1. **Euclid’s Algorithm** Euclid’s algorithm is a fast algorithm for computing the greatest common divisor of two integers. Here is an example. To compute $\gcd(16, 10)$:

\begin{align*}
16 &= 10 \times 1 + 6 & (1) \\
10 &= 6 \times 1 + 4 & \text{(notice this is a recursive call of } \gcd(10, 6)) & (2) \\
6 &= 4 \times 1 + 2 & \text{(notice this is a recursive call of } \gcd(6, 4)) & (3) \\
4 &= 2 \times 2 + 0 & \text{(notice this is a recursive call of } \gcd(4, 2)) & (4)
\end{align*}

So $\gcd(16, 10) = 2$, the last non-zero remainder. We can also back substitute to find $x, y$ such that

$$2 = 16x + 10y = \gcd(16, 10).$$

Here is how:

Rearrange (3) to get an expression for $\gcd(16, 10)$:

$$2 = 6 - 4 \times 1$$

rearrange (2) to get $4 = (10 - 6 \times 1)$

and substitute:

$$2 = 6 - (10 - 6 \times 1) \times 1$$

simplify:

$$2 = -10 + 6 \times 2$$

now rearrange (1) to get

$$6 = (16 - 10 \times 1)$$

and substitute:

$$2 = -10 + (16 - 10 \times 1) \times 2$$

simplify:

$$2 = 16 \times 2 - 10 \times 3$$

So $x = 2$ and $y = -3$.

Run Euclid’s algorithm for to determine the greatest common divisor for the following:

1. $a = 8, b = 22$
2. $a = 13, b = 21$

3. $a = 140, b = 38$