Efficiency

Announcements

Measuring Efficiency

Efficiency

A measure of how much resource consumption a computational task takes.

An analysis of computer programs rather than a technique for writing them.

In computer science, we are concerned with **time** and **space efficiency**

The time efficiency of could determine how long a user has to wait for a webpage to load.

The space efficiency of your algorithm could determine how much **memory** running your application takes

Orders of Growth

Prepend

```python
def prepend(lst, val):
    """Add VAL to the front of LST."""
    lst.insert(0, val)
```

List Size	Operations
1	
10	
100	
1000	

Exponentiation

Exponentiation

Goal: one more multiplication lets us double the problem size

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ b \cdot b^{n-1} & \text{otherwise} \end{cases}$$

n	Operations
1	
10	
100	
1000	

Exponentiation

Goal: one more multiplication lets us double the problem size

```
def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)
```

```
def square(x):
    return x * x
```

$$b^n = \begin{cases} 1 & \text{if } n = 0 \\ (b^{\frac{1}{2}n})^2 & \text{if } n \text{ is even} \\ b \cdot b^{n-1} & \text{if } n \text{ is odd} \end{cases}$$

n	Operations	
1	~1	
16	~5	
512	~9	
1024	~10	(Demo)

Exponentiation

Goal: one more multiplication lets us double the problem size

```
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)
```

Linear time:

- Doubling the input **doubles** the time
- 1024x the input takes 1024x as much time

```
def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)

def square(x):
    return x * x
```

Logarithmic time:

- Doubling the input **increases** the time by one step
- 1024x the input increases the time by only 10 steps

Quadratic Time

Functions that process all pairs of values in a sequence of length n take quadratic time

```
def overlap(a, b):
    count = 0
    for item in a:
        for other in b:
            if item == other:
                count += 1
    return count

overlap([3, 5, 7, 6], [4, 5, 6, 5])
```

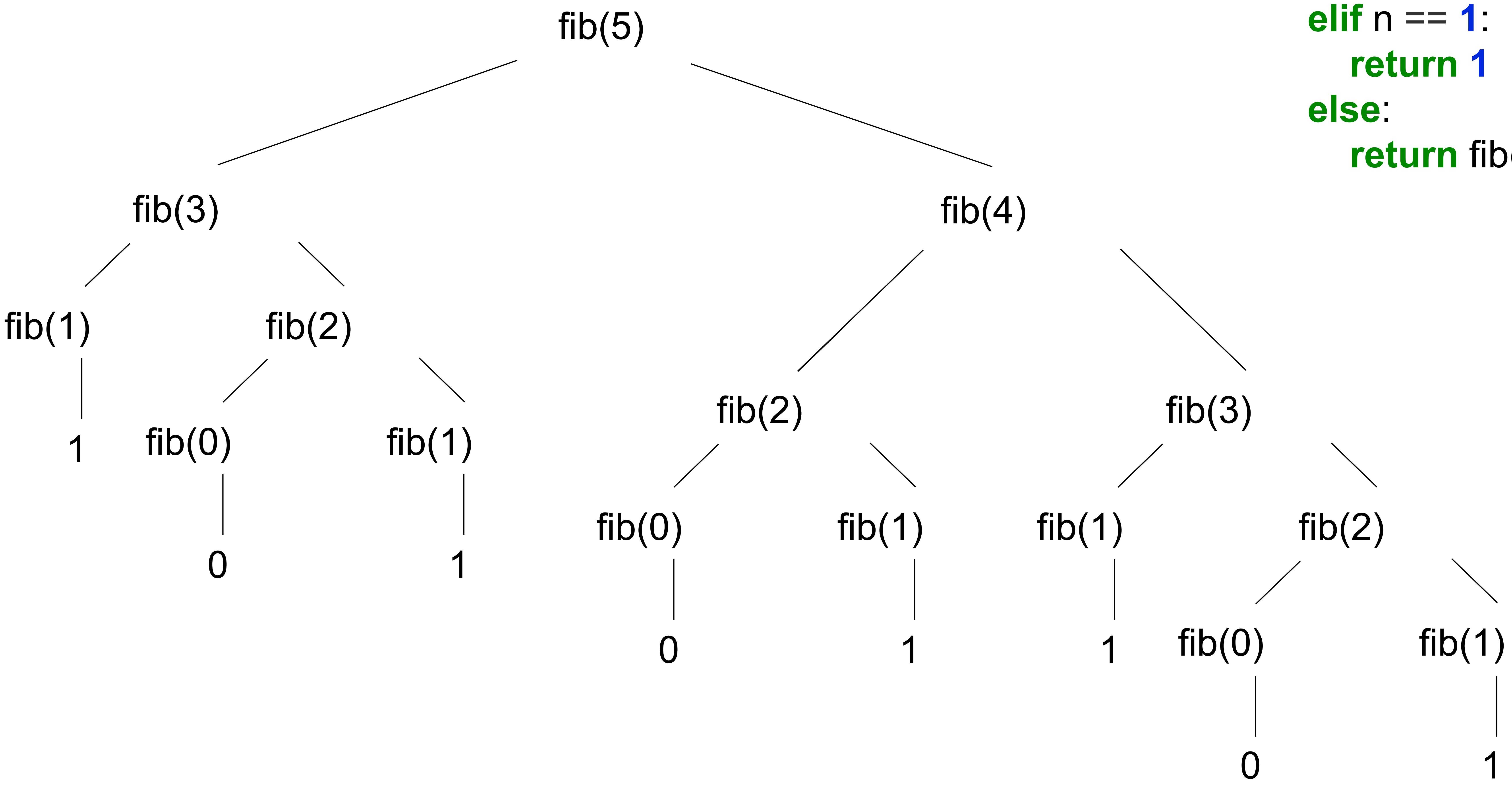
	3	5	7	6
4	0	0	0	0
5	0	1	0	0
6	0	0	0	1
5	0	1	0	0

