CS61B Lecture #23

Today:

- Range queries
- Java utilities: `SortedSet`, `Map`, etc.
- Hashing: probabilistic constant-time search.

Readings for Today: *DS(IJ)*, Chapters 6 and 7

Readings for Next Topic: *DS(IJ)*, Chapter 8 (Sorting)

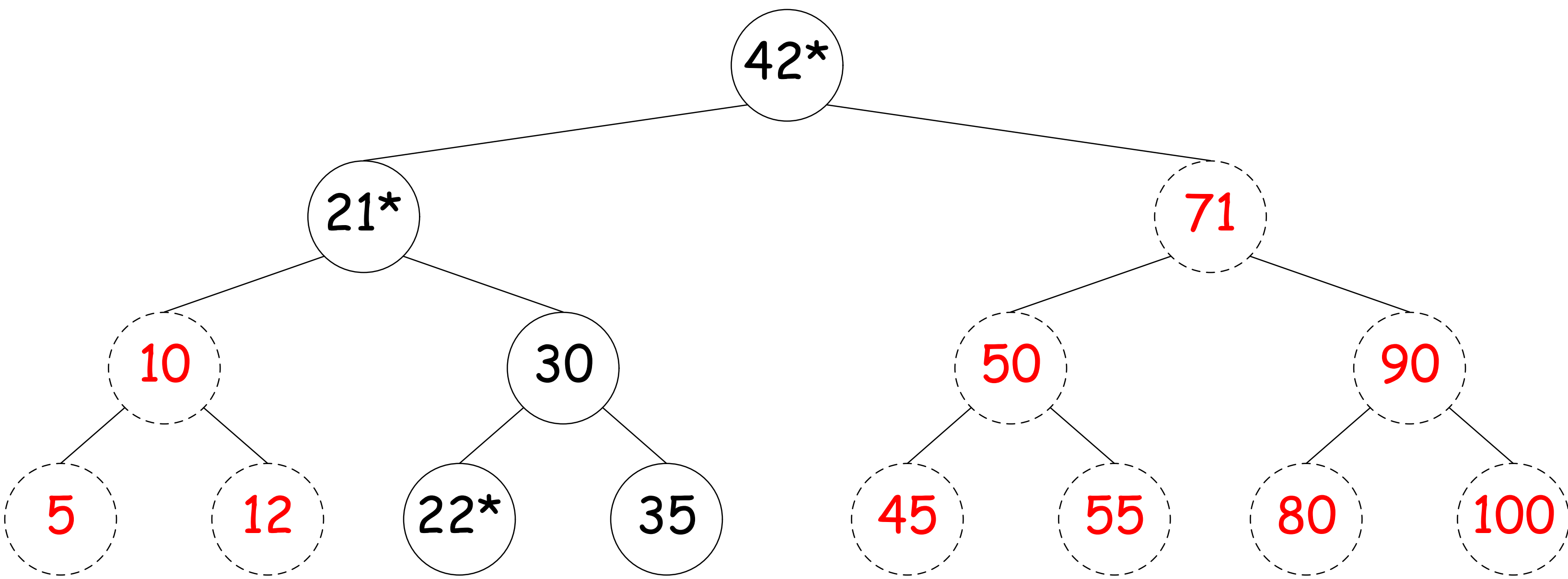
Ranges

- So far, have looked for specific items
- But for BSTs, need an ordering anyway, and can also support looking for *ranges of values*.
- Example: perform some action on all values in a BST that are within some range (in natural order):

```
/** Apply WHATTODO to all labels in T that are
 *  >= L and < U, in ascending natural order. */
static void visitRange (BST T, Comparable<Key> L, Comparable<Key> U,
                        Action whatToDo)
  if (T != null) {
    int compLeft = L.compareTo (T.label ()),
        compRight = U.compareTo (T.label ());
    if (compLeft < 0)  /* L < label */
      visitRange (T.left (), L, U, whatToDo);
    if (compLeft <= 0 && compRight > 0) /* L <= label < U */
      whatToDo.action (T);
    if (compRight > 0)  /* label < U */
      visitRange (T.right (), L, U, whatToDo);
  }
}
```

Time for Range Queries

- Time for range query $\in O(h + M)$, where h is height of tree, and M is number of data items that turn out to be in the range.
- Consider searching the tree below for all values, x, such that $25 \leq x < 40$.
- In this example, the h comes from the starred nodes; the M comes from other non-dashed nodes. Dashed nodes are never looked at.



