1. Administrivia

(1) Course Information

- Homework #10 is due this Monday

2. Variance and Independent Random Variables

For any random variables $X$ and $Y$, linearity of expectation tells us that $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$. This is not always the case for variance. The following exercise demonstrates a sufficient condition for this kind of relationship to hold.

**Exercise 1.** In the notes the fact that for independent r.v.’s $X$ and $Y$, $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ was used. Prove this result formally.

Now consider the following example which demonstrates what can go wrong when independence does not hold.

**Exercise 2.** Consider r.v.’s $X$ and $Y = X$. Find $\text{Var}(X + Y)$. Is $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$?

3. Independent and Identically Distributed Random Variables

Recall Question 5 from Homework 9:

In a certain biological experiment, a piece of DNA consisting of a linear sequence (or string) of 4001 nucleotides is subjected to bombardment by various enzymes. The effect of the bombardment is to randomly cut the string between pairs of adjacent nucleotides: each of the 4000 possible cuts occurs independently and with probability $\frac{1}{500}$. What is the expected number of pieces into which the string is cut?

Suppose that the cuts are no longer independent, but highly correlated, so that when a cut occurs in a particular place other cuts close by are much more likely. The probability of each individual cut remains $\frac{1}{500}$. Does the expected number of pieces increase, decrease, or stay the same?

**Exercise 3.** How can the indicator r.v.’s defined when solving this question remain “identically distributed” when they are no longer independent?

4. Midterm Two

Discuss the solutions to the midterm.

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