Administrivia

- Please make sure you have obtained a Unix account. If you are a concurrent enrollment student not yet on our lists, please tell a TA so that we can have you added to those eligible to receive an account.

- If you did not complete Lab #1, please try to do so over the weekend (usually, they are due Friday midnight). It is especially important to set up your central reppository.

- If you decide not to take this course after all, please tell CalCentral ASAP, so that we can adjust the waiting list accordingly.

- HW #0 now up; due next Friday at midnight. You get credit for any submission, but we suggest you give the problems a serious try.
Lecture #2: Let's Write a Program: Prime Numbers

**Problem:** want java Primes $U$ to print prime numbers through $U$.

*You type:* java Primes 101

*It types:* 2 3 5 7 11 13 17 19 23 29
            31 37 41 43 47 53 59 61 67 71
            73 79 83 89 97 101

**Definition:** A prime number is an integer greater than 1 that has no divisors smaller than itself other than 1.

**Useful Facts:**

- $k \leq \sqrt{N}$ iff $N/k \geq \sqrt{N}$, for $N, k > 0$.
- If $k$ divides $N$ then $N/k$ divides $N$.

**So:** Try all potential divisors up to and including the square root.
Plan

```java
public class Primes {
    /** Print all primes up to ARGS[0] (interpreted as an integer), 10 to a line. */
    public static void main(String[] args) {
        printPrimes(Integer.parseInt(args[0]));
    }

    /** Print all primes up to and including LIMIT, 10 to a line. */
    private static void printPrimes(int limit) {
        /*{ For every integer, x, between 2 and LIMIT, print it if isPrime(x), 10 to a line. }*/
    }

    /** True iff X is prime */
    private static boolean isPrime(int x) {
        return /*( X is prime )*/;
    }
}
```
Testing for Primes

private static boolean isPrime(int x) {
    if (x <= 1)
        return false;
    else
        return !isDivisible(x, 2); // "!" means "not"
}

/** True iff X is divisible by any positive number >=K and < X,
* given K > 1. */
private static boolean isDivisible(int x, int k) {
    if (k >= x) // a "guard"
        return false;
    else if (x % k == 0) // "/" means "remainder"
        return true;
    else // if (k < x && x % k != 0)
        return isDivisible(x, k+1);
}
Thinking Recursively

Understand and check isDivisible(13,2) by *tracing one level*.

/** True iff X is divisible by
* some number >=K and < X,
* given K > 1. */
private static boolean isDivisible...
  if (k >= x)
    return false;
  else if (x % k == 0)
    return true;
  else
    return isDivisible(x, k+1);
}

Lesson: Comments aid understanding. Make them count!

• Call assigns x=13, k=2
• Body has form ‘if (k >= x) $S_1$
  else $S_2$’.
• Since 2 < 13, we evaluate the first else.
• Check if 13 mod 2 = 0; it’s not.
• Left with isDivisible(13,3).
• Rather than tracing it, instead use the comment:
• Since 13 is not divisible by any integer in the range 3..12 (and 3 > 1), isDivisible(13,3) must be false, and we’re done!
• Sounds like that last step begs the question. Why doesn’t it?
Iteration

• isDivisible is tail recursive, and so creates an iterative process.

• Traditional “Algol family” production languages have special syntax for iteration. Four equivalent versions of isDivisible:

  ```
  if (k >= x)
    return false;
  else if (x % k == 0)
    return true;
  else
    return isDivisible(x, k+1);
  ```

  ```
  while (k < x) { // !(k >= x)
    if (x % k == 0)
      return true;
    k = k+1;
    // or k += 1, or (yuch) k++
  }
  return false;
  ```

  ```
  int k1 = k;
  while (k1 < x) {
    if (x % k1 == 0)
      return true;
    k1 += 1;
  }
  return false;
  ```

  ```
  for (int k1 = k; k1 < x; k1 += 1) {
    if (x % k1 == 0)
      return true;
  }
  return false;
  ```
Using Facts about Primes

- We haven’t used the Useful Facts from an earlier slide. Only have to check for divisors up to the square root.

- So, reimplement the iterative version of `isDivisible`:

  ```java
  /** True iff X is divisible by some number >=K and < X, 
   * given that K > 1, and that X is not divisible by 
   * any number >1 and <K. */
  private static boolean isDivisible(int x, int k) {
      int limit = (int) Math.round(Math.sqrt(x));
      for (int k1 = k; k1 <= limit; k1 += 1) {
          if (x % k1 == 0)
              return true;
      }
      return false;
  }
  
  // Why the additional (blue) condition in the comment?
  ```
Cautionary Aside: Floating Point

• In the last slide, we had

```java
int limit = (int) Math.round(Math.sqrt(x));
for (int k1 = k; k1 <= limit; k1 += 1) {
    ...
```

intending that this would check all values of k1 up to and including the square root of x.

• Since floating-point operations yield approximations to the corresponding mathematical operations, you might ask the following about `(int) Math.round(Math.sqrt(x))`:

- Is it always at least \( \lfloor \sqrt{x} \rfloor \), where \( \lfloor z \rfloor \) is the largest integer \( \leq z \)? (If not, we might miss testing \( \sqrt{x} \) if \( x \) is a perfect square.)

• As it happens, the answer is “yes” but it would not be if we had been using type `long` instead of `int`, because long integers contain more significant digits than floating point handles.

• Just an example of the sort of detail that can trip one up in edge cases.
Final Task: printPrimes (Simplified)

/** Print all primes up to and including LIMIT. */
private static void printPrimes(int limit) {

}
/** Print all primes up to and including LIMIT. */
private static void printPrimes(int limit) {
    for (int p = 2; p <= limit; p += 1) {
        if (isPrime(p)) {
            System.out.print(p + " ");
        }
    }
    System.out.println();
}
printPrimes (full version)

/** Print all primes up to and including LIMIT, 10 to * a line. */
private static void printPrimes(int limit) {
    int np;
    np = 0;
    for (int p = 2; p <= limit; p += 1) {
        if (isPrime(p)) {
            System.out.print(p + " ");
            np += 1;
            if (np % 10 == 0)
                System.out.println();
        }
    }
    if (np % 10 != 0)
        System.out.println();
}