Ordered Sets and Range Queries in Java

- Class `SortedSet` supports range queries with *views* of set:
  - `S.headSet(U)`: subset of `S` that is $<$ `U`.
  - `S.tailSet(L)`: subset that is $\geq$ `L`.
  - `S.subSet(L,U)`: subset that is $\geq$ `L`, $<$ `U`.
- Changes to views modify `S`.
- Attempts to, e.g., add to a `headSet` beyond `U` are disallowed.
- Can iterate through a view to process a range:

```
SortedSet<String> fauna = new TreeSet<String>
     (Arrays.asList ("axolotl", "elk", "dog", "hartebeest", "duck"));
for (String item : fauna.subSet ("bison", "gnu"))
    System.out.printf ("%s, ", item);
```

  would print "`dog, duck, elk,`"

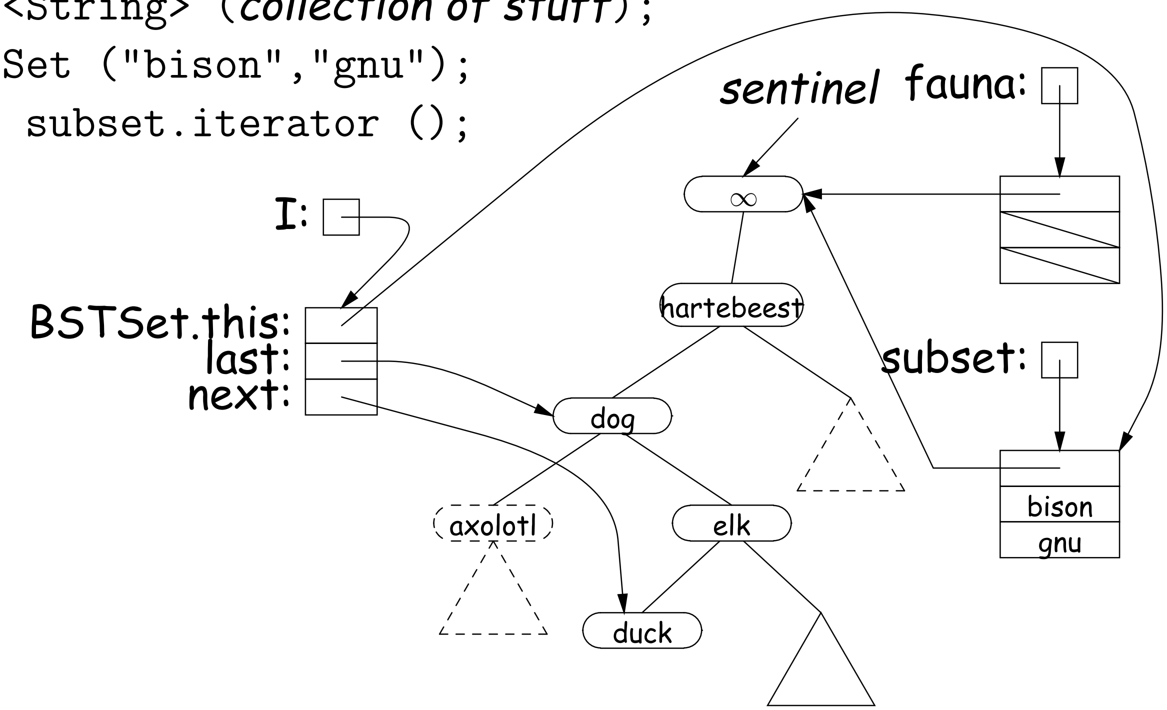
- Java library type `TreeSet<T>` requires either that `T` be `Comparable`, or that you provide a Comparator:

```
SortedSet<String> rev_fauna = new TreeSet<String> (Collections.reverseOrder());
```

