1. Counting and Probability Practice

1. A message source $M$ of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet \{0, 1, 2\}, and all such words are equally probable. What is the probability that $M$ produces a word that looks like a byte (i.e., no appearance of ‘2’)?

2. If five numbers are selected at random from the set \{1, 2, 3, \ldots, 20\}, what is the probability that their minimum is larger than 5? (A number can be chosen more than once.)

3. If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?

2. Balls in Bins: Independent? You have $k$ balls and $n$ bins labelled 1, 2, \ldots, $n$, where $n \geq 2$. You drop each ball uniformly at random into the bins.

1. What is the probability that bin $n$ is empty?

2. What is the probability that bin 1 is non-empty?

3. What is the probability that both bin 1 and bin $n$ are empty?

4. What is the probability that bin 1 is non-empty and bin $n$ is empty?

5. What is the probability that bin 1 is non-empty given that bin $n$ is empty?

3. Communication network

In the communication network shown below, link failures are independent, and each link has a probability of failure of $p$. Consider the physical situation before you write anything. A can communicate with B as long as they are connected by at least one path which contains only in-service links.
1. Given that exactly five links have failed, determine the probability that \( A \) can still communicate with \( B \).

2. Given that exactly five links have failed, determine the probability that either \( g \) or \( h \) (but not both) is still operating properly.

3. Given that \( a, d \) and \( h \) have failed (but no information about the information of the other links), determine the probability that \( A \) can communicate with \( B \).