

Combinational Logic

a) How many different two-input logic gates are possible? How many n-input logic gates?
Hint: Think about the truth table.

b) Create an inverter (NOT gate) using only NAND gates (Hint: look at the truth tables for each).

c) Minimize the following boolean expressions and draw out logic circuits for them:

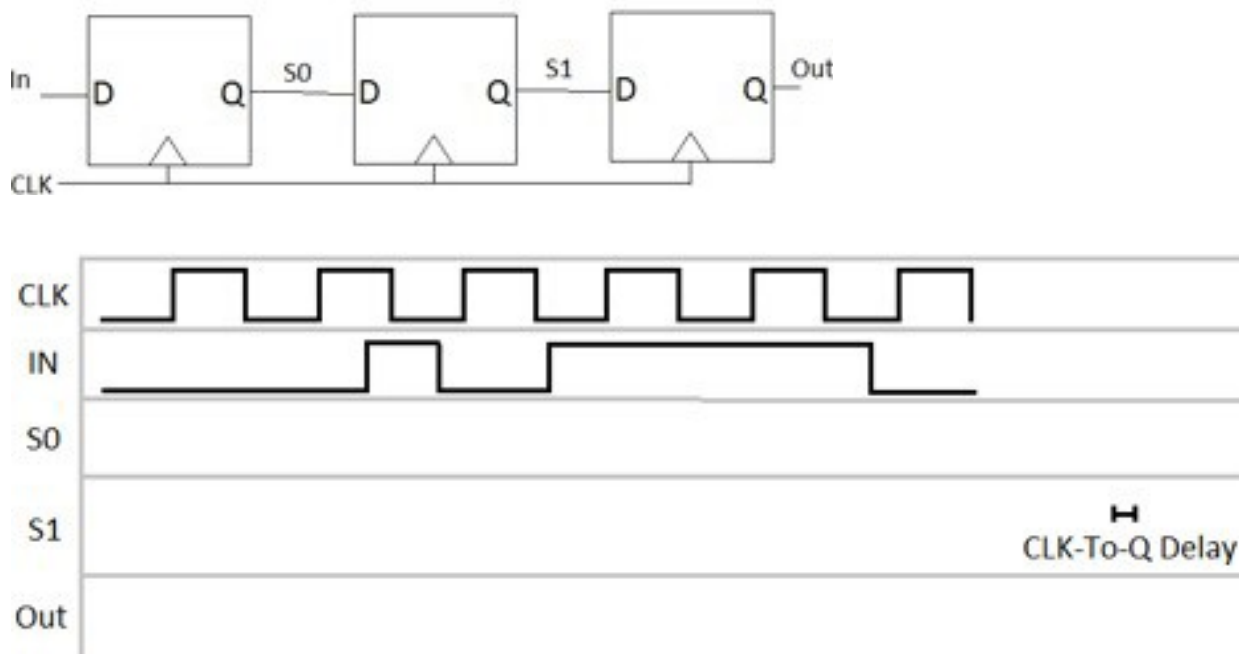
$$(A + B)(A + \bar{B})C \qquad \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + A\bar{B}C \qquad \bar{A}B + AB$$

State Elements

State elements provide a means of storing values, and controlling the flow of information in the circuit. One example is a D-type flip-flop shown below connected in series.

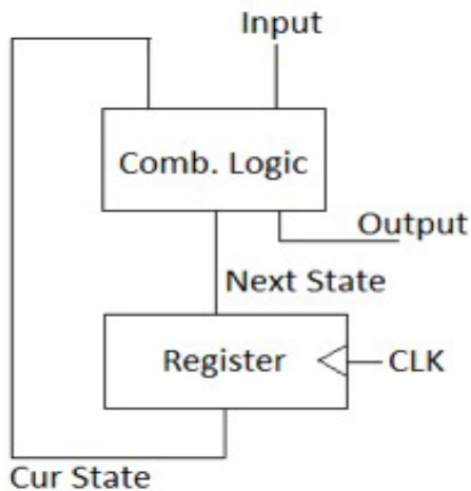
D is a single bit input, Q is a single bit output. On the rising edge of the clock, the value from D is copied to Q, after a small delay, known as the clk-to-q delay. To ensure the input value is read correctly, the input must be stable during the setup time before the edge, and during the hold time after the edge of the clock.

Fill out the timing diagrams for the circuit below.



Finite State Machines

FSMs can be useful computational models for logic circuits (as well as programs). There are a finite number of states the machine can be in and at each point in time, it is at a single state. The next state and output only depends on the current state and input value. Here's how FSMs can be implemented in hardware:



The register holds the current state (encoded as a particular combination of bits), and the combinational logic block is like a function that maps {current state, input} to {next state, output}.

Exercises

Draw a transition diagram for an FSM that can take in an input sequence one bit at a time, and after each input is received, output whether the number of 1s is divisible by 3. Write out the truth table that the combinational logic block must implement. From the truth table, you should be able to design a logic circuit.

Say you convert the FSM into a circuit like the diagram above. Given a setup time of s , a hold time of h , and a clk-to-q delay of cq , and a clock frequency of f , what is the most time the combinational logic can take?