CS 70 Discrete Mathematics and Probability Theory
Summer 2016 Dinh, Psomas, and Ye Discussion 5C

1. **(Sanity Check!)** Derive Chebyshev's inequality using Markov's inequality for random variable X.

2. **(Coin Flips)**

(a) Suppose we flip a fair coin n times and we wish to understand the probability that we get at least $3n/4$ heads. Use Markov's inequality to come up with an upper bound for this probability.

(b) Use Markov's inequality to come up with a similar upper bound on the probability that the number of heads is at least n.

(c) Find the true probability that there are at least n heads in a sequence of n fair coin flips. Is the bound you derived in the previous part tight?

3. **Working with the Law of Large Numbers**

   (a) A fair coin is tossed and you win a prize if there are more than 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

   (b) A fair coin is tossed and you win a prize if there are more than 40% heads. Which is better: 10 tosses or 100 tosses? Explain.

   (c) A coin is tossed and you win a prize if there are between 40% and 60% heads. Which is better: 10 tosses or 100 tosses? Explain.

   (d) A coin is tossed and you win a prize if there are exactly 50% heads. Which is better: 10 tosses or 100 tosses? Explain.

4. **Vegas**

   On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

   (a) Given the results of your experiment, how should you estimate p?

   (b) How many people do you need to ask to be 95% sure that your answer is off by at most 0.05?