1 Best and Worst Case

For the following functions, provide asymptotic bounds for the best case and worst case runtimes in $\Theta(\cdot)$ notation.

(a) Give the best and worst case runtimes in terms of $M$ and $N$. Assume that $\text{slam()} \in \Theta(1)$ and returns a boolean.

```java
public void comeon(int M, int N) {
    int j = 0;
    for (int i = 0; i < N; i += 1) {
        for (; j < M; j += 1) {
            if (slam(i, j))
                break;
        }
    }
    for (int k = 0; k < 1000 * N; k += 1) {
        System.out.println("space jam");
    }
}
```

(b) Extra: Give the best case and worst case runtimes for $\text{find}$ in terms of $N$, where $N$ is the length of the input array $\text{arr}$.

```java
public static boolean find(int tgt, int[] arr) {
    int N = arr.length;
    return find(tgt, arr, 0, N);
}

private static boolean find(int tgt, int[] arr, int lo, int hi) {
    if (lo == hi || lo + 1 == hi) {
        return arr[lo] == tgt;
    }
    int mid = (lo + hi) / 2;
    for (int i = 0; i < mid; i += 1) {
        System.out.println(arr[i]);
    }
    return arr[mid] == tgt || find(tgt, arr, lo, mid) || find(tgt, arr, mid, hi);
}
```
2 Best and Worst Case with Recursion

For the following recursive functions, provide asymptotic bounds for the best case and worst case runtimes in \( \Theta(\cdot) \) notation.

(a) Give the runtime in terms of \( N \).

```java
public void andslam(int N) {
    if (N > 0) {
        for (int i = 0; i < N; i += 1) {
            for (int j = 1; j < 1024; j *= 2) {
                System.out.println(i + j);
            }
        }
        andslam(N / 2);
    }
}
```

(b) Give the runtime for \( \text{andwelcome(arr, 0, N) \text{ in terms of } N} \), where \( N \) is the length of the input array \( \text{arr} \). \( \text{Math.random()} \) returns a double with a value from the range [0,1).

```java
public static void andwelcome(int[] arr, int low, int high) {
    System.out.print("[ ");
    for (int i = low; i < high; i += 1) {
        System.out.print("loyal ");
    }
    System.out.println("]");
    if (high - low > 1) {
        double coin = Math.random();
        if (coin > 0.5) {
            andwelcome(arr, low, low + (high - low) / 2);
        } else {
            andwelcome(arr, low, low + (high - low) / 2);
            andwelcome(arr, low + (high - low) / 2, high);
        }
    }
}
```
(c) Give the runtime in terms of $N$.

```java
public int tothe(int N) {
    if (N <= 1) {
        return N;
    }
    return tothe(N - 1) + tothe(N - 1) + tothe(N - 1);
}
```

(d) *Extra:* Give the runtime in terms of $N$.

```java
public static void spacejam(int N) {
    if (N == 1) {
        return;
    }
    for (int i = 0; i < N; i += 1) {
        spacejam(N-1);
    }
}
```
3 Hey you watchu gon do?

For each example below, there are two algorithms solving the same problem. Given the asymptotic runtimes for each, is one of the algorithms **guaranteed** to be faster? If so, which? And if neither is always faster, explain why. Assume the algorithms have very large input (i.e. \(N\) is very large).

(a) Algorithm 1: \(\Theta(N)\), Algorithm 2: \(\Theta(N^2)\)

(b) Algorithm 1: \(\Omega(N)\), Algorithm 2: \(\Omega(N^2)\)

(c) Algorithm 1: \(O(N)\), Algorithm 2: \(O(N^2)\)

(d) Algorithm 1: \(\Theta(N^2)\), Algorithm 2: \(O(\log N)\)

(e) Algorithm 1: \(O(N \log N)\), Algorithm 2: \(\Omega(N \log N)\)

Why do we need to assume that \(N\) is large?