Example of Representation: BSTSet

- Use binary search tree to represent set. Can use same representation for both `BSTSet` and its subsets.
- Each set has pointer to BST, plus bounds (if any).
- In this representation, `size` is rather expensive!

```
SortedSet<String>
  fauna = new BSTSet<String> (collection of stuff);
  subset = fauna.subSet ("bison","gnu");
Iterator<String> i = subset.iterator ();
```



Back to Simple Search: Hashing

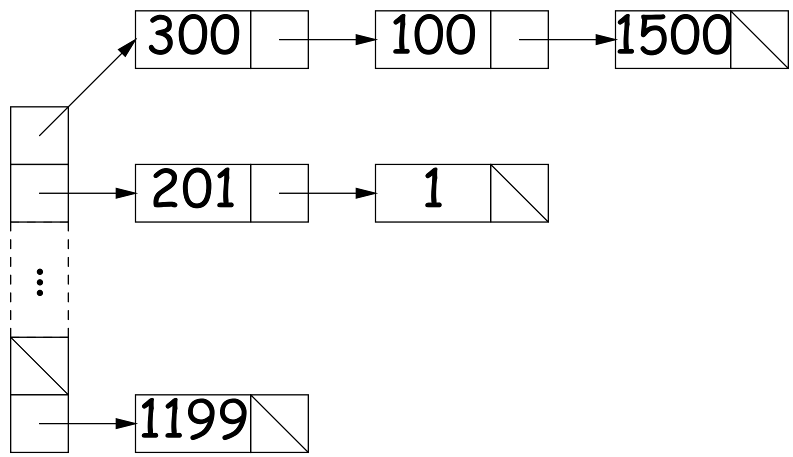
- Linear search is OK for small data sets, bad for large.
- So linear search would be OK *if* we could rapidly narrow the search to a few items.
- Suppose that in constant time could put any item in our data set into a numbered *bucket*, where # buckets stays within a constant factor of # keys.
- Suppose also that buckets contain roughly equal numbers of keys.
- Then search would be constant time.

Hash functions

- To do this, must have way to convert key to bucket number: a *hash function.*
- Example:
  - $N = 200$ data items.
  - keys are `longs`, evenly spread over the range $0..2^{63} - 1$.
  - Want to keep maximum search to $L = 2$ items.
  - Use hash function $h(K) = K\%M$, where $M = N/L = 100$ is the number of buckets: $0 \leq h(K) < M$.
  - So 100232, 433, and 10002332482 go into different buckets, but 10, 400210, and 210 all go into the same bucket.

External chaining

- Array of M buckets.
- Each bucket is a list of data items.



- Not all buckets have same length, but average is $N/M = L$, the *load factor*.
- To work well, hash function must avoid *collisions:* keys that "hash" to equal values.

Open Addressing

- Idea: Put one data item in each bucket.
- When there is a collision, and bucket is full, just use another.
- Various ways to do this:
  - Linear probes: If there is a collision at $h(K)$, try $h(K)+m$, $h(K)+2m$, etc. (wrap around at end).
  - Quadratic probes: $h(K)+m$, $h(K)+m^2$, $\ldots$
  - Double hashing: $h(K)+h'(K)$, $h(K)+2h'(K)$, etc.
- Example: $h(K) = K\%M$, with $M = 10$, linear probes with $m = 1$.
  - Add 1, 2, 11, 3, 102, 9, 18, 108, 309 to empty table.

108	1	2	11	3	102	309		18	9

- Things can get slow, even when table is far from full.
- Lots of literature on this technique, but
- Personally, I just settle for external chaining.

Filling the Table

- To get (likely to be) constant-time lookup, need to keep #buckets within constant factor of #items.
- So resize table when load factor gets higher than some limit.
- In general, must *re-hash* all table items.
- Still, this operation constant time per item,
- So by doubling table size each time, get constant *amortized* time for insertion and lookup
- (Assuming, that is, that our hash function is good).

