CS70: Discrete Math and Probability

Slides adopted from Satish Rao, CS70 Spring 2016
June 20, 2016
Introduction

Programming Computers
Programming Computers ≡ Superpower!
Programming Computers ≡ Superpower!

What are your super powerful programs doing?
Programming Computers ≡ Superpower!

What are your super powerful programs doing?
Logic and Proofs!
Programming Computers \equiv Superpower!

What are your super powerful programs doing?
  Logic and Proofs!
  Induction \equiv Recursion.
Programming Computers $\equiv$ Superpower!

What are your super powerful programs doing?
  Logic and Proofs!
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What can computers do?
Programming Computers $\equiv$ Superpower!

What are your super powerful programs doing?
   Logic and Proofs!
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What can computers do?
   Work with discrete objects.
Programming Computers $\equiv$ Superpower!

What are your super powerful programs doing?
- Logic and Proofs!
- Induction $\equiv$ Recursion.

What can computers do?
- Work with discrete objects.
  
  *Discrete Math*
Programming Computers ≡ Superpower!

What are your super powerful programs doing?
- Logic and Proofs!
- Induction ≡ Recursion.

What can computers do?
- Work with discrete objects.
  - **Discrete Math** → immense application.
Programming Computers ≡ Superpower!

What are your super powerful programs doing?
  Logic and Proofs!
  Induction ≡ Recursion.

What can computers do?
  Work with discrete objects.
  **Discrete Math** → immense application.

Computers learn and interact with the world?
Programming Computers \equiv \text{Superpower!}

What are your super powerful programs doing?
  Logic and Proofs!
  Induction \equiv \text{Recursion}.

What can computers do?
  Work with discrete objects.
  \textbf{Discrete Math} \implies \text{immense application}.

Computers learn and interact with the world?
  E.g. machine learning, data analysis.
Programming Computers ≡ Superpower!

What are your super powerful programs doing?
  Logic and Proofs!
  Induction ≡ Recursion.

What can computers do?
  Work with discrete objects.
    Discrete Math → immense application.

Computers learn and interact with the world?
  E.g. machine learning, data analysis.
  Probability!
Programming Computers $\equiv$ Superpower!

What are your super powerful programs doing?
   Logic and Proofs!
   Induction $\equiv$ Recursion.

What can computers do?
   Work with discrete objects.
      Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
   E.g. machine learning, data analysis.
      Probability!

See note 1, for more discussion.
Course Webpage: www.eecs70.org
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Explains policies, has homework/discussion worksheets, slides, exam dates, etc.
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Questions
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Questions ➔ piazza:
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[ piazza.com/berkeley/summer2016/cs70 ](http://piazza.com/berkeley/summer2016/cs70)
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Assessment:
  Homework: 20%
  Midterm 1 (07/08): 20%
  Midterm 2 (07/29): 20%
  Final (08/12): 35%
  Quiz: 4%
  Sundry: 1%
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Conflicts?
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Conflicts? Piazza pinned post.
Learning tips

Take homework seriously!
Learning tips

Take homework seriously!
Go to homework parties,
Take homework seriously!
Go to homework parties, study groups
Take homework seriously!
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VERY fast paced, start early
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Questions?
3 Co-Instructors
Just graduated,
Just graduated, from Berkeley
Just graduated, from Berkeley

Been TA for CS70 for two semesters
Just graduated, from Berkeley

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Will start working at Google as a software engineer on September
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Recently I’m climbing ... the ladder of league of legends ranking system ...
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Office hours: Monday 10-11, Tuesday 11-12 in Soda 611 or by appointment
David Dinh

Email: dinh@cs.berkeley.edu
Office Hours: M/W 3:30-5:00 (right after lecture) in 606 Soda
I just finished my first year of grad school. My research interests are numerical algorithms and complexity theory - essentially, I work on making faster algorithms for doing things like solving equations, factoring matrices, etc. (and proving that they run fast!), as well as showing that there are limits on how fast we can make these algorithms.
Also did my undergrad here at Cal - CS70 was by far my favorite lower-div.
Fun fact: I like to make ice cream.
Not here today.
Not here today. Tomorrow lecture
3 Co-Instructors
12 awesome and talented TAs.
Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
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- Consider the theory:
Wason’s experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory: “If a person travels to Chicago, he/she flies.”
Suppose we have four cards on a table:

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Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person’s destination on one side, and mode of travel.
- Consider the theory:
  “If a person travels to Chicago, he/she flies.”
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

<table>
<thead>
<tr>
<th>Alice</th>
<th>Bob</th>
<th>Charlie</th>
<th>Donna</th>
</tr>
</thead>
<tbody>
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Suppose we have four cards on a table:

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Which cards do you need to flip to test the theory?
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Answer:
Suppose we have four cards on a table:

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• Which cards do you need to flip to test the theory?

Answer: Later.
Today: Note 1.
Today: Note 1. Note 0 is background.
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The language of proofs!
Today: Note 1. Note 0 is background. Do read/skim it.

The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws
Propositions: Statements that are true or false.

\[ \sqrt{2} \text{ is irrational} \]
\[ 2+2 = 4 \]
\[ 2+2 = 3 \]
\[ 826\text{th digit of } \pi \text{ is 4} \]
\[ \text{Stephen Curry is a good player.} \]
\[ \text{All evens } > 2 \text{ are sums of 2 primes} \]
\[ 4 + 5 \]
\[ x + x \]
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Proposition Proposition
\text{True} \text{ True}
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### Propositions: Statements that are true or false.

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Propositions: Statements that are true or false.

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2+2 = 4 \quad \text{Proposition True}
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826th digit of pi is 4 \quad \text{Proposition False}

Stephen Curry is a good player. \quad \text{Not a Proposition}

All evens > 2 are sums of 2 primes \quad \text{Proposition False}

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Again: “value” of a proposition is ...
**Propositions: Statements that are true or false.**

- $\sqrt{2}$ is irrational  
  - Proposition  
  - True
- $2+2 = 4$  
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  - Proposition  
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Again: “value” of a proposition is ...  
True or False
Propositional Forms.

Put propositions together to make another...
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): $P \land Q$

Examples:

$\neg (2 + 2 = 4)$ – a proposition that is False.

$2 + 2 = 4 \land 2 + 2 = 3$ – a proposition that is False.

$2 + 2 = 3 \land 2 + 2 = 4$ – a proposition that is True.
Propositional Forms.

Put propositions together to make another...

Conjunction ("and"): \( P \land Q \)

"\( P \land Q \)" is True when both \( P \) and \( Q \) are True.
Propositional Forms.

