CS 70 Discrete Mathematics and Probability Theory Fall 2016 Seshia and Walrand DIS 11b

1. Uniform Probability Space

Let $\Omega = \{1, 2, 3, 4, 5, 6\}$ be a uniform probability space. Let also $X(\omega)$ and $Y(\omega)$, for $\omega \in \Omega$, be the random variables defined as follows:

ω	1	2	3	4	5	6
$X(\boldsymbol{\omega})$	0	0	1	1	2	2
$Y(\boldsymbol{\omega})$	0	2	3	5	2	0

Table 1: The random variables *X* and *Y*.

- (a) Calculate V = L[Y|X];
- (b) Calculate W = E[Y|X];
- (c) Calculate $E[(Y-V)^2]$;
- (d) Calculate $E[(Y W)^2]$.

[*Hint:* Recall that L[Y|X] and E[Y|X] are functions of X and that you need to specify their value as a function of X.]

2. Number of Ones

In this problem, we will revisit dice-rolling, except with conditional expectation.

- (a) If we roll a die until we see a 6, how many ones should we expect to see?
- (b) If we roll a die until we see a number greater than 3, how many ones should we expect to see?

3. Marbles in a Bag

We have r red marbles, b blue marbles and g green marbles to the same bag. If we sample balls, with replacement, until we get 3 red marbles (not necessarily consecutively), how many blue marbles should we expect to see?