CS 70 Discrete Mathematics and Probability Theory
Summer 2015 Chung-Wei Lin Discussion 1B

1. **Propositions**

Write the following propositions using the notation covered in class. (Use $\mathbb{N}$ to denote the set of natural numbers and $\mathbb{Z}$ to denote the set of integers. Also write $P(n)$ for the proposition "n is odd".)

(a) For all natural numbers n, $2n$ is even.

(b) For all natural numbers n, n is odd if n^2 is odd.

(c) There is no integer solution to the equation $x^2 - y^2 = 10$.

Answer:

(a) $\forall\, n \in \mathbb{N},\ \neg P(2n)$.

(b) $\forall\, n \in \mathbb{N},\ P(n^2) \implies P(n)$.

(c) $\neg(\exists\, x, y \in \mathbb{Z},\ x^2 - y^2 = 10)$ or $\forall\, x, y \in \mathbb{Z},\ x^2 - y^2 \neq 10$.

2. **Truth Tables**

Use truth tables to prove the following equivalences, which are known as De Morgan's Laws.

(a) $\neg(A \vee B) \equiv \neg A \wedge \neg B$

(b) $\neg(A \wedge B) \equiv \neg A \vee \neg B$

Answer:

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$\neg A \wedge \neg B$	$A \wedge B$	$\neg(A \wedge B)$	$\neg A \vee \neg B$
T	T	F	F	T	F	F	T	F	F
T	F	F	T	T	F	F	F	T	T
F	T	T	F	T	F	F	F	T	T
F	F	T	T	F	T	T	F	T	T

3. Wason Selection Task

You are shown a set of four cards placed on a table, each of which has a number on one side and a colored patch on the other side. The visible faces of the cards show 3, 8, red, and brown. Which card(s) must you turn over in order to test the truth of the proposition that "if a card shows an even number on one face, then its opposite face is red?" You can check http://en.wikipedia.org/wiki/Wason_selection_task

(a) Write the proposition using the notation covered in class.

(b) Which card(s) must you turn over in order to test the truth of the proposition?

Answer:

(a) $P \implies Q$, where P denotes an even number on the numbered side, and Q denotes red on the colored side.

(b) 8 (P is true) and brown ($\neg Q$ is true). The proposition is always true with 3 ($\neg P$ is true) and red (Q is true).

4. Quantifiers

Which of the following propositions are true? Let $Q(n)$ be the proposition "n is divisible by 2." $\mathbb{N}$ denotes the set of natural numbers.

(a) $\exists k \in \mathbb{N}, Q(k) \wedge Q(k+1)$.

(b) $\forall k \in \mathbb{N}, Q(k) \implies Q(k^2)$.

(c) $\exists x \in \mathbb{N}, \neg(\exists y \in \mathbb{N}, y < x)$.

(d) $\exists x \in \mathbb{N}, \neg(\exists y \in \mathbb{N}, y \leq x)$.

Answer:

(a) False. One of k and $k+1$ must be odd.

(b) True. If k is even, then k^2 is also even.

(c) True. $x = 0$ and there is no natural number smaller than 0.

(d) False. The propositions means that "there exists a natural number x such that there does not exist a natural number y which is smaller than or equal to x," but there exists x itself satisfying it. Note that the proposition is equivalent to $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, y > x$. It is also equivalent to $\neg(\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y \leq x)$.