## 1. Uniform Probability Space

Let $\Omega=\{1,2,3,4,5,6\}$ be a uniform probability space. Let also $X(\omega)$ and $Y(\omega)$, for $\omega \in \Omega$, be the random variables defined as follows:

Table 1: The random variables $X$ and $Y$.

| $\omega$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X(\omega)$ | 0 | 0 | 1 | 1 | 2 | 2 |
| $Y(\omega)$ | 0 | 2 | 3 | 5 | 2 | 0 |

(a) Calculate $V=L[Y \mid X]$;
(b) Calculate $W=E[Y \mid X]$;
(c) Calculate $E\left[(Y-V)^{2}\right]$;
(d) Calculate $E\left[(Y-W)^{2}\right]$.
[Hint: Recall that $L[Y \mid X]$ and $E[Y \mid X]$ are functions of $X$ and that you need to specify their value as a function of $X$.]

## 2. Number of Ones

In this problem, we will revisit dice-rolling, except with conditional expectation.
(a) If we roll a die until we see a 6 , how many ones should we expect to see?
(b) If we roll a die until we see a number greater than 3 , how many ones should we expect to see?

## 3. Marbles in a Bag

We have $r$ red marbles, $b$ blue marbles and $g$ green marbles to the same bag. If we sample balls, with replacement, until we get 3 red marbles (not necessarily consecutively), how many blue marbles should we expect to see?

