1 Balanced Search Trees
(a) Convert the red-black tree into a 2-4 tree. (Solid nodes are black.)

(b) Insert 13 into the 2-4 tree.

(c) Convert the resulting 2-4 tree into a valid red-black tree.
(d) Given a (2, 4) tree containing N keys, how would you obtain the keys in sorted order in worst case O(N) time? We don’t need actual code—pseudo code or an unambiguous description will do.

(e) If a (2,4) tree has depth h (that is, the leaves are at distance h from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

2 Tries

(a) List the words encoded by the following trie.

```
          I
         / \   
        N   C
       / \   /   
      D   F H   O
     / \ / \   /   
    X   E   X   X
```

(b) Draw the trie after inserting the words indent, inches, and trie.
3 Skip Lists

Draw the resulting skip list after adding the following numbers at the specified random height. Then highlight the links used to find 148.

<table>
<thead>
<tr>
<th>Number</th>
<th>41</th>
<th>48</th>
<th>59</th>
<th>77</th>
<th>40</th>
<th>131</th>
<th>148</th>
<th>54</th>
<th>139</th>
<th>179</th>
<th>43</th>
<th>128</th>
<th>161</th>
<th>189</th>
<th>170</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
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<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>