CS 70 Discrete Mathematics and Probability Theory Fall 2016 Seshia and Walrand Discussion 9B

1. Rain and Wind

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in EECS70, you are curious to play around with these numbers. Find the probability that:

- (a) A given day is both windy and rainy.
- (b) A given day is rainy.
- (c) For a given pair of days, exactly one of the two days is rainy.
- (d) A given day that is non-rainy is also non-windy.

2. Balls and Bins

Throw n balls into n bins.

- (a) What is the probability that the first bin is empty?
- (b) What is the probability that the first *k* bins are empty?
- (c) What is the probability that the second bin is empty given that the first one is empty?
- (d) Are the events that "the first bin is empty" and "the first two bins are empty" independent?
- (e) Are the events that "the first bin is empty" and "the second bin is empty" independent?

3. Birthdays

Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded. (Assume there are 365 days in a year.)

- (a) What is the probability that after the first 3 people's birthdays are recorded, no match has occurred (i.e. each person has a unique birthday)?
- (b) What is the probability that the first 3 people all share the same birthday?
- (c) What is the probability that it takes more than 20 people for a match to occur?
- (d) What is the probability that it takes exactly 20 people for a match to occur?
- (e) Suppose instead that you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?