Hash Functions: Strings

- For String, "$s_0 s_1 \cdots s_{n-1}$" want function that takes all characters and their positions into account.
- What's wrong with $s_0 + s_1 + \ldots + s_{n-1}$?
- For strings, Java uses

$$h(s) = s_0 \cdot 31^{n-1} + s_1 \cdot 31^{n-2} + \ldots + s_{n-1}$$

  computed modulo 2^{32} as in Java `int` arithmetic.
- To convert to a table index in $0..N-1$, compute `h(s)%N` (but *don't* use table size that is multiple of 31!)
- Not as hard to compute as you might think; don't even need multiplication!

```
int r; r = 0;
for (int i = 0; i < s.length (); i += 1)
   r = (r << 5) - r + s.charAt (i);
```

Hash Functions: Other Data Structures I

- Lists (`ArrayList`, `LinkedList`, etc.) are analagous to strings: *e.g.*, Java uses

```
hashCode = 1; Iterator i = list.iterator();
while (i.hasNext()) {
   Object obj = i.next();
   hashCode =
     31*hashCode
     + (obj==null ? 0 : obj.hashCode());
}
```

- Can limit time spent computing hash function by not looking at entire list. For example: look only at first few items (if dealing with a `List` or `SortedSet`).
- Causes more collisions, but does *not* cause equal things to go to different buckets.

Hash Functions: Other Data Structures II

- Recursively defined data structures ⇒ recursively defined hash functions.
- For example, on a binary tree, one can use something like

```
hash(T):
    if (T == null)
       return 0;
    else return someHashFunction (T.label ())
                + 255 * hash(T.left ())
                + 255*255 * hash(T.right ());
```

- Can use address of object ("hash on identity") if distinct (`!=`) objects are never considered equal.
- But careful! Won't work for Strings, because `.equal` Strings could be in different buckets:

```
String H  = "Hello",
       S1 = H + ", world!",
       S2 = "Hello, world!";
```

- Here `S1.equals(S2)`, but `S1 != S2`.

What Java Provides

- In class `Object`, is function `hashCode()`.
- By default, returns address of `this`, or something similar.
- Can override it for your particular type.
- For reasons given on last slide, is overridden for type `String`, as well as many types in the Java library, like all kinds of `List`.
- The types `Hashtable`, `HashSet`, and `HashMap` use `hashCode` to give you fast look-up of objects.

```
HashMap<KeyType,ValueType> map =
  new HashMap<KeyType,ValueType> (approximate size, load factor);

map.put (key, value);  // Map KEY -> VALUE.
// VALUE last mapped to by SOMEKEY.
... map.get (someKey)
       // VALUE last mapped to by SOMEKEY.
... map.containsKey (someKey)
       // Is SOMEKEY mapped?
... map.keySet () // All keys in MAP (a Set)
```

Characteristics

- Assuming good hash function, add, lookup, deletion take $\Theta(1)$ time, amortized.
- Good for cases where one looks up *equal* keys.
- Usually bad for *range queries:* "Give me every name between Martin and Napoli." [Why?]
- But sometimes OK, if hash function is monotonic (i.e., when key $k_1 > k_2$, then $h(k_1) \geq h(k_2)$. For example,
  - Items are time-stamped records; key is the time.
  - Hashing function is to have one bucket for every hour.
- Hashing is probably not a good idea for small sets that you rapidly create and discard [why?]

Comparing Search Structures

Here, N is #items, k is #answers to query.

Function	Unordered List	Sorted Array	Bushy Search Tree	"Good" Hash Table	Heap
find	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(N)$
add	$\Theta(1)$	$\Theta(N)$	$\Theta(\lg N)$	$\Theta(1)$	$\Theta(\lg N)$
range query	$\Theta(N)$	$\Theta(k + \lg N)$	$\Theta(k + \lg N)$	$\Theta(N)$	$\Theta(N)$
find largest	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(1)$
remove largest	$\Theta(N)$	$\Theta(1)$	$\Theta(\lg N)$	$\Theta(N)$	$\Theta(\lg N)$