(Demo)

Exponential Time

Tree-recursive functions can take exponential time

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



Time for n+n

Time for input n+1

Time for input n

Common Orders of Growth

Exponential growth. E.g., recursive fib

Incrementing *n* multiplies *time* by a constant

$$a \cdot b^{n+1} = (a \cdot b^n) \cdot b$$

Quadratic growth. E.g., overlap

Incrementing *n* increases *time* by *n* times a constant

$$a \cdot (n+1)^2 = (a \cdot n^2) + a \cdot (2n+1)$$

Linear growth. E.g., slow exp

Incrementing *n* increases *time* by a constant

$$a \cdot (n+1) = (a \cdot n) + a$$

Logarithmic growth. E.g., exp_fast

Doubling *n* only increments *time* by a constant

$$a \cdot \ln(2 \cdot n) = (a \cdot \ln n) + a \cdot \ln 2$$

Constant growth. Increasing *n* doesn't affect time

Order of Growth Notation

Big Theta and Big O Notation for Orders of Growth

Exponential growth. E.g., recursive fib — $\Theta(b^n)$ — $O(b^n)$

Incrementing *n* multiplies *time* by a constant

Quadratic growth. E.g., overlap — $\Theta(n^2)$ — $O(n^2)$

Incrementing *n* increases *time* by *n* times a constant

Linear growth. E.g., slow exp — $\Theta(n)$ — $O(n)$

Incrementing *n* increases *time* by a constant

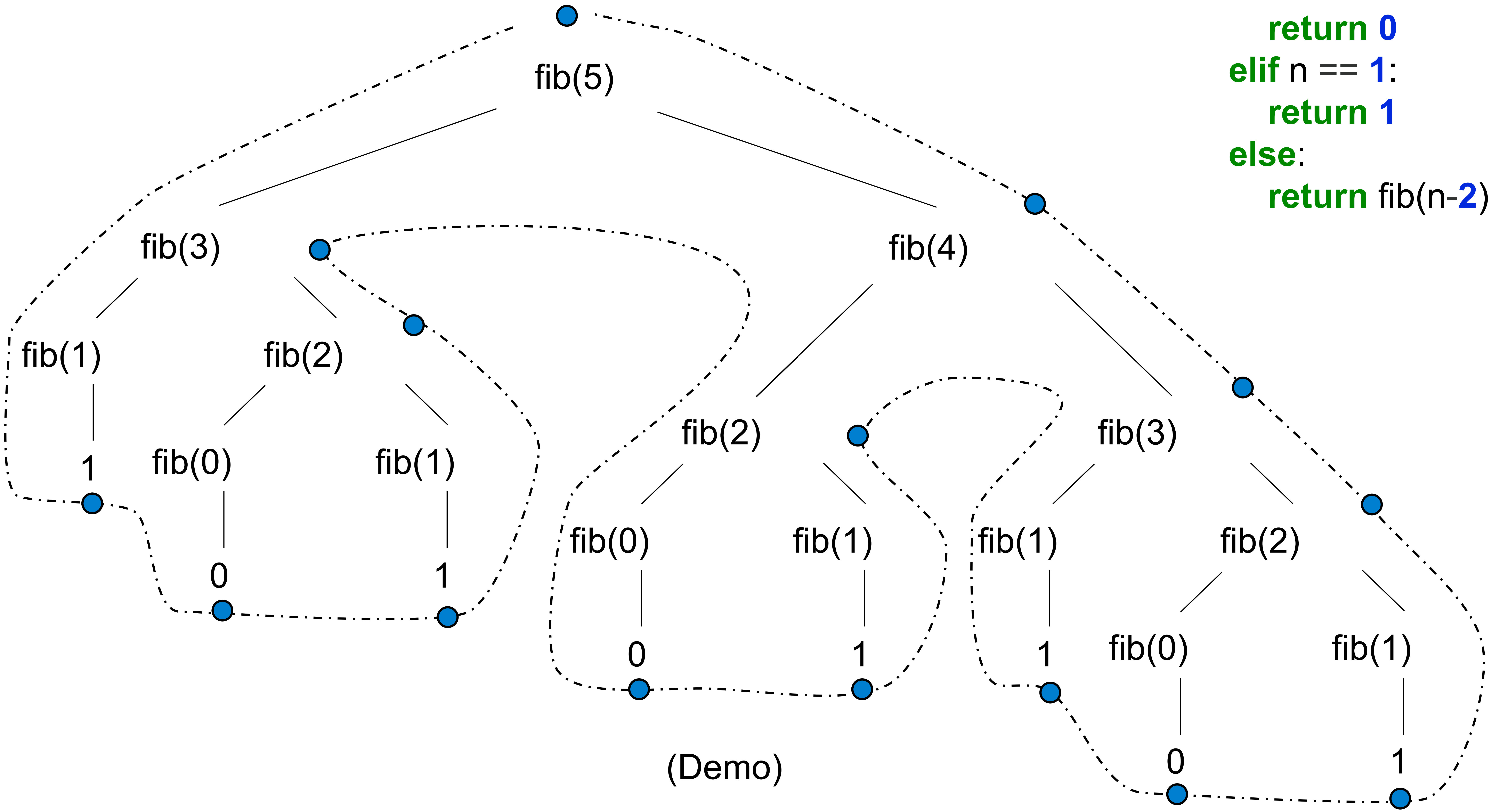
Logarithmic growth. E.g., exp_fast — $\Theta(\log n)$ — $O(\log n)$

