CS 70 Discrete Mathematics and Probability Theory Spring 2017 Satish Rao DIS Oa

1. Summations

- (a) What is $\sum_{i=4}^{7} i$?
- (b) Let $S = \{1, 2, 20\}$, what is $\sum_{x \in S} x^2$?
- (c) Let $S = \{1, 2, 4\}$, is $1 + \sum_{x \in S} x$ divisible by 2?
- (d) Let $S = \{0, ..., n\}$, is $1 + \sum_{x \in S} 2^x$ divisible by 2? $(n \ge 0.)$
- (e) Let $S = \{0, ..., n\}$, is $1 + \prod_{x \in S} 2^x$ divisible by 2? $(n \ge 0)$
- (f) Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$, what is $\sum_{x \in A \cup B} x + \prod_{x \in A \cap B} x$?
- (g) Let $A = \{1, 2, 3\}, B = \{2, 3, 4, 5\}, \text{ and } C = \{4, 5, 6\} \text{ what is } |(A \cap B) \cup (B \cap C)|$?

2. Writing in propositional logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

- (a) The square of a nonzero integer is positive.
- (b) There are no integer solutions to the equation $x^2 y^2 = 10$.
- (c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.
- (d) For any two distinct real numbers, we can find a rational number in between them.

3. Implication

Which of the following implications are true? Give a counterexample for each false assertion.

- (a) $\forall x, \forall y, P(x, y) \implies \forall y, \forall x, P(x, y).$
- (b) $\exists x, \exists y, P(x, y) \implies \exists y, \exists x, P(x, y).$
- (c) $\forall x, \exists y, P(x, y) \implies \exists y, \forall x, P(x, y).$
- (d) $\exists x, \forall y, P(x, y) \implies \forall y, \exists x, P(x, y).$