## 1. Summations

(a) What is $\sum_{i=4}^{7} i$ ?
(b) Let $S=\{1,2,20\}$, what is $\sum_{x \in S} x^{2}$ ?
(c) Let $S=\{1,2,4\}$, is $1+\sum_{x \in S} x$ divisible by 2 ?
(d) Let $S=\{0, \ldots, n\}$, is $1+\sum_{x \in S} 2^{x}$ divisible by 2 ? $(n \geq 0$.)
(e) Let $S=\{0, \ldots, n\}$, is $1+\Pi_{x \in S} 2^{x}$ divisible by 2 ? $(n \geq 0)$
(f) Let $A=\{1,2,3\}$ and $B=\{2,3,4,5\}$, what is $\sum_{x \in A \cup B} x+\Pi_{x \in A \cap B} x$ ?
(g) Let $A=\{1,2,3\}, B=\{2,3,4,5\}$, and $C=\{4,5,6\}$ what is $|(A \cap B) \cup(B \cap C)|$ ?

## 2. Writing in propositional logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.
(a) The square of a nonzero integer is positive.
(b) There are no integer solutions to the equation $x^{2}-y^{2}=10$.
(c) There is one and only one real solution to the equation $x^{3}+x+1=0$.
(d) For any two distinct real numbers, we can find a rational number in between them.

## 3. Implication

Which of the following implications are true? Give a counterexample for each false assertion.
(a) $\forall x, \forall y, P(x, y) \Longrightarrow \forall y, \forall x, P(x, y)$.
(b) $\exists x, \exists y, P(x, y) \Longrightarrow \exists y, \exists x, P(x, y)$.
(c) $\forall x, \exists y, P(x, y) \Longrightarrow \exists y, \forall x, P(x, y)$.
(d) $\exists x, \forall y, P(x, y) \Longrightarrow \forall y, \exists x, P(x, y)$.