Doubling *n* only increments *time* by a constant

Constant growth. Increasing *n* doesn't affect time — $\Theta(1)$ — $O(1)$

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:




```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

(Demo)

Memoization Revisited

(Demo)

Memoization

Idea: Remember the results that have been computed before

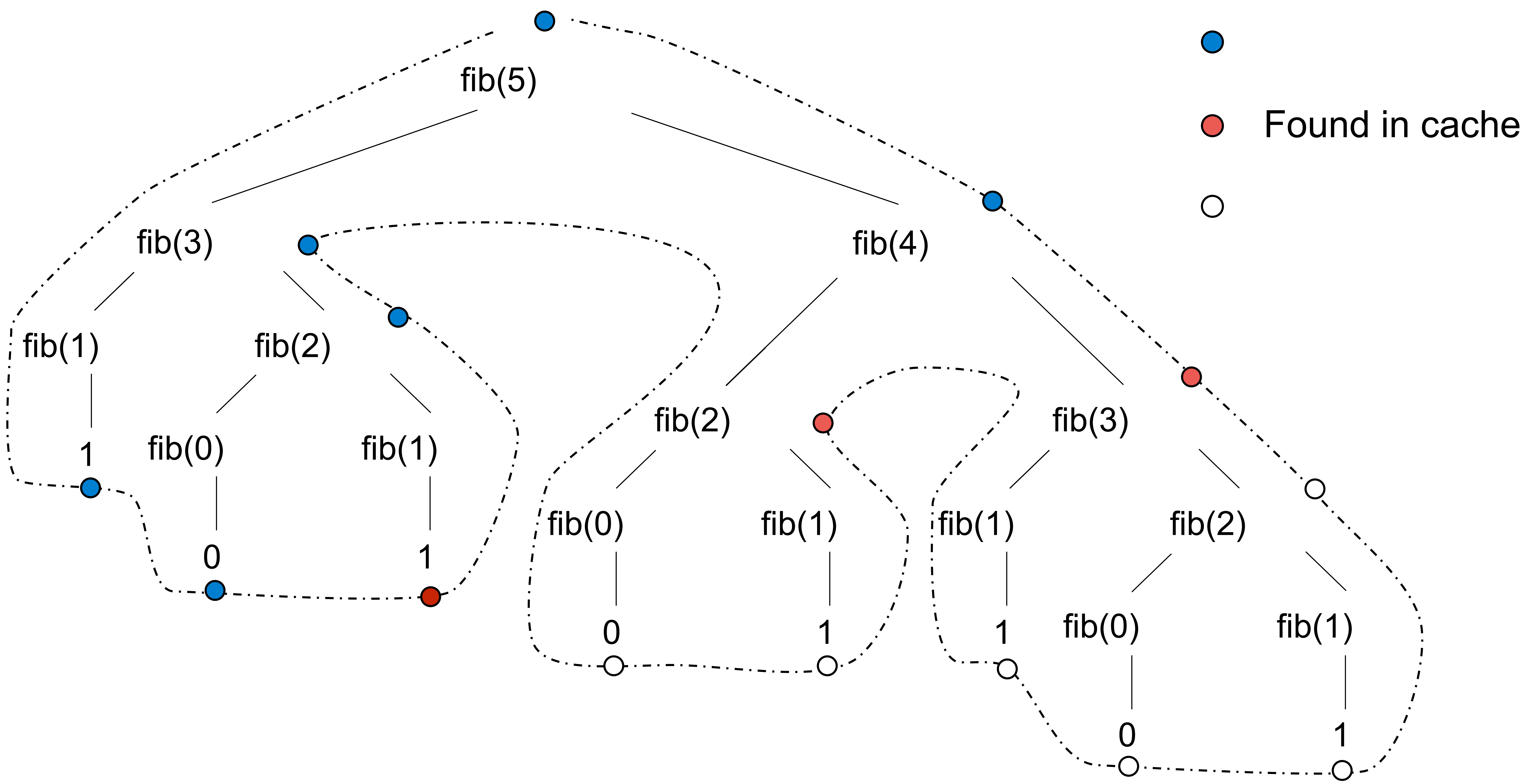
```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

Keys are arguments that map to return values

Same behavior as f, if f is a pure function

(Demo)

Memoized Tree Recursion



Break

Revisiting Past Problems

Space

Space and Environments

Which environment frames do we need to keep during evaluation?

