

1. Warm-Up (Markov's Inequality)

Let X, Y, Z be non-negative random variables such that $Z = X + 10Y$.

- (a) Using Markov's inequality, upper bound the chance that X takes on a value higher than $k\mu$, where $\mu = \mathbb{E}(X)$.

- (b) If $\mathbb{E}(X) = 10$ and $\mathbb{E}(Y) = 4$, upper bound $\Pr[Z \geq 100]$.

- (c) If $\mathbb{E}(Z) = 100$ and $\mathbb{E}(Y) = 4$, upper bound $\Pr[X \geq 10]$.

- (d) What does this tell you about Markov's inequality?

- (e) Suppose $Y = (X - \mu)^2$, where $\mu = \mathbb{E}(X)$. Using Markov's inequality, upper bound the chance that Y takes on a value higher than α^2 .

2. Getting Warmer (Chebyshev)

Suppose we want to use a low-cost thermometer for measuring the weather. The thermometer has a standard deviation of 2 degrees from the actual temperature.

- (a) Upper bound the chance that the thermometer is more than 5 degrees from the actual temperature.

- (b) If we had two identical thermometers, bound the chance that at least one thermometer stays within 5 degrees of the actual temperature.
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- (c) If we had n identical thermometers, each with standard deviation σ degrees from the actual temperature, lower bound the chance that at least one thermometer stays within k degrees of the actual temperature.
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3. Summertime

You're a DJ for a jazz club and you have r songs, each of which lasts 5 minutes. You can't attend work tonight, so your assistant randomly selects songs (with replacement) before the live band arrives.

- (a) The live band arrives 2 hours after the club opens. What's the chance that at least one song is repeated?

- (b) What's the expected number of repeats? A repeat is when you hear a song on two different occasions. (Hint: think about collisions)

- (c) Let's define X to be the number of repeats. At what threshold of repeats c can you ensure that the probability of having more than c repeats is less than 10%? Assume you have the answer to part (b).

4. Hard Summer

We asked 500 people to independently rate this year's Hard Summer on a scale from 0 to 10. The poll average was an 8.5. Upper bound the variance of the poll. What's the probability that the poll is accurate to within an error of 0.4?