Put propositions together to make another...

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"\( P \land Q \)" is \textbf{True} when both \( P \) and \( Q \) are \textbf{True} . Else \textbf{False} .
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Conjunction (“and”): \( P \land Q \)

“\( P \land Q \)” is True when both \( P \) and \( Q \) are True. Else False.

Disjunction (“or”): \( P \lor Q \)
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Negation ("not"): $\neg P$
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Examples:

\( \neg “(2 + 2 = 4)” \) – a proposition that is ...
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"\( \neg P \)" is True when \( P \) is False. Else False.

Examples:

\( \neg "(2 + 2 = 4)" \) – a proposition that is ... False

"2 + 2 = 3" \( \land "2 + 2 = 4" \) – a proposition that is ...
Propositional Forms.

Put propositions together to make another...

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Examples:

\( \neg \) “\((2 + 2 = 4)\)” – a proposition that is ... False

“\( 2 + 2 = 3 \) \( \land \) “\( 2 + 2 = 4 \)” – a proposition that is ... False
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$\neg \left( 2 + 2 = 4 \right)$ – a proposition that is ... False

“$2 + 2 = 3$” $\land$ “$2 + 2 = 4$” – a proposition that is ... False

“$2 + 2 = 3$” $\lor$ “$2 + 2 = 4$” – a proposition that is ...


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$\neg \ "(2 + 2 = 4)"$ – a proposition that is ... False

"$2 + 2 = 3$" $\land$ "$2 + 2 = 4$" – a proposition that is ... False

"$2 + 2 = 3$" $\lor$ "$2 + 2 = 4$" – a proposition that is ... True
Propositional Forms: quick check!

\[ P = \text{“\(\sqrt{2}\) is rational”} \]

\[ Q = \text{“826th digit of pi is 2”} \]

P is False.

Q is True.

\[ P \land Q \text{ is False} \]

\[ P \lor Q \text{ is True} \]

\[ \neg P \text{ is True} \]
Propositional Forms: quick check!

$P = \text{“} \sqrt{2} \text{ is rational”}$

$Q = \text{“} 826\text{th digit of } \pi \text{ is 2”}$
Propositional Forms: quick check!

\[ P \equiv \text{“}\sqrt{2}\text{ is rational”} \]
\[ Q \equiv \text{“826th digit of pi is 2”} \]
Propositional Forms: quick check!

$P = \text{“}\sqrt{2} \text{ is rational”}$

$Q = \text{“826th digit of pi is 2”}$

$P$ is ...
$P = \text{“}\sqrt{2} \text{ is rational”}$

$Q = \text{“}826\text{th digit of pi is 2”}$

$P$ is ...False .
Propositional Forms: quick check!

\[ P = \text{“}\sqrt{2} \text{ is rational”} \]
\[ Q = \text{“826th digit of pi is 2”} \]

\[ P \text{ is ...} \text{False} \text{ .} \]
\[ Q \text{ is ...} \]
Propositional Forms: quick check!

\[ P = \text{"} \sqrt{2} \text{ is rational} \]
\[ Q = \text{"} 826\text{th digit of pi is 2"} \]

\[ P \text{ is } \text{False} . \]
\[ Q \text{ is } \text{True} . \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2”} \]

\[ P \text{ is ...} \text{False} \ . \]
\[ Q \text{ is ...} \text{True} \ . \]

\[ P \land Q \ldots \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2”} \]

\[ P \text{ is } \text{False} \ . \]
\[ Q \text{ is } \text{True} \ . \]

\[ P \land Q \text{ ... False} \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational} \]
\[ Q = \text{“} 826\text{th digit of pi is 2} \]

- \( P \) is ... False.
- \( Q \) is ... True.

\[ P \land Q \ldots \text{False} \]
\[ P \lor Q \ldots \]
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2”} \]

\( P \) is \(...\text{False}.. \)
\( Q \) is \(...\text{True}.. \)

\( P \land Q \) \(...\text{False}.. \)
\( P \lor Q \) \(...\text{True}.. \)
Propositional Forms: quick check!

\[ P = \text{“} \sqrt{2} \text{ is rational”} \]
\[ Q = \text{“} 826\text{th digit of pi is 2”} \]

\[ P \text{ is } \text{False} \]
\[ Q \text{ is } \text{True} \]

\[ P \land Q \text{ ... False} \]
\[ P \lor Q \text{ ... True} \]
\[ \neg P \text{ ...} \]
Propositional Forms: quick check!

$P = \text{“}\sqrt{2} \text{ is rational”}$
$Q = \text{“826th digit of pi is 2”}$

$P$ is $\text{False}$.
$Q$ is $\text{True}$.

$P \wedge Q$ ... $\text{False}$
$P \vee Q$ ... $\text{True}$
$\neg P$ ... $\text{True}$
Propositions:

$C_1$ - Take class 1
Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2
Propositions:

\( C_1 \) - Take class 1
\( C_2 \) - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

\[
( (C_1 \land \neg C_2) \land (C_3 \land \neg C_4) ) \land ((C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4))
\]
Put them together.

Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.
Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2

....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:
Propositions:

\( C_1 \) - Take class 1  
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....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

\[
\left( (C_1 \lor C_2) \land (C_3 \lor C_4) \right) \lor \left( (C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4) \right)
\]
Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2
...

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

$$((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor (((C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4))$$

Can you take class 1?
Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

$((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor (((C_2 \land C_3) \land (C_5 \lor C_6)) \land \neg C_4))$

Can you take class 1?
Can you take class 1 and class 5 together?
Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2
....

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

\[
((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor ((C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4))
\]

Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...
Put them together..

Propositions:

$C_1$ - Take class 1
$C_2$ - Take class 2

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

$((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor ((C_2 \land C_3) \land (C_5 \lor C_6) \land \neg C_4))$

Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...complicated.
Propositions:

- $C_1$ - Take class 1
- $C_2$ - Take class 2

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

$$((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor (((C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4))$$

Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...complicated.

We can program!!!!
Propositions:

\[ C_1 - \text{Take class 1} \]
\[ C_2 - \text{Take class 2} \]

You can only take one of class 1, class 2 and one of class 3, class 4, or take both class 2, class 3 and take either class 5 or class 6, as long as you are not taking class 4 at the same time.

Propositional Form:

\[ ((C_1 \lor C_2) \land (C_3 \lor C_4)) \lor ((C_2 \land C_3) \land (C_5 \lor C_6) \land (\neg C_4)) \]

Can you take class 1?
Can you take class 1 and class 5 together?

