CS 70 Discrete Mathematics and Probability Theory
Summer 2016 Dinh, Psomas, and Ye Discussion 6C

1. **Markov Chain Terminology**

   In this question, we will walk you through terms related to Markov chains. Keep in mind the following theorems and ideas as you work.

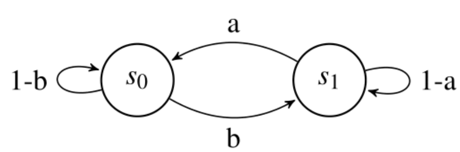
   (a) (Irreducibility) A Markov chain is irreducible if it can go from every state i to every other state j, possibly in multiple steps.

   (b) (Periodicity) $d(i) :=$ g.c.d$\{n > 0 \mid P^n(i,i) = Pr[X_n = i | X_0 = i] > 0\}, i \in \mathscr{X}$ where $d(i) = 1$ if and only if the Markov chain is aperiodic.

   (c) (Matrix Representation) Define the transition probability matrix P by filling entry i, j with probability $P(i, j)$, where $X_n = i, X_{n+1} = j$.

   (d) (Invariance) A distribution π is invariant for the transition probability matrix P if it satisfies the following balance equation: $\pi = \pi P$.

   Use the above theorems and the Markov chain below to answer the following questions.



   (a) For what values of a and b is the above Markov chain irreducible? Reducible?

   (b) For $a = 1, b = 1$, prove that the above Markov chain is periodic.

   (c) For $0 < a < 1, 0 < b < 1$, prove that the above Markov chain is aperiodic.

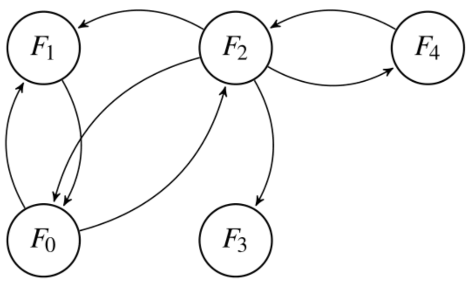
   (d) Construct a transition probability matrix using the above Markov chain.

   (e) Write the balance equations for this Markov chain. If $a = 0, b = 0$, is this distribution invariant?

2. **The Dwinelle Labyrinth**

You have decided to take a humanities class this semester, a French class to be specific. Instead of a final exam, your professor has issued a final paper. You must turn in this paper *before* noon to the professor's office on floor 3 in Dwinelle, and it's currently 11:48 a.m.

Let Dwinelle be modeled by the following Markov chain. Instead of rushing to turn it in, we will spend valuable time computing whether or not we *could have* made it. Suppose walking between floors takes 1 minute.



(a) Will you make it in time, if you choose a floor to transition to, uniformly at random? (i.e., If F_0 is the number of steps needed to get to F_3, is $E[F_0] < 12$?)

(b) Will you make it in time, if for every floor, you order all accessible floors and are twice as likely to take higher floors? (i.e., If you are considering 1, 2, or 3, you will take each with probabilities $\frac{1}{7}, \frac{2}{7}, \frac{4}{7}$, respectively.)