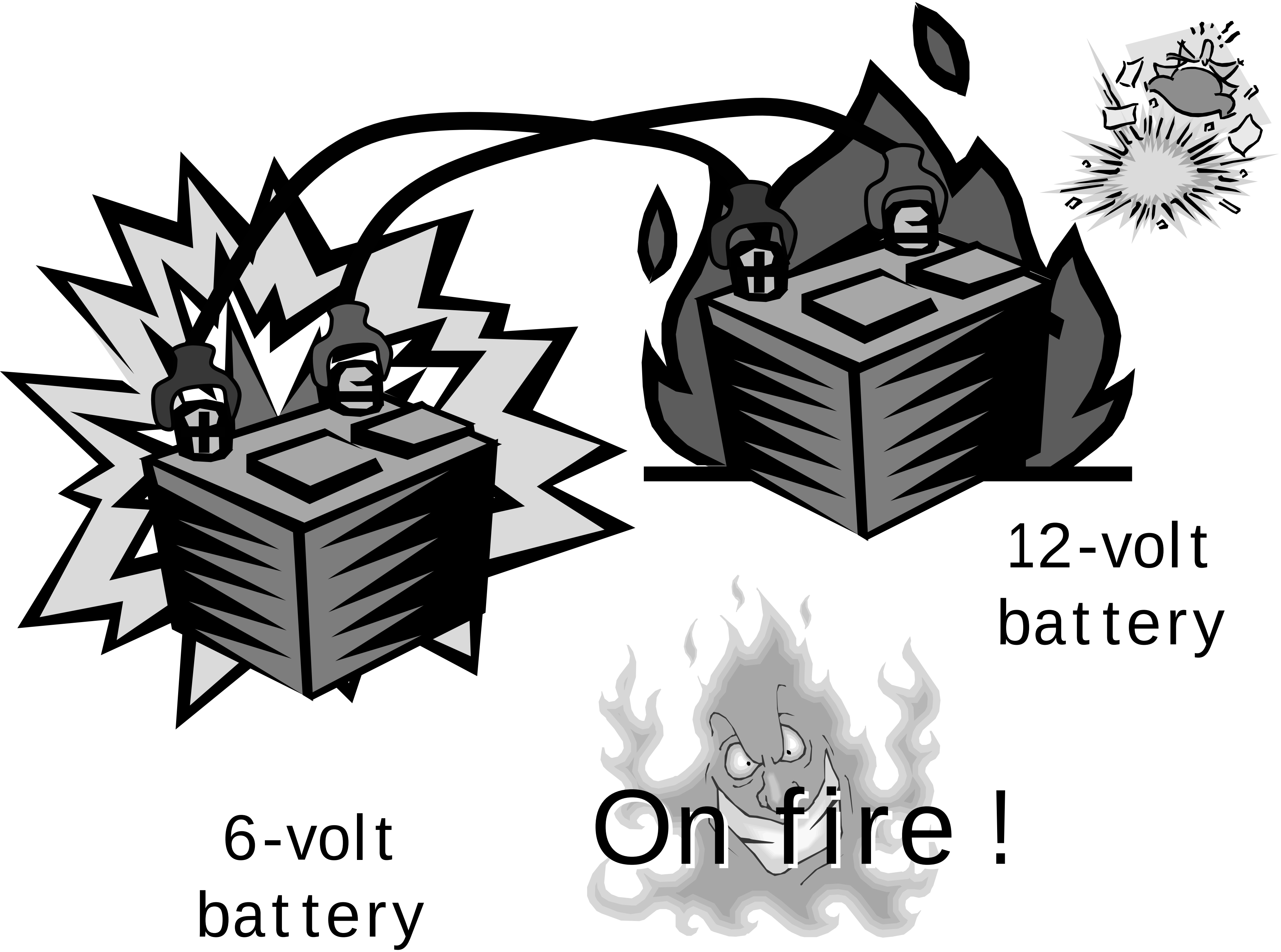


$$v_1'' = v_2'' = \frac{0.1}{0.3}(12) = 4V$$

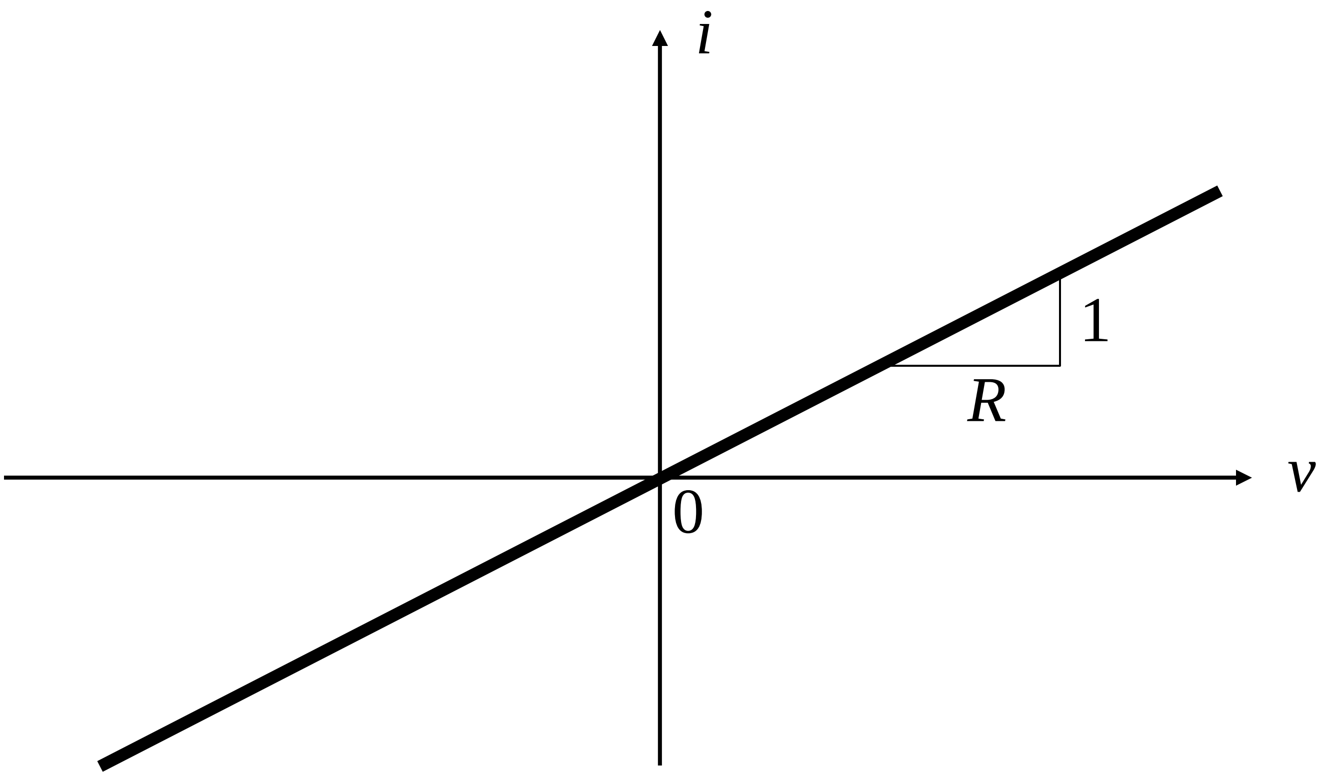
∴ By superposition,

$$v_1 = v_2 = 4 + 4 = 8V$$


12-volt
battery
6-volt
battery


6-volt
battery
12-volt
battery
On fire!

