CS 70 Discrete Mathematics and Probability Theory
Summer 2015 Chung-Wei Lin Discussion 6A

1. Variance

Prove or give a counterexample: for any random variables X and Y, $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$.

2. Cookies

TAs are giving out cookies to students in section! To make it more interesting, we give exactly 10 cookies to a student with probability $\frac{1}{2}$, otherwise we give either 1, 2, or 3 cookies randomly with equal probability. What is the expected number of cookies you'd get? What is its variance?

3. Bernoulli and Binomial Distribution

A random variable X is called a Bernoulli random variable with parameter p if $X = 1$ with probability p and $X = 0$ with probability $1 - p$.

(a) Calculate $\mathbb{E}(X)$ and $\text{Var}(X)$.

(b) A Binomial random variable with parameters n and p is defined to be the sum of n independent, identically distributed Bernoulli random variables with parameter p. If Z is a Binomial random variable with parameters n and p, what are $\mathbb{E}(Z)$ and $\text{Var}(Z)$?

4. Will I Get My Package?

A deceitful delivery dude is out transporting n packages to n customers. Not only does he hand a random package to each customer, but he also opens a package before delivering with probability $\frac{1}{2}$ (independently of the choice of the package). Let X be the number of customers who receive their own packages unopened.

(a) Compute the expectation $\mathbb{E}(X)$.

(b) Calculate the probability that two particular customers i, j receive their own packages unopened.

(c) Compute $\text{Var}(X)$.