4.14

The circuit element can be modeled as

\[ \text{Calculate } V_L \text{ in the following circuit:} \]

First notice \( V_{gc} = V_s \)

Also we simply have a voltage divider between \( R_p \) and \( R_L \).

\[ V_L = (-\mu V_{gc}) \left( \frac{R_L}{R_p + R_L} \right) \]

\[ V_L = -\frac{\mu V_s R_L}{R_p + R_L} \]

4.30

We want an Op-Amp Circuit with \( R_i' > 10M\Omega \), \( A' = -10 \) and \( R_0' < 10\Omega \).

Given: \( R_i = 10^5 \Omega \), \( R_0 = 50\Omega \), \( A = 10^5 \)

This can be done with an inverting amplifier

\[ R_i' \approx R_i \]
\[ R_0' \approx \frac{R_0}{A} \left( \frac{R_i + R_2}{R_i} \right) \]
\[ A' \approx -\frac{R_2}{R_i} \]

The conditions are satisfied when \( R_i = 20M\Omega \) and \( R_2 = 200M\Omega \)

\[ R_i' = 20M\Omega \quad A' = -10 \quad R_0' = 0.0055\Omega \]

Note: \( R_i \) can be any value above 10M\Omega, I chose an easy number.
Suppose you don’t have extremely large megohm resistors. Then you could consider inserting a voltage buffer in front of the inverting amplifier. The buffer has “infinite” input resistance, giving you greater flexibility in choosing $R_1$ and $R_2$.

For this design, you can choose any values for $R_2$ and $R_1$ so long as $\frac{R_2}{R_1} = 10$. 
5.3
Prove that \( L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \)

\[
\begin{align*}
I_{\text{tot}} &= I_1 + I_2 \\
\frac{1}{L_{eq}} \int V(t) \, dt &= \frac{1}{L_1} \int V(t) \, dt + \frac{1}{L_2} \int V(t) \, dt \\
\frac{1}{L_{eq}} &= \frac{1}{L_1} + \frac{1}{L_2} \\
\Rightarrow L_{eq} &= \frac{L_1 L_2}{L_1 + L_2}
\end{align*}
\]

First, we compare the total current to the individual currents.

\[
I_{\text{tot}} = \frac{1}{L_{eq}} \int V(t) \, dt, \quad I_1 = \frac{1}{L_1} \int V(t) \, dt, \quad I_2 = \frac{1}{L_2} \int V(t) \, dt
\]

\( V(t) \) is the same for all because of the parallel connection.

5.6

Given the above integrator circuit, we would like to find an equation for \( V_{\text{out}} \).

Using the ideal Op-Amp Technique, \( V^{(+)}, V^{(-)} = 0 \) and \( I^{(+)}, I^{(-)} = 0 \).

Now \( I_R = -I_C \)

\[
\begin{align*}
\frac{V_{\text{in}} - 0}{R} &= -C \frac{dV}{dt} \\
\int_{0}^{t} \frac{V_{\text{in}}(t)}{RC} \, dt &= \int_{0}^{t} \frac{V_{\text{out}}(t)}{RC} \, dt \\
\Rightarrow V_{\text{out}}(t) &= -\frac{1}{RC} \int_{0}^{t} V_{\text{in}}(t') \, dt'
\end{align*}
\]

The \( t' \) was added simply to distinguish between the differential variable and the time variable.
5.8

D) Find the single Capacitor equivalent for

\[ C_{SE} = \frac{C_1 C_2}{C_1 + C_2} \]

\[ C_{PAR} = C_1 + C_2 \]

\[
\frac{(0.015 \mu F)(0.032 \mu F)}{(0.015 \mu F + 0.032 \mu F)} = 10.2 \text{nF}
\]

\[ 10.2 \text{nF} \]

\[ \]

D) Find the single inductor equivalent for

\[ L_{SE} = L_1 + L_2 \]

\[ L_{PAR} = \frac{L_1 L_2}{L_1 + L_2} \]

\[
\frac{(3.5 \text{mH})(3 \text{mH})}{(3.5 \text{mH} + 3 \text{mH})} = 1.6 \text{mH}
\]

\[ 1.6 \text{mH} \]

5.10

C = 1 \mu F

Find the current through this capacitor when the given voltage waveform is applied.

\[ i = C \frac{dv}{dt} \]

\[ \begin{align*}
0 < t < 1 \text{msec} & \quad (\frac{dv}{dt})_1 = 10^4 \text{ V/sec} \\
1 \text{msec} < t < 2 \text{msec} & \quad (\frac{dv}{dt})_2 = -10^4 \text{ V/sec}
\end{align*} \]

\[ i_1 = C \left(\frac{dv}{dt}\right)_1 = 10 \text{mA} \]

\[ i_2 = C \left(\frac{dv}{dt}\right)_2 = -10 \text{mA} \]