Linear Resistor



Ohm's Law

$$v = R\,i$$

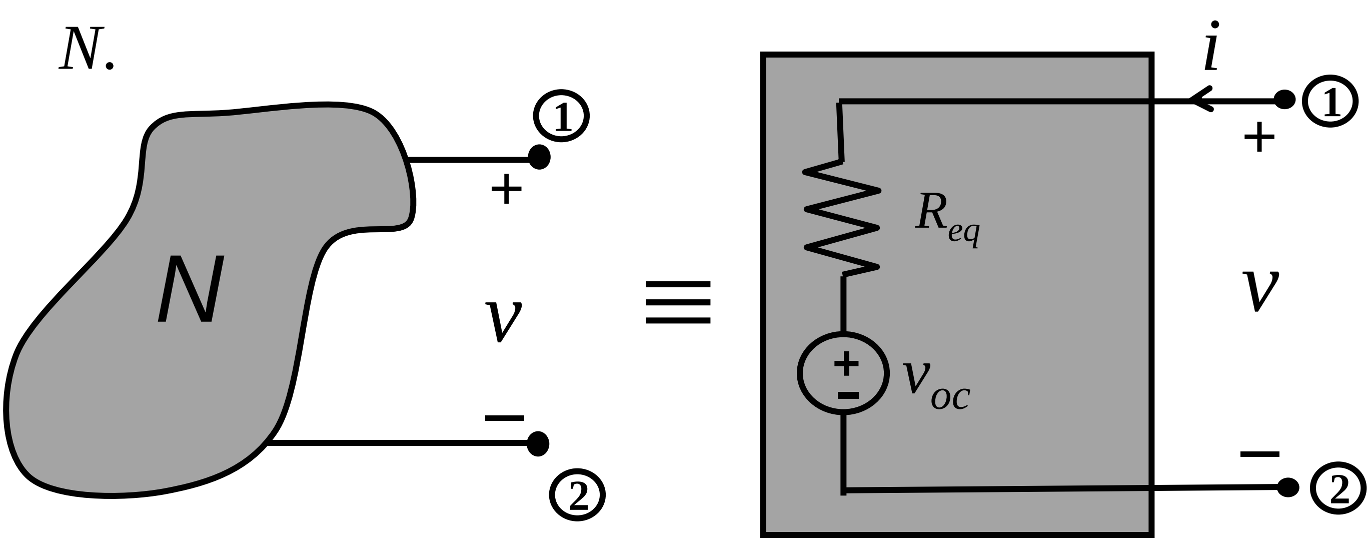
The constant *R* is called the **RESISTANCE**.

$$i = G\,v$$

The constant $G = 1/R$ is called the **CONDUCTANCE**.

Thevenin's Theorem

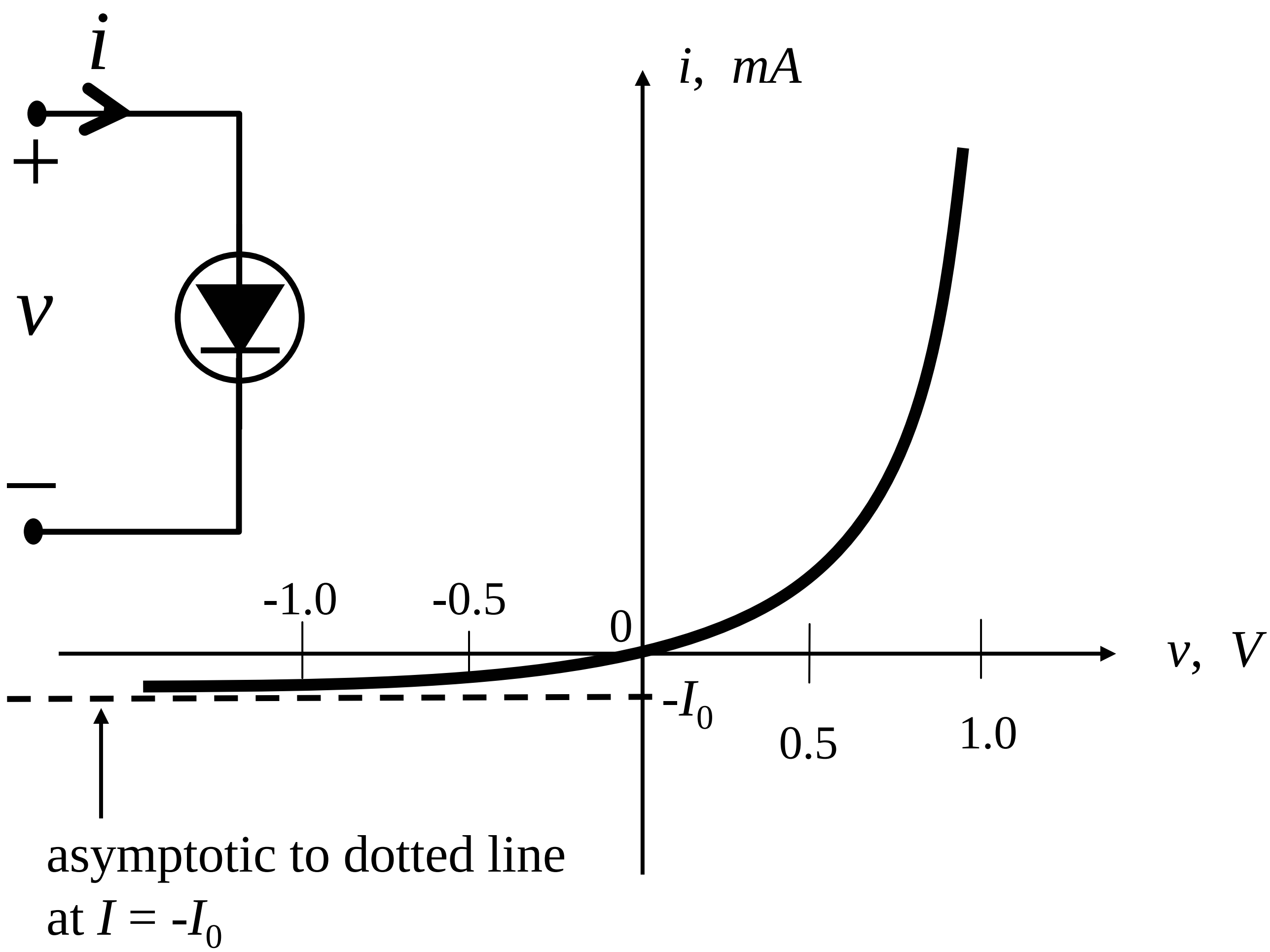
We can substitute the 2-terminal box N with an **equivalent** one-port called the **Thevenin Equivalent Circuit** made of a **linear resistance** R_{eq}, called the Thevenin **equivalent resistance**, **in series** with an independent voltage source v_{oc}, called the Thevenin **open-circuit voltage**, without affecting the solutions inside ***any*** external circuit N_{ext} connected across N.