This seems ...complicated.

We can program!!!!We need a way to keep track!
### Truth Tables for Propositional Forms.

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$P \land Q$</th>
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<tbody>
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Notice: $\land$ and $\lor$ are commutative.
Truth Tables for Propositional Forms.

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Notice: $\land$ and $\lor$ are commutative.
Truth Tables for Propositional Forms.

\[
\begin{array}{|c|c|c|}
\hline
P & Q & P \land Q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\hline
\end{array}
\]

Notice: $\land$ and $\lor$ are commutative.
Truth Tables for Propositional Forms.

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One use for truth tables: Logical Equivalence of propositional forms!

Example:

$\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$...because the two propositional forms have the same...Truth Table!
Truth Tables for Propositional Forms.

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Notice: $\land$ and $\lor$ are commutative.
Truth Tables for Propositional Forms.

\[
\begin{array}{c|c|c}
P & Q & P \land Q \\
\hline
T & T & T \\
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F & F & F \\
\end{array}
\]

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\begin{array}{c|c|c}
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Notice: \( \land \) and \( \lor \) are commutative.
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Notice: $\land$ and $\lor$ are commutative.
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Notice: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!
Truth Tables for Propositional Forms.

\[
\begin{array}{c|c|c}
P & Q & P \land Q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\quad \quad \quad
\begin{array}{c|c|c}
P & Q & P \lor Q \\
\hline
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]

Notice: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$
Truth Tables for Propositional Forms.

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</table>

Notice: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$

...because the two propositional forms have the same...
Truth Tables for Propositional Forms.

\[ P \quad Q \quad P \land Q \]
\[
\begin{array}{ccc}
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

\[ P \quad Q \quad P \lor Q \]
\[
\begin{array}{ccc}
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]

Notice: \( \land \) and \( \lor \) are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \lnot (P \land Q) \) logically equivalent to \( \lnot P \lor \lnot Q \)

...because the two propositional forms have the same...

....Truth Table!
Truth Tables for Propositional Forms.

<table>
<thead>
<tr>
<th>$P$</th>
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<th>$P \land Q$</th>
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Notice: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$

...because the two propositional forms have the same...

....Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg (P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
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One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \land Q)$ logically equivalent to $\neg P \lor \neg Q$

...because the two propositional forms have the same...

...Truth Table!

<table>
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<th>$P$</th>
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Notice: ∧ and ∨ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: \( \neg(P ∧ Q) \) logically equivalent to \( \neg P ∨ \neg Q \)

...because the two propositional forms have the same...

....Truth Table!

<table>
<thead>
<tr>
<th>P</th>
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DeMorgan’s Law’s for Negation: distribute and flip!

\( \neg(P ∧ Q) \)
Truth Tables for Propositional Forms.

<table>
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<tr>
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Notice: \( \land \) and \( \lor \) are commutative.

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<th>Q</th>
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<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Notice: $\land$ and $\lor$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg (P \land Q)$ logically equivalent to $\neg P \lor \neg Q$

...because the two propositional forms have the same...

...Truth Table!

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$\neg (P \lor Q)$</th>
<th>$\neg P \land \neg Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

DeMorgan’s Law’s for Negation: distribute and flip!

$\neg (P \land Q) \equiv \neg P \lor \neg Q$

$\neg (P \lor Q) \equiv \neg P \land \neg Q$
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q,\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:
\[ P \text{ is } \text{True}. \]
LHS: \(T \land (Q \lor R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q \), \( (F \land Q) \equiv F \).

Cases:

\( P \) is **True**.

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:
\(P\) is True.
- LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
- RHS: \((T \land Q) \lor (T \land R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]?

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

- **\(P\) is True**.
  - LHS: \((T \land (Q \lor R)) \equiv (Q \lor R)\).
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:
  
  - **P is True**.
    
    LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
    
    RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R). \)
  
  - **P is False**.
\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

\(P\) is True.

\(\text{LHS: } T \land (Q \lor R) \equiv (Q \lor R).\)
\(\text{RHS: } (T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False.

\(\text{LHS: } F \land (Q \lor R)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:
- \( P \) is \textbf{True} .
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)
- \( P \) is \textbf{False} .
  - LHS: \( F \land (Q \lor R) \equiv F. \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:
- \(P \) is True.
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R) \).
- \(P \) is False.
  - LHS: \( F \land (Q \lor R) \equiv F \).
  - RHS: \((F \land Q) \lor (F \land R) \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True.

LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).
RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

\(P\) is False.

LHS: \(F \land (Q \lor R) \equiv F\).
RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F)\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

\(P\) is True.

- LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
- RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

\(P\) is False.

- LHS: \(F \land (Q \lor R) \equiv F.\)
- RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:
- \( P \) is True .
  - LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
  - RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)
- \( P \) is False .
  - LHS: \( F \land (Q \lor R) \equiv F. \)
  - RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)
- \(P\) is False.
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) ? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:

\[ P \text{ is True} . \]

LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)

RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)

\[ P \text{ is False} . \]

LHS: \( F \land (Q \lor R) \equiv F. \)

RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) ? \]

Simplify: \( T \lor Q \equiv T, \)
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]?

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F\).

Cases:

- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R)\).
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R)\).

- \(P\) is False.
  - LHS: \(F \land (Q \lor R) \equiv F\).
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F\).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) \]?

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q\).
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).

Cases:

\( P \) is **True**.

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).

\( P \) is **False**.

LHS: \( F \land (Q \lor R) \equiv F \).
RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F \).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) \? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q \).

Foil 1:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F. \)

Cases:
\[ P \text{ is True}. \]
LHS: \( T \land (Q \lor R) \equiv (Q \lor R). \)
RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R). \)

\[ P \text{ is False}. \]
LHS: \( F \land (Q \lor R) \equiv F. \)
RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F. \)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q. \)

Foil 1:
\[ (A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)? \]
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \( (T \land Q) \equiv Q, (F \land Q) \equiv F \).

Cases:

\emph{P} is True .

LHS: \( T \land (Q \lor R) \equiv (Q \lor R) \).
RHS: \( (T \land Q) \lor (T \land R) \equiv (Q \lor R) \).

\emph{P} is False .

LHS: \( F \land (Q \lor R) \equiv F \).
RHS: \( (F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F \).

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \( T \lor Q \equiv T, F \lor Q \equiv Q \).

