

CS 70 FALL 2006 — DISCUSSION #11

D. GARMIRE, L. ORECCHIA & B. RUBINSTEIN

1. ADMINISTRIVIA

- (1) Course Information
 - Homework #10 is due this Monday

2. VARIANCE AND INDEPENDENT RANDOM VARIABLES

For *any* random variables X and Y , linearity of expectation tells us that $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$. This is not always the case for variance. The following exercise demonstrates a sufficient condition for this kind of relationship to hold.

Exercise 1. In the notes the fact that for independent r.v.'s X and Y , $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ was used. Prove this result formally.

Now consider the following example which demonstrates what can go wrong when independence does not hold.

Exercise 2. Consider r.v.'s X and $Y = X$. Find $\text{Var}(X + Y)$. Is $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$?

3. INDEPENDENT AND IDENTICALLY DISTRIBUTED RANDOM VARIABLES

Recall Question 5 from Homework 9:

In a certain biological experiment, a piece of DNA consisting of a linear sequence (or string) of 4001 nucleotides is subjected to bombardment by various enzymes. The effect of the bombardment is to randomly cut the string between pairs of adjacent nucleotides: each of the 4000 possible cuts occurs independently and with probability $\frac{1}{500}$. What is the expected number of pieces into which the string is cut?

Suppose that the cuts are no longer independent, but highly correlated, so that when a cut occurs in a particular place other cuts close by are much more likely. The probability of each individual cut remains $\frac{1}{500}$. Does the expected number of pieces increase, decrease, or stay the same?

Exercise 3. How can the indicator r.v.'s defined when solving this question remain “identically distributed” when they are no longer independent?

4. MIDTERM TWO

Discuss the solutions to the midterm.

Date: November 8, 2006.

The authors gratefully acknowledge Chris Crutchfield and Amir Kamil for the use of their previous notes, which form part of the basis for this handout.