Linear Resistive One-port

Thevenin Equivalent One-port

pn junction diode

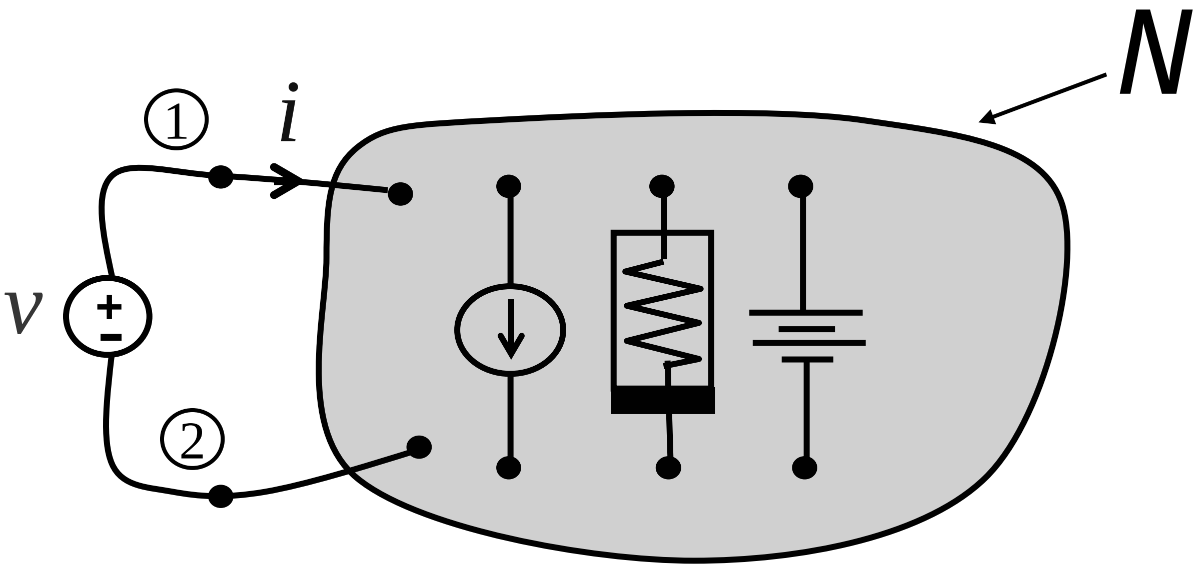


approximating equation:

