1 Basic Algorithmic Analysis

For each of the following function pairs $f$ and $g$, list out the $\Theta, \Omega, O$ relationships between $f$ and $g$, if any such relationship exists. For example, $f(x) \in O(g(x))$.

1. $f(x) = x^2$, $g(x) = x^2 + x$
2. $f(x) = 500000x^3$, $g(x) = x^5$
3. $f(x) = \log(x)$, $g(x) = 5x$
4. $f(x) = e^x$, $g(x) = x^5$
5. $f(x) = \log(5^x)$, $g(x) = x$

2 Practice with Runtime

For each of the following functions, find the Big-Theta expression for the runtime of the function in terms of the input variable $n$.

1. For this problem, assume that the static method `constant` runs in $\Theta(1)$ time.

   ```java
   public static void bars(int n) {
       for (int i = 0; i < n; i += 1) {
           for (int j = 0; j < i; j += 1) {
               System.out.println(i + j);
           }
       }
       for (int k = 0; k < n; k += 1) {
           constant(k);
       }
   }
   ``

2. ```java
   public static void barsRearranged(int n) {
       for (int i = 1; i <= n; i *= 2) {
           for (int j = 0; j < i; j += 1) {
               System.out.println("mooove");
           }
       }
   ```
# A Bit with some Bits

Complete the following method. When given a list of integers, `bitVote` returns an integer such that the $i^{th}$ bit of the return value is 1 if and only if more than half of the integers in the list have 1 in the $i^{th}$ bit. Keep in mind that Java `int`s are 32 bits long!

For example, if `bitList` was `[1,3]`, then in binary this would be $[(01)_2,(11)_2]$ (with 30 more zeros in front of each number), and the result would be $(01)_2$ → 1, since the right-most digit was 1 for more than half the numbers, but the second-from-the-right digit was not 1 for more than half the numbers.

Note: the solution to this question isn’t very complicated, but it’s not short! Try breaking it down into components, and ask your neighbors for help!

```java
public static int bitVote(int[] bitList) {

    for (int i = 0; i < 32; i++) {
        // For each bit index

            for (int k : bitList) {
                // For each integer

            }

    }

}
```