CS61B Lecture #20

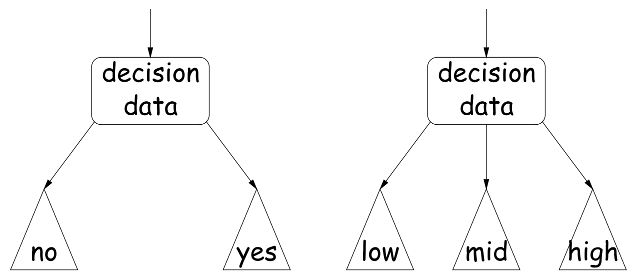
Administrative:

- Watch the newsgroup for extra TA office hours.
- All requests for alternative times should now be in.
- Midterm 6:30–8:30 Tuesday, 19 October, in 100 Genetics and Plant Biology Bldg.

Today: Trees as Search Structures (*DS(IJ) Chapter 6*).

Divide and Conquer

- Much (most?) computation is devoted to finding things in response to various forms of query.
- Linear search for response can be expensive, especially when data set is too large for primary memory.
- Preferable to have criteria for *dividing* data to be searched into pieces recursively
- Remember figure for $\lg N$ algorithms: at 1μsec per comparison, could process 10^{300000} items in 1 sec.
- Tree is a natural framework for the representation:



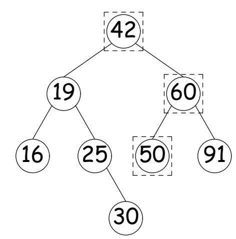
Binary Search Trees

Binary Search Property:

- Tree nodes contain *keys*, and possibly other data.
- All nodes in left subtree of node have *smaller* keys.
- All nodes in right subtree of node have *larger* keys.
- "Smaller" means any complete transitive, anti-symmetric ordering on keys:
  - exactly one of $x \prec y$ and $y \prec x$ true.
  - $x \prec y$ and $y \prec z$ imply $x \prec z$.
  - (To simplify, won't allow duplicate keys this semester).
- E.g., in dictionary database, node label would be (*word, definition*): *word* is the key.

Finding

- Searching for 50 and 49:

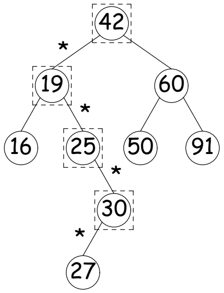



```
/** Node in T containing L,
 *  or null if none */
static BST find(BST T, Object L) {
  if (T == null)
    return T;
  if (L.keyequals (T.label()))
    return T;
  else if (L ≺ T.label())
    return find(T.left(), L);
  else
    return find(T.right (), L);
}
```

- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.

Inserting

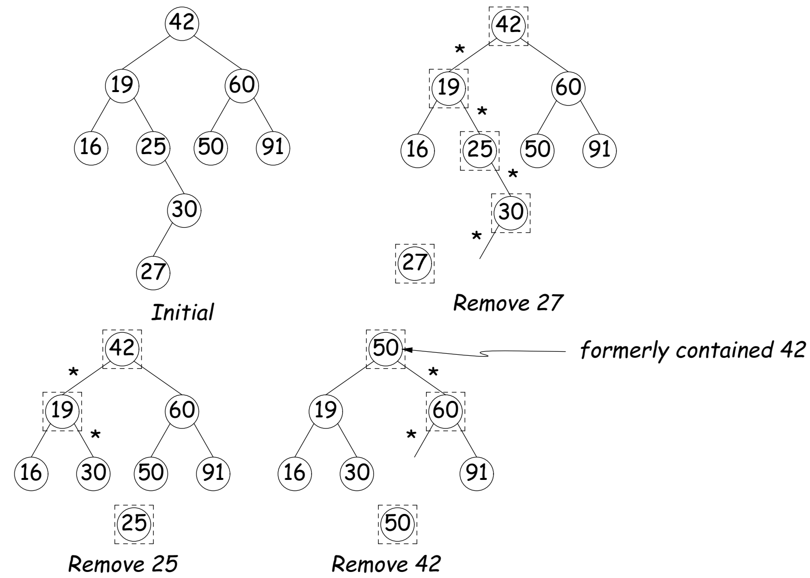
- Inserting 27




```
/** Insert L in T, replacing existing
 *  value if present, and returning
 *  new tree. */
BST insert(BST T, Object L) {
  if (T == null)
    return new BST(L);
  if (L.keyequals (T.label()))
    T.setLabel (L);
  else if (L ≺ T.label())
    T.setLeft(insert (T.left (), L));
  else
    T.setRight(insert (T.right (), L));
  return T;
}
```

- Starred edges are set (to themselves, unless initially null).
- Again, time proportional to height.