$$i = I_0\left(e^{\frac{v}{V_T}} - 1\right)$$

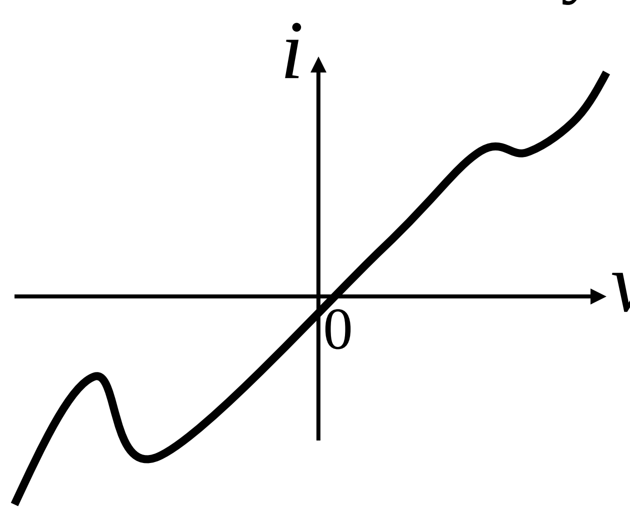
where I_0 and V_T are device parameters.

Driving-Point Characteristic



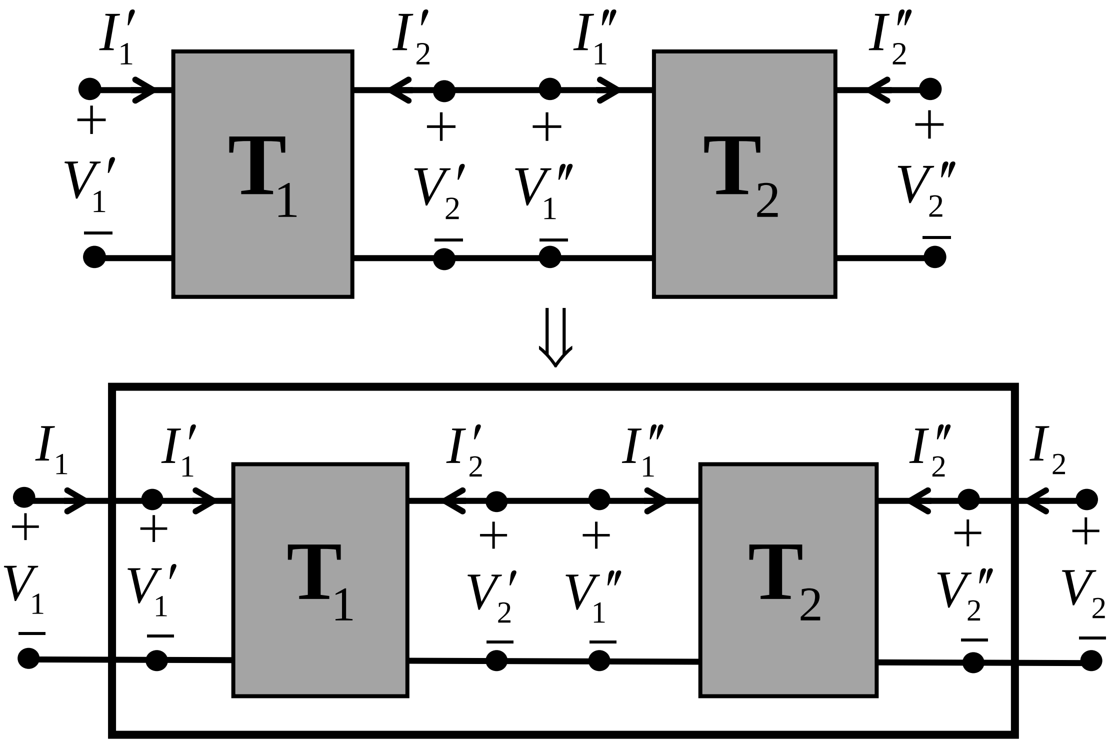
The 2 nodes { ① , ② } where the voltage source is connected are called **driving-point terminals.**

The *i*-vs.-*v* driving-point **characteristic** is the set of all (*i*,*v*) which simultaneously satisfy:



1. KCL
2. KVL
3. Constitutive Relation of all elements inside ***N***

Cascading Two-Ports



Let us describe the above two 2-ports by their Transmission Representations $\mathbf{T}_1$ and $\mathbf{T}_2$:

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{T}_1 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \quad , \quad \begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{T}_2 \end{bmatrix} \begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

$$V_1'' = V_2' \quad , \quad I_1'' = -I_2'$$

$$\Rightarrow \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{T} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\quad \mathbf{T} = \mathbf{T}_1 \, \mathbf{T}_2$