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Village with just 1 barber, all men clean-shaven.

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Case 1: It's the barber. Case 2: Somebody else.

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Cannot answer that question in either case!

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Cannot answer that question in either case! Paradox!!!

Naive Set Theory: Any definable collection is a set.

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$$\exists y \ \forall x \ (x \in y \iff P(x)) \tag{1}$$

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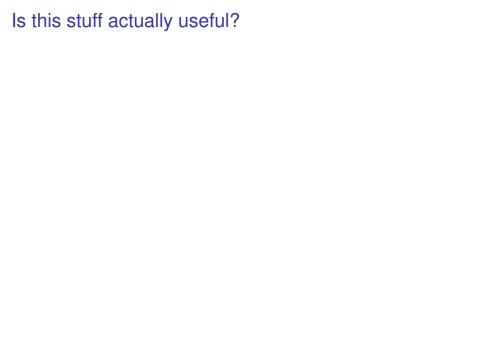
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Oops!



Verify that my program is correct!

Verify that my program is correct! Check that the compiler works!

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Check that the compiler works!

How about.. Check that the compiler terminates on a certain input.

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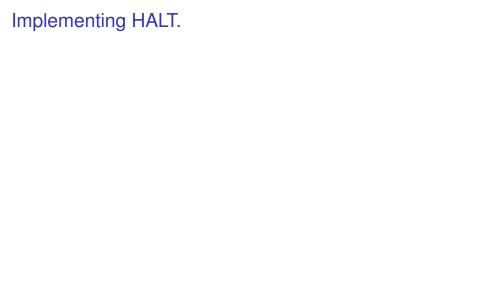
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Implementing HALT.

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Run P on I and check!

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How long do you wait?



HALT(P, I)

HALT(P, I)P - program

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Theorem: There is no program HALT.

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Theorem: There is no program HALT.

Proof Idea: Proof by contradiction, use self-reference.

Proof:

Proof: Assume there is a program $HALT(\cdot, \cdot)$.

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Turing(P)

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Does Turing(Turing) halt?

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Contradiction.

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Contradiction. Program HALT does not exist!

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Any program is a fixed length string.

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	P_1	P_2	P_3	• • • •
P ₁ P ₂ P ₃	H L L	H L H	L H H	
÷	:	:	:	٠.

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	P_1	P_2	P_3	• • • •
P ₁ P ₂ P ₃	Н	Н	L	
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Halt(P,P) - diagonal.				

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P.	Н	Н	ı	
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- an (infinite) tape with characters

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Now that's a computer!

Church proved an equivalent theorem. (Previously.)

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....natural number!!!!

Computer Programs are interesting objects.

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Proof Idea: Diagonalization.

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Program: Turing (or DIAGONAL) takes P.

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