Foil 1:

\((A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)?\)

Foil 2:
Distributive?

\[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)? \]

Simplify: \((T \land Q) \equiv Q, (F \land Q) \equiv F.\)

Cases:

- \(P\) is True.
  - LHS: \(T \land (Q \lor R) \equiv (Q \lor R).\)
  - RHS: \((T \land Q) \lor (T \land R) \equiv (Q \lor R).\)

- \(P\) is False.
  - LHS: \(F \land (Q \lor R) \equiv F.\)
  - RHS: \((F \land Q) \lor (F \land R) \equiv (F \lor F) \equiv F.\)

\[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)? \]

Simplify: \(T \lor Q \equiv T, F \lor Q \equiv Q.\)

Foil 1:
\[ (A \lor B) \land (C \lor D) \equiv (A \land C) \lor (A \land D) \lor (B \land C) \lor (B \land D)? \]

Foil 2:
\[ (A \land B) \lor (C \land D) \equiv (A \lor C) \land (A \lor D) \land (B \lor C) \land (B \lor D)? \]
Implication.

\[ P \implies Q \] interpreted as

"If \( P \), then \( Q \)."
Implication.

$P \implies Q$ interpreted as

If $P$, then $Q$. 

Examples:

Statement: If you stand in the rain, then you'll get wet.
$P = \text{"you stand in the rain"}$
$Q = \text{"you will get wet"}$

Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Statement: If a right triangle has sidelengths $a, b, c$, then $a^2 + b^2 = c^2$.

$P = \text{"a right triangle has sidelengths} \ a, b, c",$
$Q = \text{"}a^2 + b^2 = c^2\text{"}$. 
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
\( P \) = "you stand in the rain"
\( Q \) = "you will get wet"
Can conclude: "you'll get wet."

Statement: If a right triangle has sidelengths \( a \), \( b \), \( c \), then \( a^2 + b^2 = c^2 \).
\( P \) = "a right triangle has sidelengths \( a \), \( b \), \( c \)"
\( Q \) = "\( a^2 + b^2 = c^2 \)"

Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:
**Implication.**

$P \implies Q$ interpreted as

If $P$, then $Q$.

True Statements: $P, P \implies Q$.
Conclude: $Q$ is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
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If $P$, then $Q$.

True Statements: $P, P \implies Q$.
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Examples:

Statement: If you stand in the rain, then you’ll get wet.
$P = \text{“you stand in the rain”}$
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, \ P \implies Q \).
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Examples:

Statement: If you stand in the rain, then you’ll get wet.
\( P = \) “you stand in the rain”
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\[ P \implies Q \] interpreted as

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True Statements: \( P, P \implies Q \).
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Examples:

Statement: If you stand in the rain, then you’ll get wet.
\[ P = "you stand in the rain" \]
\[ Q = "you will get wet" \]

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement: If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).
$P \implies Q$ interpreted as

If $P$, then $Q$.

True Statements: $P, P \implies Q$.
Conclude: $Q$ is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
   $P$ = “you stand in the rain”
   $Q$ = “you will get wet”

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement: If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

$P$ = “a right triangle has sidelengths $a \leq b \leq c$”,
Implication.

\[ P \implies Q \] interpreted as

If \( P \), then \( Q \).

True Statements: \( P, P \implies Q \).
Conclude: \( Q \) is true.

Examples:

Statement: If you stand in the rain, then you’ll get wet.
\[ P = \text{“you stand in the rain”} \]
\[ Q = \text{“you will get wet”} \]

Statement: “Stand in the rain”
Can conclude: “you’ll get wet.”

Statement: If a right triangle has sidelengths \( a \leq b \leq c \), then \( a^2 + b^2 = c^2 \).

\[ P = \text{“a right triangle has sidelengths } a \leq b \leq c\text{”}, \]
\[ Q = \text{“} a^2 + b^2 = c^2 \text{”}. \]
The statement “$P \implies Q$”
The statement “$P \implies Q$” only is \textbf{False} if $P$ is \textbf{True} and $Q$ is \textbf{False}.
The statement “\( P \implies Q \)”

- only is False if \( P \) is True and \( Q \) is False.
- False implies nothing

If chemical plant pollutes river, fish die.

If fish die, did chemical plant pollute river?

Not necessarily.

\( P \implies Q \) and \( Q \) are True does not mean \( P \) is True.

Be careful!

Instead we have: \( P \implies Q \) and \( P \) are True does mean \( Q \) is True.

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?

\( (P \implies Q) \land P \implies Q \).
The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
$P$ False means
The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing

P False means $Q$ can be True
The statement “$P \implies Q$”

only is \textbf{False} if $P$ is \textbf{True} and $Q$ is \textbf{False}.

False implies nothing

$P$ \textbf{False} means $Q$ can be \textbf{True} or \textbf{False}
The statement “\( P \implies Q \)” only is \textbf{False} if \( P \) is \textbf{True} and \( Q \) is \textbf{False}.

False implies nothing

\( P \) \textbf{False} means \( Q \) can be \textbf{True} or \textbf{False}

Anything implies true.
The statement “$P \implies Q$”

only is \textbf{False} if $P$ is \textbf{True} and $Q$ is \textbf{False}.

False implies nothing
P \textbf{False} means $Q$ can be \textbf{True} or \textbf{False}
Anything implies true.
$P$ can be \textbf{True} or \textbf{False} when
The statement \( P \implies Q \)

only is \textbf{False} if \( P \) is \textbf{True} and \( Q \) is \textbf{False}.

False implies nothing
P \textbf{False} means \( Q \) can be \textbf{True} or \textbf{False}
Anything implies true.
P \textbf{can be} \textbf{True} or \textbf{False} when \( Q \) is \textbf{True}
The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing

$P$ False means $Q$ can be True or False

Anything implies true.

$P$ can be True or False when $Q$ is True

If chemical plant pollutes river, fish die.
The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing

$P$ **False** means $Q$ can be **True** or **False**

Anything implies true.

$P$ can be **True** or **False** when $Q$ is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?
The statement “\( P \implies Q \)”

only is **False** if \( P \) is **True** and \( Q \) is **False**.

False implies nothing
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Non-Consequences/consequences of Implication

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False implies nothing
P False means $Q$ can be True or False
Anything implies true.
P can be True or False when $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True
Non-Consequences/consequences of Implication

The statement "\( P \implies Q \)"

only is \textbf{False} if \( P \) is \textbf{True} and \( Q \) is \textbf{False}.

False implies nothing

\( P \) \textbf{False} means \( Q \) can be \textbf{True} or \textbf{False}

Anything implies \textbf{true}.

