

**Lecture 23 – Combinational Logic Blocks**



2010-03-17

Hello to Casey Holgado listening from Oklahoma State!

Lecturer SOE Dan Garcia

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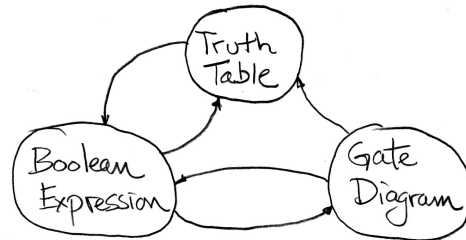
National Broadband Plan ⇒

The FCC delivered their plan to Congress yesterday, "with an ambitious agenda to connect by 2020 all corners of the nation & transform the economy and society with the communications network of the future". 100 Million households @ 100 Mbit/s!! The Obama administration put \$7.2 billion toward this...  
[broadband.gov](http://broadband.gov)



**Review**

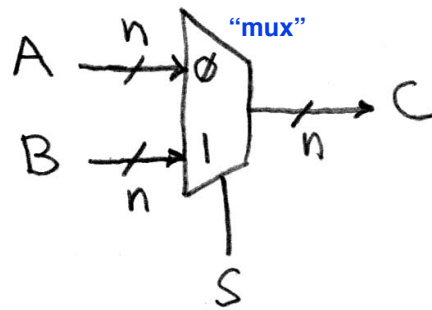
- Use this table and techniques we learned to transform from 1 to another



**Today**

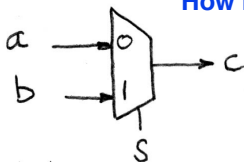
- Data Multiplexors
- Arithmetic and Logic Unit
- Adder/Subtractor

**Data Multiplexor (here 2-to-1, n-bit-wide)**



**N instances of 1-bit-wide mux**

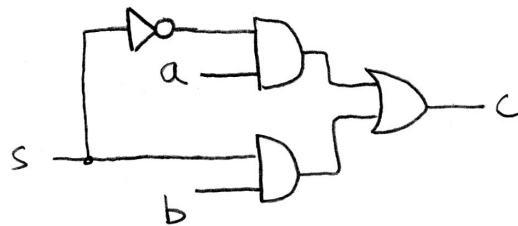
How many rows in TT?



$$\begin{aligned}
 c &= \bar{s}a\bar{b} + \bar{s}ab + s\bar{a}b + sab \\
 &= \bar{s}(a\bar{b} + ab) + s(\bar{a}b + ab) \\
 &= \bar{s}(a(\bar{b} + b)) + s((\bar{a} + a)b) \\
 &= \bar{s}(a(1)) + s((1)b) \\
 &= \bar{s}a + sb
 \end{aligned}$$

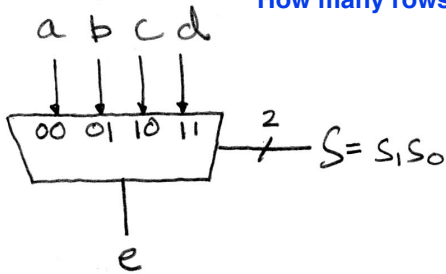
**How do we build a 1-bit-wide mux?**

$$\bar{s}a + sb$$



### 4-to-1 Multiplexor?

How many rows in TT?



$$e = \bar{s}_1\bar{s}_0a + \bar{s}_1s_0b + s_1\bar{s}_0c + s_1s_0d$$

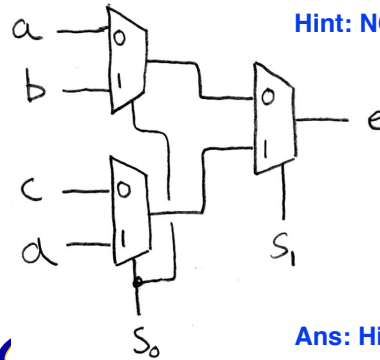


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### Is there any other way to do it?

Hint: NCAA tourney!



Ans: Hierarchically!



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### Administrivia

- Midterm discussion, moving forward
- If you want a regrade for your midterm, staple a paper with your explanation (clearly indicating on what question you want more points) to your exam and turn it in to your TA in lab this week
  - We'll regrade the exam and your score MIGHT go down...

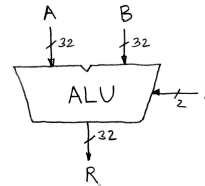


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### Arithmetic and Logic Unit

- Most processors contain a special logic block called "Arithmetic and Logic Unit" (ALU)
- We'll show you an easy one that does ADD, SUB, bitwise AND, bitwise OR



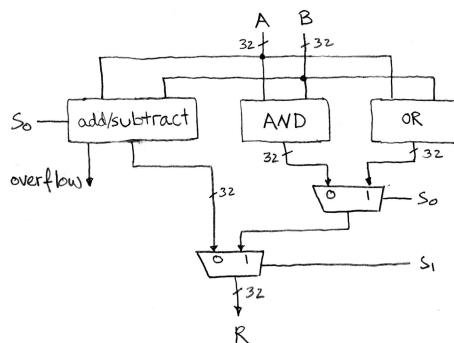
when  $S=00$ ,  $R=A+B$   
 when  $S=01$ ,  $R=A-B$   
 when  $S=10$ ,  $R=A \text{ AND } B$   
 when  $S=11$ ,  $R=A \text{ OR } B$



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### Our simple ALU



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### Adder/Subtractor Design -- how?

