CS 70 Discrete Mathematics and Probability Theory Fall 2016 Seshia and Walrand Discussion 8b

1. Communication network

In the communication network shown below, link failures are independent, and each link has a probability of failure of p. Consider the physical situation before you write anything. A can communicate with B as long as they are connected by at least one path which contains only in-service links.



- (a) Given that exactly five links have failed, determine the probability that *A* can still communicate with *B*.
- (b) Given that exactly five links have failed, determine the probability that either *g* or *h* (*but not both*) is still operating properly.
- (c) Given that *a*, *d* and *h* have failed (but no information about the information of the other links), determine the probability that *A* can communicate with *B*.

2. Boy or Girl Paradox

You know Mr. Smith has two children, at least one of whom is a boy. Assume that gender is independent and uniformly distributed, so for any child, the probability that they are a boy is the same as the probability they are a girl, which is $\frac{1}{2}$.

- (a) What is the probability that both children are boys?
- (b) Now suppose you knock on Mr. Smith's front door and you are greeted by a boy who you correctly deduce to be Mr. Smith's son. What is the probability that he has two boys? Compare your answer to the answer in part (a).

3. Bayes Rule - Man Speaks Truth

- (a) A man speaks the truth 3 out of 4 times. He flips a biased coin that comes up Heads $\frac{1}{3}$ of the time and reports that it is Heads. What is the probability it is Heads?
- (b) A man speaks the truth 3 out of 4 times. He rolls a fair 6-sided dice and reports it comes up 6. What is the probability it is really 6?

4. Disease diagnosis

You have a high fever and go to the doctor to identify the cause. 1% of the people have H1N1, 10% of the people have the flu, and 89% have neither. Assume that no person has both. Suppose that 100% of the H1N1 people have a high fever, 30% of the flu people have a high fever, and 2% of the people who have neither, have a high fever. Is it more likely that you have H1N1, the flu, or neither?