\( P \) can be \textbf{True} or \textbf{False} when \( Q \) is \textbf{True}

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

\( P \implies Q \) and \( Q \) are \textbf{True} does not mean \( P \) is \textbf{True}

Be careful!
Non-Consequences/consequences of Implication

The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing

- $P$ **False** means $Q$ can be **True** or **False**
- Anything implies true.
- $P$ can be **True** or **False** when $Q$ is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

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Be careful!

Instead we have:
The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing

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Anything implies true.

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Non-Consequences/consequences of Implication

The statement “\( P \implies Q \)”

only is **False** if \( P \) is **True** and \( Q \) is **False**.

False implies nothing

\( P \text{ False} \) means \( Q \) can be **True** or **False**

Anything implies true.

\( P \text{ can be True or False when } Q \text{ is True} \)

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

\( P \implies Q \) and \( Q \) are **True** does not mean \( P \) is **True**

Be careful!

Instead we have:

\( P \implies Q \) and \( P \) are **True** does **not** mean \( Q \) is **True**.

The chemical plant pollutes river.
The statement “$P \implies Q$”

only is **False** if $P$ is **True** and $Q$ is **False**.

False implies nothing
P **False** means $Q$ can be **True** or **False**
Anything implies true.
$P$ can be **True** or **False** when $Q$ is **True**

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

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Be careful!

Instead we have:
$P \implies Q$ and $P$ are **True** *does* mean $Q$ is **True**.

The chemical plant pollutes river. Can we conclude fish die?
The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means $Q$ can be True or False
Anything implies true.
P can be True or False when $Q$ is True

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are True does not mean $P$ is True

Be careful!

Instead we have:

$P \implies Q$ and $P$ are True does mean $Q$ is True.

The chemical plant pollutes river. Can we conclude fish die?
The statement “$P \implies Q$”

only is False if $P$ is True and $Q$ is False.

False implies nothing
P False means $Q$ can be True or False
Anything implies true.
$P$ can be True or False when $Q$ is True

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$P \implies Q$ and $P$ are True does mean $Q$ is True.

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?
The statement “$P \implies Q$” only is \textbf{False} if $P$ is \textbf{True} and $Q$ is \textbf{False}.

- False implies nothing
- $P$ \textbf{False} means $Q$ can be \textbf{True} or \textbf{False}
- Anything implies true.
- $P$ can be \textbf{True} or \textbf{False} when $Q$ is \textbf{True}

If chemical plant pollutes river, fish die.
If fish die, did chemical plant pollute river?

Not necessarily.

$P \implies Q$ and $Q$ are \textbf{True} does not mean $P$ is \textbf{True}

Be careful!

Instead we have:

$P \implies Q$ and $P$ are \textbf{True} \textit{does} mean $Q$ is \textbf{True}.

The chemical plant pollutes river. Can we conclude fish die?

Some Fun: use propositional formulas to describe implication?

$$((P \implies Q) \land P) \implies Q.$$
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
Implication and English.

\[ P \implies Q \]

- If \( P \), then \( Q \).
- \( Q \) if \( P \).
  Just reversing the order.

- \( P \) only if \( Q \).
  Remember if \( P \) is true then \( Q \) must be true.
  This suggests that \( P \) can only be true if \( Q \) is true.
  Since if \( Q \) is false \( P \) must have been false.

- \( P \) is sufficient for \( Q \).
  This means that proving \( P \) allows you to conclude that \( Q \) is true.

- \( Q \) is necessary for \( P \).
  For \( P \) to be true it is necessary that \( Q \) is true.
  Or if \( Q \) is false then we know that \( P \) is false.
Implication and English.

\[ P \implies Q \]

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These two propositional forms are logically equivalent!
Truth Table: implication.

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$\neg P \lor Q \equiv P \implies Q$.

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$\neg P \lor Q \equiv P \implies Q$.

These two propositional forms are logically equivalent!
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$. 
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.

If fish die the plant pollutes. Not logically equivalent!

Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$. (Logically Equivalent: $\iff$.)
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  - If the plant pollutes, fish die.
  - If the fish don’t die, the plant does not pollute.
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
  • If the fish don’t die, the plant does not pollute.
    (contrapositive)
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
  • If the fish don’t die, the plant does not pollute.
    (contrapositive)
  
  • If you stand in the rain, you get wet.
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
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    (contrapositive)

• If you stand in the rain, you get wet.
• If you did not stand in the rain, you did not get wet.
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
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  • If you stand in the rain, you get wet.
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    (not contrapositive!)

Definition: If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P$ (logically equivalent: $\equiv$).
Contrapositive, Converse

• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
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  • If you did not get wet, you did not stand in the rain.
Contrapositive, Converse

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Logically equivalent! Notation: $\equiv$. 
Contrapositive, Converse

- **Contrapositive of** $P \implies Q$ **is** $\neg Q \implies \neg P$.
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Logically equivalent! Notation: $\equiv$.

$P \implies Q$
Contrapositive, Converse

• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
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    (not contrapositive!)
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    (contrapositive.)

Logically equivalent! Notation: $\equiv$. 

\[
P \implies Q \equiv \neg P \lor Q
\]
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
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    (not contrapositive!)
  • If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

\[
P \implies Q \equiv \neg P \lor Q \equiv \neg(\neg Q) \lor \neg P
\]
• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
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  • If you stand in the rain, you get wet.
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    (not contrapositive!)
  • If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

\[ P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P. \]
Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
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    (contrapositive)
  - If you stand in the rain, you get wet.
  - If you did not stand in the rain, you did not get wet.
    (not contrapositive!)
  - If you did not get wet, you did not stand in the rain.
    (contrapositive.)

  Logically equivalent! Notation: $\equiv$.

  $P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P$.

- Converse of $P \implies Q$ is $Q \implies P$. 
Contrapositive, Converse

• Contrapositive of \( P \implies Q \) is \( \neg Q \implies \neg P \).
  • If the plant pollutes, fish die.
  • If the fish don’t die, the plant does not pollute.
    (contrapositive)

• If you stand in the rain, you get wet.
• If you did not stand in the rain, you did not get wet.
  (not contrapositive!)
• If you did not get wet, you did not stand in the rain.
  (contrapositive.)

Logically equivalent! Notation: \( \equiv \).
\[
P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.
\]

• Converse of \( P \implies Q \) is \( Q \implies P \).
If fish die the plant pollutes.
Contrapositive, Converse

• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  • If the plant pollutes, fish die.
  • If the fish don’t die, the plant does not pollute.
    (contrapositive)

• If you stand in the rain, you get wet.
  • If you did not stand in the rain, you did not get wet.
    (not contrapositive!) converse!
  • If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

$P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P$.