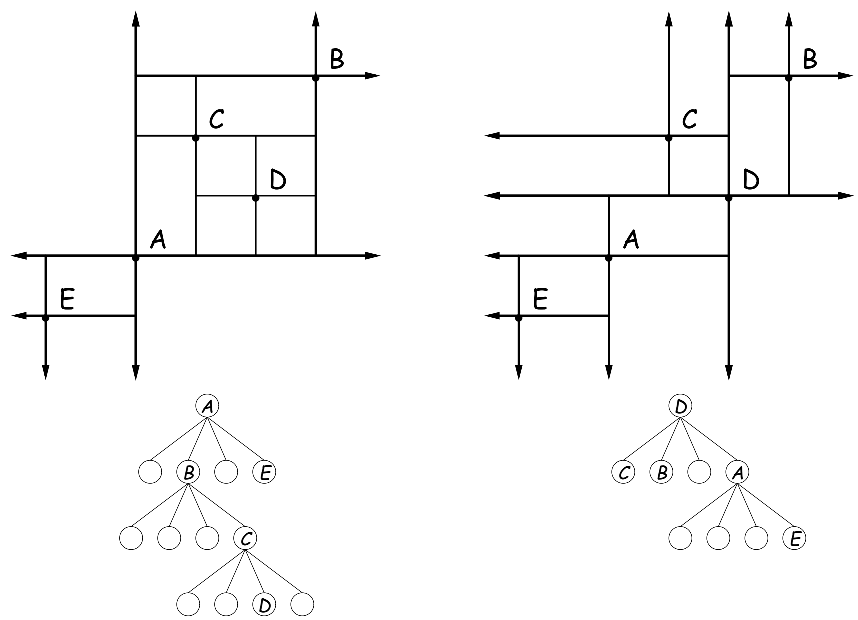
Deletion



A Leap Ahead: Quad Trees

- Want to *index* information about locations so that items can be retrieved by position.
- Quad Trees do so using standard data-structuring trick: *Divide and Conquer*.
- Idea: divide (2D) space into four *quadrants,* and store items in the appropriate quadrant. Repeat this recursively with each quadrant that contains more than one item.
- Original definition: a quad tree is either
  - Empty, or
  - An item at some position (x, y), called the root, plus
  - four quad trees, each containing only items that are northwest, northeast, southwest, and southeast of (x, y).
- Big idea is that if you are looking for point (x', y') and the root is not the point you are looking for, you can narrow down which of the four subtrees of the root to look in by comparing coordinates (x, y) with (x', y').

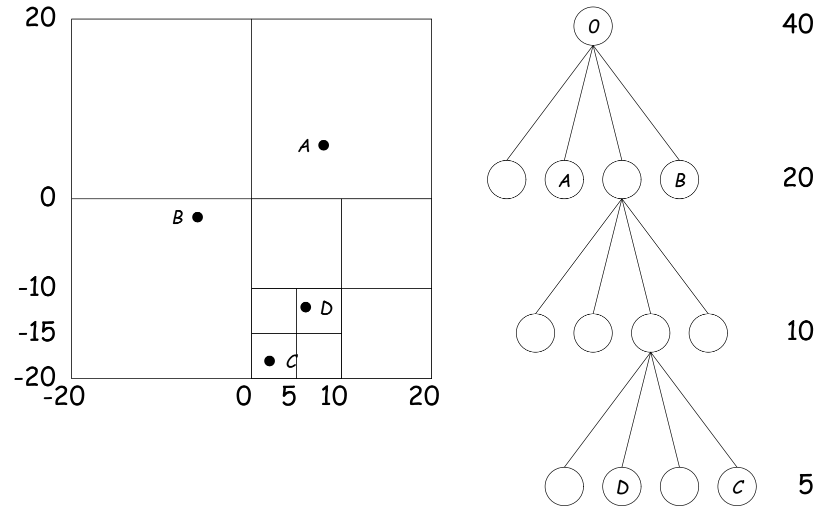
Classical Quad Tree: Example



Another Kind of Quad Tree

- For our project, it is good to be able to *delete* items from a tree: when a particle moves, the subtree that it goes in may change.
- Difficult to do with the classical data structure above, so we'll define instead:
- A quadtree consists of a bounding rectangle, B and either
  - Nothing (an empty quadtree), or
  - An item that lies in that rectangle, or
  - Four quadtrees whose bounding rectangles are the four quadrants of B (all of equal size).
- A completely empty quad tree can have an arbitrary bounding rectangle, or you can wait for the first point to be inserted.

Example of "Our" Quad Tree



Navigating Our Quad Trees

- To find an item at (x, y) in quad tree T,
  1. If (x, y) is outside the bounding rectangle of T, or T is empty, then (x, y) is not in T.
  2. Otherwise, if T contains a single item, compare it to (x, y).
  3. Otherwise, T consists of four quad trees. Recursively look for (x, y) in each (however, step #1 above will cause all but one of these bounding boxes to reject the point immediately).
- Similar procedure works when looking for all items within some rectangle, R:
  1. If R does not intersect the bounding rectangle of T, or T is empty, then there are no items in R.
  2. Otherwise, if T contains a single item, return it if it is in R, and otherwise nothing.
  3. Otherwise, T consists of four quad trees. Recursively look for points in R in each one of them.

Insertion into Our Quad Trees

Various cases for inserting a new point N, showing initial state $\Longrightarrow$ final state.