1 Basic Algorithmic Analysis

For each of the following function pairs \( f \) and \( g \), list out the \( \Theta, \Omega, O \) relationships between \( f \) and \( g \), if any such relationship exists. The log function here denotes the natural logarithm.

1. \( f(x) = x^2, g(x) = x^2 + x \)
2. \( f(x) = 5000000x^3, g(x) = x^5 \)
3. \( f(x) = \log(x), g(x) = 5x \)
4. \( f(x) = e^x, g(x) = x^5 \) (hint: \( 5 > e \))
5. \( f(x) = \log(5^x), g(x) = x \)

2 Practice with Runtime

For each of the following functions, find the Big-Theta expression for the runtime of the function in terms of the input variable \( n \).

1. For this problem, you may assume that the static method \( \text{constant} \) runs in \( \Theta(1) \) time.

```java
public static void thisIsANestedLoop(int n) {
    for (int i = 0; i < n; i += 1) {
        for (int j = 0; j < i; j += 1) {
            System.out.println(i + j);
        }
    }
    for (int k = 0; k < n; k += 1) {
        constant(k);
    }
}
```

2. ```java
public static void thisIsMoreConfusing(int n) {
    for (int i = 1; i <= n; i *= 2) {
        for (int j = 0; j < i; j += 1) {
            System.out.println("moo");
        }
    }
}
```
3  A Bit with some Bits

Complete the following method such that it does what it is intended to do: given a list of integers, it returns an integer such that the i-th bit of the return value is 1 if and only if more than half of the integers in the list have 1 in the i-th bit. Keep in mind that Java int s are 32 bits long!

Note: the solution to this question isn’t very complicated, but it’s not short! Try breaking it down into components, and ask your neighbors for help!

```java
public static int bitVote(int[] bitList) {

    for (int i = 0; i < 32; i++) { // For each bit index
        int count = 0;
        for (int k : bitList) { // For each integer
            if ((k >> i) & 1) count++;
        }
        if (count > bitList.length / 2) return (1 << i);
    }
}
```