

## Problem Set 13

### 1. Power Law Distributions

- (a) Show that the power law has infinite mean iff  $a \leq 2$ .
- (b) Show that the power law has infinite variance iff  $a \leq 3$ .

### 2. A paradox in conditional probability?

Here is some on-time arrival data for two airlines, A and B, into the airports of Los Angeles and Chicago. (Predictably, both airlines perform better in LA, which is subject to less flight congestion and less bad weather.)

	Airline A		Airline B	
	# flights	# on time	# flights	# on time
Los Angeles	600	534	200	188
Chicago	250	176	900	685

- (a) Which of the two airlines has a better chance of arriving on time into Los Angeles? What about Chicago?
- (b) Which of the two airlines has a better chance of arriving on time overall?
- (c) Do the results of parts (a) and (b) surprise you? Explain the apparent paradox, and interpret it in terms of conditional probabilities.

### 3. Cardinality

Recall that a real number which is not a rational number is called irrational. Are the irrationals countably infinite or uncountably infinite? Prove your answer. (You may use the fact that the reals are uncountable).

### 4. Bijections

Show that the set of real numbers  $\{x : 0 < x \leq 1\}$  has the same cardinality as the set of real numbers  $\{x : x \geq 1\}$ .

### 5. Undecidability

We say that two programs are equivalent if they give the same output on every input. Prove that it is impossible to write a computer program that takes as input two pieces of code  $code_1$  and  $code_2$  and tests whether the two programs are equivalent.