CS 70 FALL 2006 — DISCUSSION #11

D. GARMIRE, L. ORECCHIA & B. RUBINSTEIN

1. Administrivia

(1) Course Information
  - Homework #10 is due this Monday

2. Variance and Independent Random Variables

For *any* random variables X and Y, linearity of expectation tells us that $\mathbb{E}[X+Y] = \mathbb{E}[X] + \mathbb{E}[Y]$. This is not always the case for variance. The following exercise demonstrates a sufficient condition for this kind of relationship to hold.

Exercise 1. In the notes the fact that for independent r.v.'s X and Y, $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ was used. Prove this result formally.

Now consider the following example which demonstrates what can go wrong when independence does not hold.

Exercise 2. Consider r.v.'s X and $Y = X$. Find $\mathrm{Var}(X+Y)$. Is $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$?

3. Independent and Identically Distributed Random Variables

Recall Question 5 from Homework 9:

> *In a certain biological experiment, a piece of DNA consisting of a linear sequence (or string) of* 4001 *nucleotides is subjected to bombardment by various enzymes. The effect of the bombardment is to randomly cut the string between pairs of adjacent nucleotides: each of the* 4000 *possible cuts occurs independently and with probability* $\frac{1}{500}$*. What is the expected number of pieces into which the string is cut?*
>
> *Suppose that the cuts are no longer independent, but highly correlated, so that when a cut occurs in a particular place other cuts close by are much more likely. The probability of each individual cut remains* $\frac{1}{500}$*. Does the expected number of pieces increase, decrease, or stay the same?*

Exercise 3. How can the indicator r.v.'s defined when solving this question remain "identically distributed" when they are no longer independent?

4. Midterm Two

Discuss the solutions to the midterm.

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