At any moment there is a set of active environments

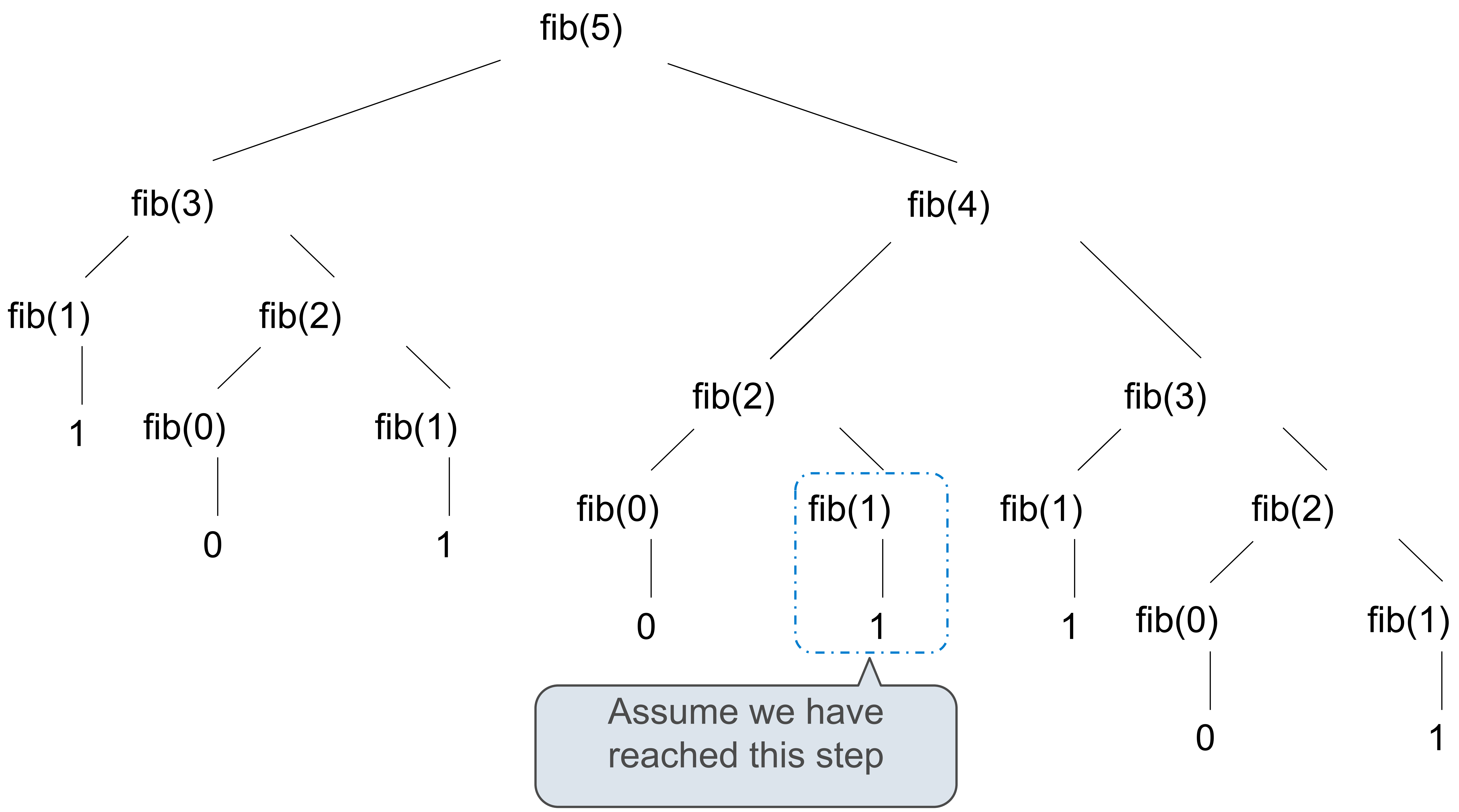
Values and frames in active environments consume memory