• Converse of $P \implies Q$ is $Q \implies P$.
  If fish die the plant pollutes.
Contrapositive, Converse

• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  
  • If the plant pollutes, fish die.
  • If the fish don’t die, the plant does not pollute.
    (contrapositive)
  
  • If you stand in the rain, you get wet.
  • If you did not stand in the rain, you did not get wet.
    (not contrapositive!) converse!
  • If you did not get wet, you did not stand in the rain.
    (contrapositive.)

Logically equivalent! Notation: $\equiv$.

$P \implies Q \equiv \neg P \lor Q \equiv \neg(\neg Q) \lor \neg P \equiv \neg Q \implies \neg P$.

• Converse of $P \implies Q$ is $Q \implies P$.
  If fish die the plant pollutes.
  Not logically equivalent!
Contrapositive, Converse

• Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.
  • If the plant pollutes, fish die.
  • If the fish don’t die, the plant does not pollute.
    (contrapositive)

  • If you stand in the rain, you get wet.
  • If you did not stand in the rain, you did not get wet.
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Logically equivalent! Notation: $\equiv$.

$P \implies Q \equiv \neg P \lor Q \equiv \neg (\neg Q) \lor \neg P \equiv \neg Q \implies \neg P.$

• Converse of $P \implies Q$ is $Q \implies P$.
  If fish die the plant pollutes.
  Not logically equivalent!

• **Definition:** If $P \implies Q$ and $Q \implies P$ is $P$ if and only if $Q$ or $P \iff Q$.
  (Logically Equivalent: $\iff$.)
Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
Propositions?

- \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
- \( x > 2 \)
Variables.

Propositions?

• \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).
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• \( n \) is even and the sum of two primes
Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $x > 2$
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No. They have a free variable.
Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
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No. They have a free variable.

We call them predicates, e.g., $Q(x) = “x$ is even”
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Same as boolean valued functions from 61A or 61AS!
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Same as boolean valued functions from 61A or 61AS!

• $P(n) = \text{"} \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{"}$
• $R(x) = \text{"} x > 2 \text{"}$
Propositions?

- $\Sigma_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $x > 2$
- $n$ is even and the sum of two primes

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- $P(n) = “\Sigma_{i=1}^{n} i = \frac{n(n+1)}{2}.”$
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• Remember Wason’s experiment!
  $F(x) = \text{“}\text{Person } x \text{ flew}\text{”}$
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Variables.

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Next:
Variables.

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  If person \( x \) goes to Chicago then person \( x \) flew.

Next: Statements about boolean valued functions!!
There exists quantifier:

$(\exists x \in S)(P(x))$ means "There exists an $x$ in $S$ where $P(x)$ is true."

For example:

$(\exists x \in N)(x = x^2)$

Equivalent to "$(0 = 0) \land (1 = 1) \land (2 = 4) \land ...$"

Much shorter to use a quantifier!

For all quantifier:

$(\forall x \in S)(P(x))$ means "For all $x$ in $S$, we have $P(x)$ is True."

Examples:

"Adding 1 makes a bigger number."

$(\forall x \in N)(x + 1 > x)"

"the square of a number is always non-negative"

$(\forall x \in N)(x^2 \geq 0)"

Wait!

What is $N$?
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\((\exists x \in S)(P(x))\) means "There exists an \(x\) in \(S\) where \(P(x)\) is true."
Quantifiers.

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For example:

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Equivalent to "(0 = 0)"
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Equivalent to “$0 = 0 \lor 1 = 1$”
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Equivalent to "$(0 = 0) \lor (1 = 1) \lor (2 = 4)$"
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Wait! What is \(\mathbb{N}\)?
Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$”

Proposition has universe:
Proposition: “For all natural numbers $n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.”

Proposition has universe: “the natural numbers”.

Universe examples include...
- $\mathbb{N} = \{0, 1, 2, \ldots\}$ (natural numbers)
- $\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ (integers)
- $\mathbb{Z}^+ = \{0, 1, 2, \ldots\}$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{\text{Alice}, \text{Bob}, \text{Charlie}, \text{Donna}\}$
- See note 0 for more!
Proposition: “For all natural numbers \( n \), \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).”

Proposition has **universe**: “the natural numbers”.

Universe examples include..

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- See note 0 for more!
Theory:

“If a person travels to Chicago, he/she flies.”

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

P(x) = “Person x went to Chicago.”

Q(x) = “Person x flew”

Statement/theory:

8 x 2 {A, B, C, D}, P(A) = False. Do we care about Q(A)?

No.

P(A) = Q(A), when P(A) is False, Q(A) can be anything.

Q(B) = False. Do we care about P(B)?

Yes. P(B) = Q(B) ⌃ ¬Q(B) = ¬P(B).

So P(Bob) must be False.

P(C) = True. Do we care about P(C)?

Yes. P(C) = Q(C) means Q(C) must be true.

Q(D) = True. Do we care about P(D)?

No. P(D) = Q(D) holds whatever P(D) is when Q(D) is true.

Only have to turn over cards for Bob and Charlie.
Theory:
“If a person travels to Chicago, he/she flies.”

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

Statement/theory:

\[
P(A) = \text{“Person A went to Chicago.”}
\]

\[
Q(x) = \text{“Person x flew.”}
\]

\[
\{A, B, C, D\}, P(A) = \text{False}.
\]

Do we care about \(Q(A)\)?
No.

\[
P(B) = \text{false, so } Q(B) = \text{false.}
\]

Do we care about \(P(B)\)?
Yes.

\[
P(C) = \text{true, so } Q(C) = \text{true.}
\]

Do we care about \(P(C)\)?
Yes.

\[
P(D) = \text{true, so } Q(D) = \text{true.}
\]

Do we care about \(P(D)\)?
No.

Only have to turn over cards for Bob and Charlie.
Theory:
“If a person travels to Chicago, he/she flies.”
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Statement/theory: \( \forall x \in \{A, B, C, D\}, P(x) \)
Theory:
“If a person travels to Chicago, he/she flies.”

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Statement/theory: \( \forall x \in \{A, B, C, D\}, P(x) \implies Q(x) \)

Only have to turn over cards for Bob and Charlie.
Theory:
“If a person travels to Chicago, he/she flies.”

Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.

