1 A* Search

For the graph below, let \( g(u, v) \) be the weight of the edge between any nodes \( u \) and \( v \). Let \( h(u, v) \) be the value returned by the heuristic for any nodes \( u \) and \( v \). Remember the heuristic serves to estimate the distance between two nodes \( u \) and \( v \).

\[
\begin{array}{|c|c|}
\hline
\text{Edge weights:} & \text{Heuristics:} \\
\hline
\text{g}(A, B) = 1 & \text{h}(A, G) = 8 \\
\text{g}(B, C) = 3 & \text{h}(B, G) = 6 \\
\text{g}(C, F) = 2 & \text{h}(C, G) = 5 \\
\text{g}(C, G) = 4 & \text{h}(F, G) = 1 \\
\text{g}(F, G) = 1 & \text{h}(D, G) = 6 \\
\text{g}(A, D) = 2 & \text{h}(E, G) = 3 \\
\text{g}(D, E) = 3 & \\
\text{g}(E, G) = 3 & \\
\hline
\end{array}
\]

a) Given the weights and heuristic values for the graph above, what path would A* search return, starting from A and with G as a goal?

b) Is the heuristic admissible? Why or why not? A heuristic is admissible if it never overestimates the distance it is estimating.
a) Perform Prim’s algorithm to find a minimum spanning tree of the graph above. Pick A as the initial node. Whenever there are multiple nodes with the same cost, process them in alphabetical order.

b) Use Kruskal’s algorithm to find a minimum spanning tree of the graph above. Will Prim’s and Kruskal’s always return the same MST?

c) Draw the final state of the tree that results from union find operations executed when running Kruskal’s on the graph above. When there are ties, choose the root to be the alphabetically first node.