## 1. Communication network

In the communication network shown below, link failures are independent, and each link has a probability of failure of $p$. Consider the physical situation before you write anything. $A$ can communicate with $B$ as long as they are connected by at least one path which contains only in-service links.

(a) Given that exactly five links have failed, determine the probability that $A$ can still communicate with $B$.
(b) Given that exactly five links have failed, determine the probability that either $g$ or $h$ (but not both) is still operating properly.
(c) Given that $a, d$ and $h$ have failed (but no information about the information of the other links), determine the probability that $A$ can communicate with $B$.

## 2. Boy or Girl Paradox

You know Mr. Smith has two children, at least one of whom is a boy. Assume that gender is independent and uniformly distributed, so for any child, the probability that they are a boy is the same as the probability they are a girl, which is $\frac{1}{2}$.
(a) What is the probability that both children are boys?
(b) Now suppose you knock on Mr. Smith's front door and you are greeted by a boy who you correctly deduce to be Mr. Smith's son. What is the probability that he has two boys? Compare your answer to the answer in part (a).

## 3. Bayes Rule - Man Speaks Truth

(a) A man speaks the truth 3 out of 4 times. He flips a biased coin that comes up Heads $\frac{1}{3}$ of the time and reports that it is Heads. What is the probability it is Heads?
(b) A man speaks the truth 3 out of 4 times. He rolls a fair 6-sided dice and reports it comes up 6 . What is the probability it is really 6 ?

## 4. Disease diagnosis

You have a high fever and go to the doctor to identify the cause. $1 \%$ of the people have H1N1, $10 \%$ of the people have the flu, and $89 \%$ have neither. Assume that no person has both. Suppose that $100 \%$ of the H1N1 people have a high fever, $30 \%$ of the flu people have a high fever, and $2 \%$ of the people who have neither, have a high fever. Is it more likely that you have H1N1, the flu, or neither?