Which cards do you need to flip to test the theory?

\[ P(x) = \text{“Person x went to Chicago.”} \quad Q(x) = \text{“Person x flew”} \]

Statement/theory: \( \forall x \in \{A, B, C, D\}, P(x) \implies Q(x) \)

\( P(A) = \text{False} \).
Theory:
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\( Q(B) = \text{False} \). Do we care about \( P(B) \)?
Yes. \( P(B) \implies Q(B) \iff \neg Q(B) \implies \neg P(B) \).
So \( P(\text{Bob}) \) must be False.
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\( Q(B) = \text{False} \). Do we care about \( P(B) \)?
Yes. \( P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B) \).
So \( P(\text{Bob}) \) must be False.

\( P(C) = \text{True} \).
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Yes. \( P(C) \implies Q(C) \) means \( Q(C) \) must be true.
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So \( P(Bob) \) must be \text{False}.

\( P(C) = \text{True} \). Do we care about \( P(C) \)?
Yes. \( P(C) \implies Q(C) \) means \( Q(C) \) must be true.

\( Q(D) = \text{True} \).
Theory:
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Theory:
“If a person travels to Chicago, he/she flies.”

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   Yes. \( P(B) \implies Q(B) \equiv \neg Q(B) \implies \neg P(B) \).
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Theory:
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Yes. \( P(C) \implies Q(C) \) means \( Q(C) \) must be true.

\( Q(D) = \text{True} \). Do we care about \( P(D) \)?
No. \( P(D) \implies Q(D) \) holds whatever \( P(D) \) is when \( Q(D) \) is true.
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Only have to turn over cards for Bob and Charlie.
More for all quantifiers examples.

- "Doubling a natural number always makes it larger"\[8x2\ N \geq x\] False
  
  Consider \(x = 0\) can fix statement...

- "Square of any natural number greater than 5 is greater than 25."\[8x2 \ N \geq x > 5 \Rightarrow x^2 > 25\]. Idea alert: restrict domain using implication. Note that we may omit universe if clear from context.
More for all quantifiers examples.

• “doubling a natural number always makes it larger”
More for all quantifiers examples.

• “doubling a natural number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x)$$
More for all quantifiers examples.

- “doubling a natural number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \quad \text{False}\]
• “doubling a natural number always makes it larger”

\[(\forall x \in N) (2x > x)\]  False \text{Consider} \ x = 0
• “doubling a natural number always makes it larger”

\[(\forall x \in \mathbb{N}) \, (2x > x) \quad \text{False} \quad \text{Consider } x = 0\]

Can fix statement...
More for all quantifiers examples.

• “doubling a natural number always makes it larger”

\[(\forall x \in N) (2x > x)\]  False  Consider  \(x = 0\)

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\[(\forall x \in N) \ (2x > x) \quad \text{False Consider } x = 0\]

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\[(\forall x \in N) \ (2x \geq x) \quad \text{True}\]
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Can fix statement...

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• “Square of any natural number greater than 5 is greater than 25.”
• “doubling a natural number always makes it larger”

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\[(\forall x \in N) \]
• “doubling a natural number always makes it larger”

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• “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in N)(x > 5)\]
More for all quantifiers examples.

• “doubling a natural number always makes it larger”

\[(\forall x \in N) (2x > x) \text{ False Consider } x = 0\]

Can fix statement...

\[(\forall x \in N) (2x \geq x) \text{ True}\]

• “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in N)(x > 5 \implies\)
More for all quantifiers examples.

• “doubling a natural number always makes it larger”

$$(\forall x \in \mathbb{N}) (2x > x) \quad \text{False} \quad \text{Consider } x = 0$$

Can fix statement...

$$(\forall x \in \mathbb{N}) (2x \geq x) \quad \text{True}$$

• “Square of any natural number greater than 5 is greater than 25.”

$$(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).$$
More for all quantifiers examples.

• “doubling a natural number always makes it larger”

\[(\forall x \in N) (2x > x)\] False \textbf{Consider} \(x = 0\)

Can fix statement...

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Idea alert:
• “doubling a natural number always makes it larger”

\[(\forall x \in \mathbb{N}) (2x > x) \] False Consider \( x = 0\)

Can fix statement...

\[(\forall x \in \mathbb{N}) (2x \geq x) \] True

• “Square of any natural number greater than 5 is greater than 25.”

\[(\forall x \in \mathbb{N})(x > 5 \implies x^2 > 25).\]

Idea alert: Restrict domain using implication.
• “doubling a natural number always makes it larger”

\((\forall x \in N)\ (2x > x)\) \quad \text{False Consider } x = 0

Can fix statement...

\((\forall x \in N)\ (2x \geq x)\) \quad \text{True}

• “Square of any natural number greater than 5 is greater than 25.”

\((\forall x \in N)(x > 5 \implies x^2 > 25).\)

Idea alert: Restrict domain using implication.

Note that we may omit universe if clear from context.
• In English: “there is a natural number that is the square of every natural number”.

Quantifiers..not commutative.
Quantifiers..not commutative.

• In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N)\]

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• In English: “there is a natural number that is the square of every natural number”.

\((\exists y \in N) (\forall x \in N)\)
• In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) \ (\forall x \in N) \ (y = x^2)\]
Quantifiers..not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\]  False
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\]  \text{False}

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- In English: “there is a natural number that is the square of every natural number”.

\[ (\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2) \quad \text{False} \]

- In English: “the square of every natural number is a natural number.”

\[ (\forall x \in \mathbb{N}) \]
Quantifiers...not commutative.

- In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in \mathbb{N}) (\forall x \in \mathbb{N}) (y = x^2)\] False

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\[(\exists y \in N) (\forall x \in N) (y = x^2)\quad \text{False}\]

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Quantifiers...not commutative.

• In English: “there is a natural number that is the square of every natural number”.

\[(\exists y \in N) (\forall x \in N) (y = x^2)\] False

• In English: “the square of every natural number is a natural number.”

\[(\forall x \in N)(\exists y \in N) (y = x^2)\] True
Consider

\[ \neg (\forall x \in S) (P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is, \( \neg (\forall x \in S) (P(x)) \implies \exists x (x \notin S) (\neg P(x)) \).
Consider

\[ \neg (\forall x \in S)(P(x)), \]

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.
Consider

\neg (\forall x \in S)(P(x))

English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

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English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,

\[ \neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)). \]
Consider

$$\neg(\forall x \in S)(P(x)),$$

English: there is an \(x\) in \(S\) where \(P(x)\) does not hold.