Memory that is used for other values and frames can be recycled

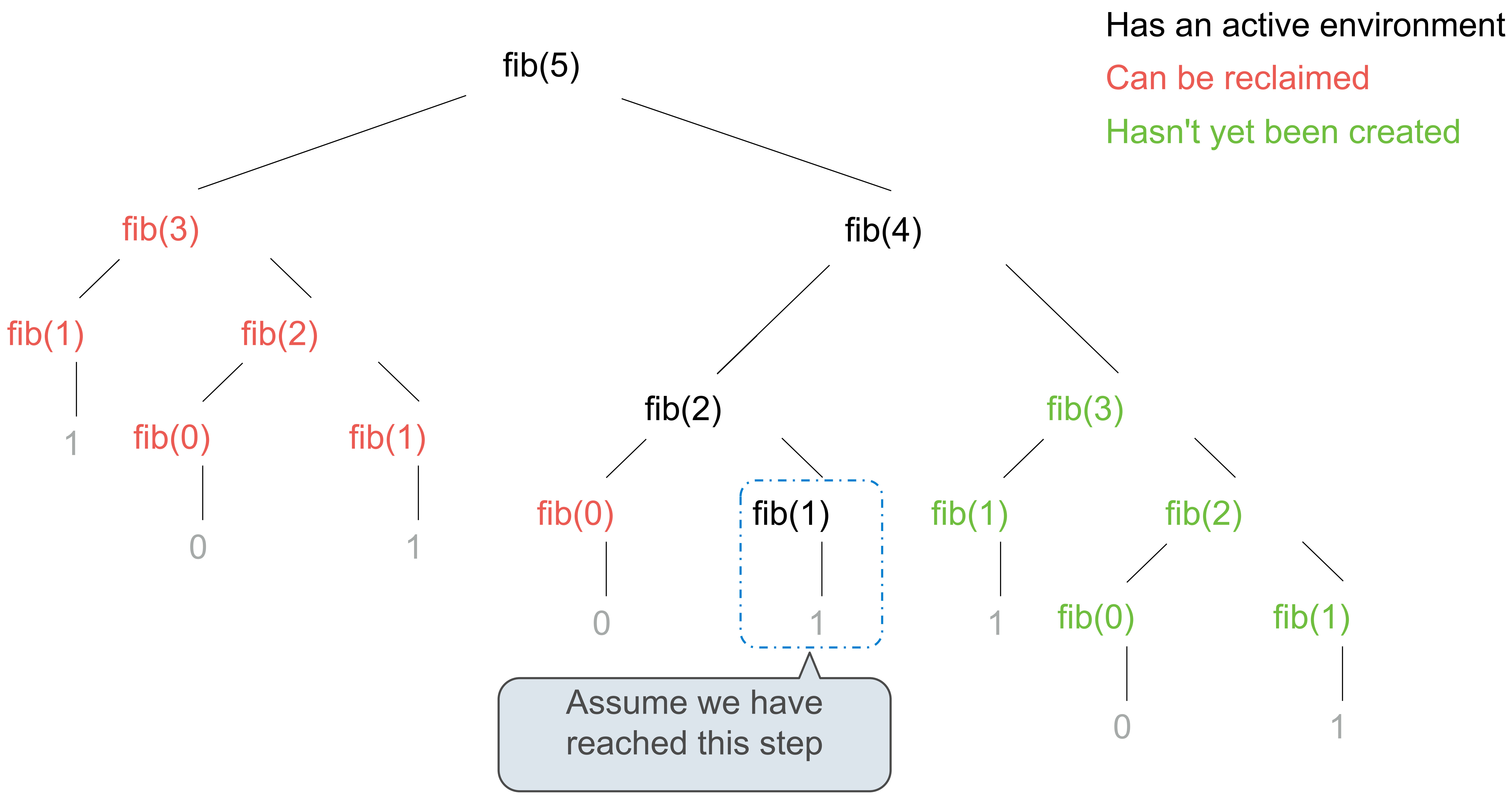
Active environments:

- Environments for any function calls currently being evaluated
- Parent environments of functions named in active environments

Fibonacci Space Consumption



Fibonacci Space Consumption



Generators and Space

(Demo)