1 Heaps of fun®

(a) Assume that we have a binary min-heap (smallest value on top) data structure called Heap that stores integers and has properly implemented insert and removeMin methods. Draw the heap and its array representation after each of the operations below:

```java
Heap h = new Heap(5); // Creates a min-heap with 5 as the root
h.insert(7);
h.insert(3);
h.insert(1);
h.insert(2);
h.removeMin();
h.removeMin();
```

(b) Consider an array-based min-heap with N elements. What is the worst case asymptotic run time of each of the following operations if we ignore resizing? What is the worst case asymptotic run time if we take into account resizing? What are the advantages of using an array-based heap vs. using a node-based heap?

<table>
<thead>
<tr>
<th>Operation</th>
<th>Ignore Resize</th>
<th>With Resize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insert</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Find Min</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remove Min</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) You are tasked to implement a max-heap data structure of integers using only a min-heap of integers. Could you complete the task? If so, describe your approach. If not, explain why it’s impossible.

2 HashMap Modification (from 61BL SU2010 MT2)

(a) When you modify a key that has been inserted into a HashMap, will you be able to retrieve that entry again? Explain.

□ Always □ Sometimes □ Never
(b) When you modify a value that has been inserted into a HashMap, will you be able to retrieve that entry again? Explain.

□ Always □ Sometimes □ Never

3 Hash Functions

(a) Here are four potential implementations of the Integer’s hashCode() function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw or disadvantage.

A few notes: A “valid” hashCode() means that any two Integers that are .equals() to each other should also return the same hash code value. In addition, the Integer class extends the Number class, a direct subclass of Object. The Number class’ hashCode() method directly calls the Object class’ hashCode() method.

(1) public int hashCode() {
    return -1;
}

(2) public int hashCode() {
    return intValue() * intValue();
}

(3) public int hashCode() {
    Random rand = new Random();
    return rand.nextInt();
}

(4) public int hashCode() {
    return super.hashCode();
}

(b) Suppose that we represent Tic-Tac-Toe boards as 3 by 3 arrays of integers (with each integer in the range 0 to 2 to represent blank, ‘X’, and ‘O’ respectively). Describe a hash function for Tic-Tac-Toe boards that are represented in this way such that boards that are not equal will never have the same hash code.

(c) Is it possible to add arbitrarily many Strings to a Java HashSet with no collisions? If not, what is the minimum number of distinct Strings you need to add to a HashSet to guarantee a collision?

(A few useful hints: Java arrays have a maximum size of $2^{31} - 1$, and Java HashSet’s underlying array’s size is always a power of 2.)