That is,

$$\neg(\forall x \in S)(P(x)) \iff \exists(x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.
Consider

$$\neg (\forall x \in S)(P(x)),$$

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That is,

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**What we do in this course! We consider claims.**

**Claim:** $(\forall x) P(x)$
Consider

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English: there is an $x$ in $S$ where $P(x)$ does not hold.

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$$\neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)).$$

What we do in this course! We consider claims.

**Claim:** $(\forall x) P(x)$  “For all inputs $x$ the program works.”
Consider

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That is,

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What we do in this course! We consider claims.

**Claim:** $(\forall x) P(x)$ “For all inputs $x$ the program works.”

For False, find $x$, where $\neg P(x)$. 
Consider
\[ \neg(\forall x \in S)(P(x)), \]
English: there is an \( x \) in \( S \) where \( P(x) \) does not hold.

That is,
\[ \neg(\forall x \in S)(P(x)) \iff \exists (x \in S)(\neg P(x)). \]

What we do in this course! We consider claims.

**Claim:** \((\forall x) P(x)\) “For all inputs \( x \) the program works.”

For **False**, find \( x \), where \( \neg P(x) \).

Counterexample.
Consider

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- Counterexample.
- Bad input.
- Case that illustrates bug.

For **True**: prove claim. Next lectures...
Negation of exists.

Consider

\neg (\exists x \in S) (P(x))

English: means that for all \( x \) in \( S \), \( P(x) \) does not hold. That is, \( \neg (\exists x \in S) (P(x)) = \forall x \in S \neg P(x) \).
Negation of exists.

Consider

\[ \neg(\exists x \in S)(P(x)) \]
Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that for all $x$ in $S$, $P(x)$ does not hold.
Negation of exists.

Consider

$$\neg(\exists x \in S)(P(x))$$

English: means that for all $x$ in $S$, $P(x)$ does not hold.

That is,

$$\neg(\exists x \in S)(P(x)) \iff \forall (x \in S) \neg P(x).$$
Theorem: No three positive integers \( a, b, c \) satisfy the equation \( a^n + b^n = c^n \) for any integer \( n \) strictly larger than two.
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Which Theorem?
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Fermat’s Last Theorem!
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How to express this theorem using propositions?
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Which Theorem?

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How to express this theorem using propositions?

$(\forall n \in N)$;
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Which Theorem?

Fermat’s Last Theorem!

How to express this theorem using propositions?

$(\forall n \in N); \neg (\exists a, b, c \in N);$
Theorem: No three positive integers $a, b, c$ satisfy the equation $a^n + b^n = c^b$ for any integer $n$ strictly larger than two.

Which Theorem?

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How to express this theorem using propositions?

$(\forall n \in N); \neg(\exists a, b, c \in N);(n \geq 3 \implies a^n + b^n = c^n)$
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Which Theorem?

Fermat’s Last Theorem!

How to express this theorem using propositions?

$$(\forall n \in N); \neg(\exists a, b, c \in N);(n \geq 3 \implies a^n + b^n = c^n)$$

Using implication to state edge case restrictions (for any integer strictly greater than two)
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Which Theorem?

Fermat’s Last Theorem!

How to express this theorem using propositions?

\[(\forall n \in \mathbb{N}); \neg(\exists a, b, c \in \mathbb{N});(n \geq 3 \implies a^n + b^n = c^n)\]

Using implication to state edge case restrictions (for any integer strictly greater than two)

DeMorgan Restatement:
Theorem: No three positive integers $a, b, c$ satisfy the equation $a^n + b^n = c^n$ for any integer $n$ strictly larger than two.

Which Theorem?

Fermat’s Last Theorem!

How to express this theorem using propositions?

$$(\forall n \in N); \neg(\exists a, b, c \in N);(n \geq 3 \implies a^n + b^n = c^n)$$

Using implication to state edge case restrictions (for any integer strictly greater than two)

DeMorgan Restatement:

Theorem: $\neg(\exists n \in N) (\exists a, b, c \in N) (n \geq 3 \implies a^n + b^n = c^n)$
Propositions are statements that are true or false.
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land$, $\lor$, $\neg$.

DeMorgan's Laws: "Flip and Distribute negation"

$\neg (P \lor Q)$ is equivalent to $\neg P \land \neg Q$.

$\neg \forall x P(x)$ is equivalent to $\exists x \neg P(x)$.

$\neg \exists x P(x)$ is equivalent to $\forall x \neg P(x)$.
Propositions are statements that are true or false.

Propositional forms use $\land$, $\lor$, $\neg$.

Propositional forms correspond to truth tables.
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Propositional forms use $\land, \lor, \neg$.

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Logical equivalence of forms means same truth tables.
Summary.

Propositions are statements that are true or false.

Propositional forms use \( \land, \lor, \neg \).

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: \( P \implies Q \)
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Propositional forms use $\land$, $\lor$, $\neg$.

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Implication: $P \implies Q \iff \neg P \lor Q$. 

DeMorgan's Laws: "Flip and Distribute negation"

$\neg (P \land Q) \iff \neg P \lor \neg Q$

$\neg \forall x P(x) \iff \exists x \neg P(x)$

$\neg \exists x P(x) \iff \forall x \neg P(x)$. 

Next Time: proofs!
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land$, $\lor$, $\neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$
Propositions are statements that are true or false.

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Implication: $P \implies Q \iff \neg P \lor Q$.

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Converse: $Q \implies P$
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Predicates: Statements with “free” variables.
Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

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Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x)$, $\exists y \ Q(y)$
Propositions are statements that are true or false.

Propositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \vee Q$.

Contrapositive: $\neg Q \implies \neg P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \ \exists y \ Q(y)$

Now can state theorems!
Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \neg P \lor Q$.

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Now can state theorems! And disprove false ones!
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DeMorgans Laws: “Flip and Distribute negation”
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DeMorgans Laws: “Flip and Distribute negation”

$\neg (P \lor Q) \iff (\neg P \land \neg Q)$
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Summary.

Propositions are statements that are true or false.

Propositional forms use $\land, \lor, \lnot$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

Implication: $P \implies Q \iff \lnot P \lor Q$.

Contrapositive: $\lnot Q \implies \lnot P$
Converse: $Q \implies P$

Predicates: Statements with “free” variables.

Quantifiers: $\forall x \ P(x), \exists y \ Q(y)$

Now can state theorems! And disprove false ones!

DeMorgan’s Laws: “Flip and Distribute negation”

$\lnot(P \lor Q) \iff (\lnot P \land \lnot Q)$
$\lnot\forall x \ P(x) \iff \exists x \ \lnot P(x)$.
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Propositions are statements that are true or false.

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