Power in Sinusoidal Signal

$$
\begin{aligned}
& < P > = \\
& \frac{1}{T} \int_0^T V_0 \cos(\omega t + \phi_1) I_0 \cos(\omega t + \phi_2) dt \\
= \quad & V_0 I_0 \frac{1}{T} \int_0^T \Big(\cos^2(\omega t) \cos(\phi_1) \cos(\phi_2) \\
& \quad + \sin^2(\omega t) \sin(\phi_2) \sin(\phi_1) \\
& \quad + \ldots \cos(\omega t) \sin(\omega t) \Big) dt
\end{aligned}
$$

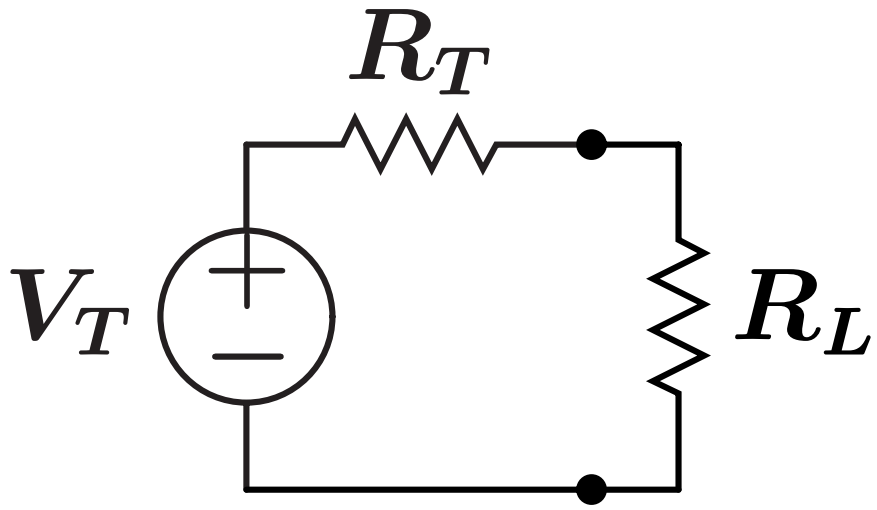
Sinusoidal Power Contd.

$$
\begin{aligned}
& < P >= \\
= \quad & \tfrac{1}{2} V_0 I_0 \Big(\cos(\phi_1) \cos(\phi_2) + \sin(\phi_1) \sin(\phi_2) \Big) \\
= \quad & \tfrac{1}{2} V_0 I_0 \cos(\phi_1 - \phi_2) \\
= \quad & V_{RMS} I_{RMS} \cos(\Delta\phi)
\end{aligned}
$$

Phasor Form of Power

$$
\begin{aligned}
\mathbf{V} &= V_0 e^{j\phi_1} \\
\mathbf{I} &= I_0 e^{j\phi_2} \\
P &= \tfrac{1}{2} V_0 I_0 \cos(\phi_1 - \phi_2) \\
&= \tfrac{1}{2} V_0 I_0 \Re e[e^{j(\phi_1 - \phi_2)}] \\
&= \tfrac{1}{2} \Re e(\mathbf{V}\mathbf{I}^*)
\end{aligned}
$$

Optimum Load: DC Case



$$P_L = V_0\left(\frac{R_L}{R_L + R_T}\right)\left(\frac{V_0}{R_L + R_T}\right)$$

DC Optimum Load Contd.

let $x = R_L/R_T$:

$$P_L = \frac{V_0^2}{R_T}\left(\frac{x}{(1+x)^2}\right)$$

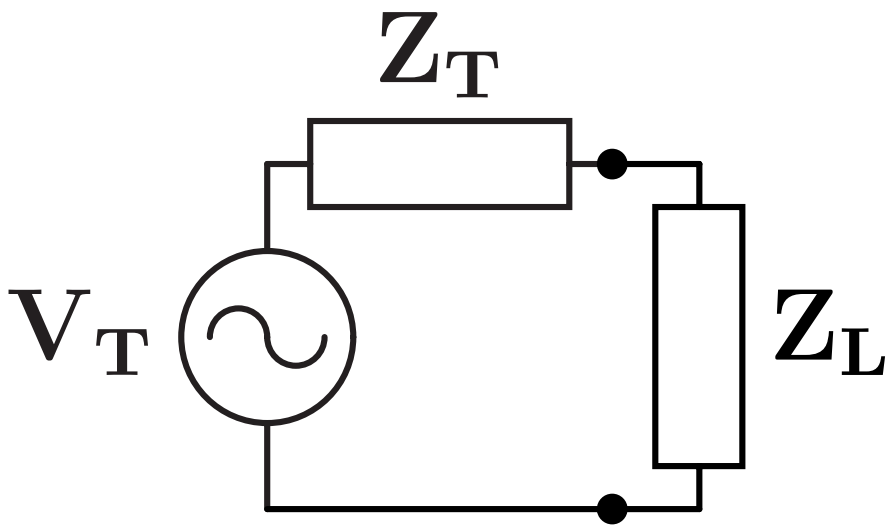
$d/dx = 0$ when:

$$\frac{1-x}{(1+x)^3} = 0 \rightarrow x = 1$$

Optimum Load is when

$$R_L = R_T$$

Optimum Load: AC Case



$$P_L = \Re e\left[\frac{V_T}{2}\left(\frac{Z_L}{Z_L + Z_T}\right)\left(\frac{V_T}{Z_L + Z_T}\right)^*\right]$$

AC Optimum Load Contd.

$$P_L = (1/2)|V_T|^2\left(\frac{R_L}{(R_L + R_T)^2 + (X_L + X_T)^2}\right)$$

Maximum when:

$$X_L = -X_T$$

Optimum Load is when

$$R_L = R_T$$

Conjugate Match Maximizes Power

$$X_L(opt) = -X_T$$

$$R_L(opt) = R_T$$

$$Z_L(opt) = Z_T^*$$

$$P_{max} = \frac{|V_T|^2}{8R_T}$$