1 Throwing Balls into a Depth-Limited Bin

Say you want to throw n balls into n bins with depth $k - 1$ (they can fit $k - 1$ balls, after that the bins overflow). Suppose that n is a large number and $k = 0.1n$. You throw the balls randomly into the bins, but you would like it if they don't overflow. You feel that you might expect not too many balls to land in each bin, but you're not sure, so you decide to investigate the probability of a bin overflowing.

(a) Focus on the first bin. Get an upper bound the number of ways that you can throw the balls into the bins such that this bin overflows. Try giving an argument about the following strategy: select k balls to put in the first bin, and then throw the remaining balls randomly.

(b) Calculate an upper bound on the probability that the first bin will overflow.

(c) Upper bound the probability that some bin will overflow. [*Hint*: Use the union bound.]

(d) How does the above probability scale as n gets really large? [*Hint*: Use the union bound.]

2 Telebears

Lydia has just started her Telebears appointment. She needs to register for a marine science class and CS 70. There are no waitlists, and she can attempt to enroll once per day in either class or both. The Telebears system is strange and picky, so the probability of enrolling in the marine science class is p_1 and the probability of enrolling in CS 70 is p_2. The probabilities are independent. Let M be the number of attempts it takes to enroll in the marine science class, and C be the number of attempts it takes to enroll in CS 70.

(a) What distribution do M and C follow? Are M and C independent?

(b) For some integer $k \geq 1$, what is $\Pr[C \geq k]$?

(c) For some integer $k \geq 1$, what is the probability that she is enrolled in both classes before attempt k?

3 Fishy Computations

Use the Poisson distribution to answer these questions:

(a) Suppose that on average, a fisherman catches 20 salmon per week. What is the probability that he will catch exactly 7 salmon this week?

(b) Suppose that on average, you go to Fisherman's Wharf twice a year. What is the probability that you will go at most once in 2018?

(c) Suppose that in March, on average, there are 5.7 boats that sail in Laguna Beach per day. What is the probability there will be *at least* 3 boats sailing throughout the *next two days* in Laguna?