- Truth-table, then determine canonical form, then minimize and implement as we've seen before
- Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer



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### Adder/Subtractor – One-bit adder LSB...

	$a_3$	$a_2$	$a_1$	$a_0$
+	$b_3$	$b_2$	$b_1$	$b_0$
	$s_3$	$s_2$	$s_1$	$s_0$

$a_0$	$b_0$	$s_0$	$c_1$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s_0 =$$

$$c_1 =$$



### Adder/Subtractor – One-bit adder (1/2)...

	$a_3$	$a_2$	$a_1$	$a_0$
+	$b_3$	$b_2$	$b_1$	$b_0$
	$s_3$	$s_2$	$s_1$	$s_0$

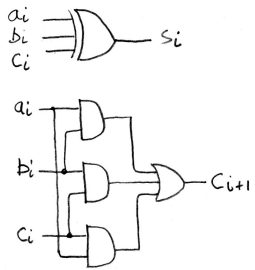
$a_i$	$b_i$	$c_i$	$s_i$	$c_{i+1}$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s_i =$$

$$c_{i+1} =$$



### Adder/Subtractor – One-bit adder (2/2)...

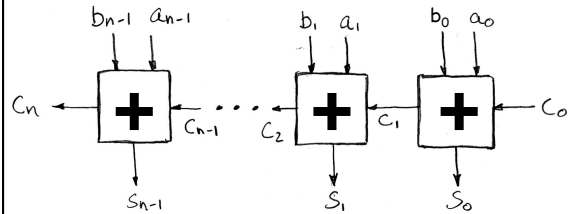


$$s_i = \text{XOR}(a_i, b_i, c_i)$$

$$c_{i+1} = \text{MAJ}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$$



### N 1-bit adders $\Rightarrow$ 1 N-bit adder



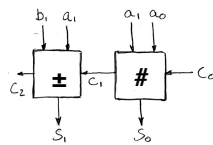
What about overflow?  
Overflow =  $c_n$ ?



### What about overflow?

#### • Consider a 2-bit signed # & overflow:

- 10 = -2 + -2 or -1
- 11 = -1 + -2 only
- 00 = 0 NOTHING!
- 01 = 1 + 1 only



#### • Highest adder

- $C_1 = \text{Carry-in} = C_{in}$ ,  $C_2 = \text{Carry-out} = C_{out}$
- No  $C_{out}$  or  $C_{in} \Rightarrow$  NO overflow!

What op? •  $C_{in}$ , and  $C_{out} \Rightarrow$  NO overflow!

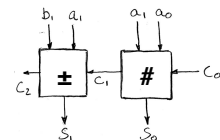
- $C_{in}$ , but no  $C_{out} \Rightarrow$  A, B both > 0, overflow!
- $C_{out}$ , but no  $C_{in} \Rightarrow$  A, B both < 0, overflow!



### What about overflow?

#### • Consider a 2-bit signed # & overflow:

- 10 = -2
- 11 = -1
- 00 = 0
- 01 = 1



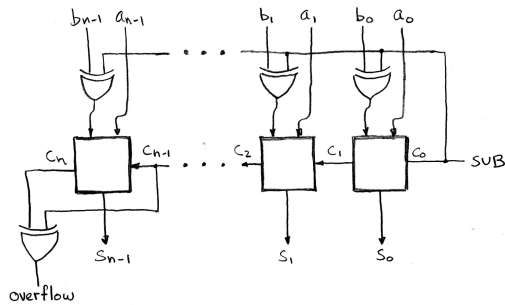
#### • Overflows when...

- $C_{in}$ , but no  $C_{out} \Rightarrow$  A, B both > 0, overflow!
- $C_{out}$ , but no  $C_{in} \Rightarrow$  A, B both < 0, overflow!

$$\text{overflow} = c_n \text{ XOR } c_{n-1}$$



### Extremely Clever Subtractor



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### Peer Instruction

- 1) Truth table for mux with 4-bits of signals has  $2^4$  rows
- 2) We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl

12
a) FF
b) FT
c) TF
d) TT



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### Peer Instruction Answer

- 1) Truth table for mux with 4-bits of signals controls 16 inputs, for a total of 20 inputs, so truth table is  $2^{20}$  rows...**FALSE**
  - 2) We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl ... **TRUE**
- 
- 1) Truth table for mux with 4-bits of signals is  $2^4$  rows long
  - 2) We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl

12
a) FF
b) FT
c) TF
d) TT



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### “And In conclusion...”

- Use muxes to select among input
  - S input bits selects  $2^S$  inputs
  - Each input can be n-bits wide, indep of S
- Can implement muxes hierarchically
- ALU can be implemented using a mux
  - Coupled with basic block elements
- N-bit adder-subtractor done using N 1-bit adders with XOR gates on input
  - XOR serves as conditional inverter



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