61A Lecture 22

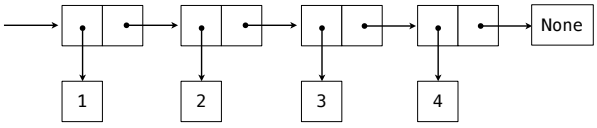
Wednesday, October 19

Closure Property of Data

A tuple can contain another tuple as an element.

Pairs are sufficient to represent sequences.

Recursive list representation of the sequence 1, 2, 3, 4:



Recursive lists are recursive: the rest of the list is a list.

Recursive List Class

Methods can be recursive as well!

This part was all in Homework 6

```
class Rlist(object):

    class EmptyList(object):
        def __len__(self):
            return 0

    empty = EmptyList()

    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest

    def __len__(self):
        return 1 + len(self.rest)

    def __getitem__(self, i):
        if i == 0:
            return self.first
        return self.rest[i-1]
```

There's the base case!

Yes, this call is recursive

Demo

Recursive Operations on Recursive Lists

Recursive list processing almost always involves a recursive call on the rest of the list.

```
>>> s = Rlist(1, Rlist(2, Rlist(3)))

>>> s.rest
Rlist(2, Rlist(3))

>>> extend_rlist(s.rest, s)
Rlist(2, Rlist(3, Rlist(1, Rlist(2, Rlist(3)))))


def extend_rlist(s1, s2):
    if s1 is Rlist.empty:
        return s2
    return Rlist(s1.first, extend_rlist(s1.rest, s2))
```


Map and Filter on Recursive Lists

We want operations on a whole list, not an element at a time.

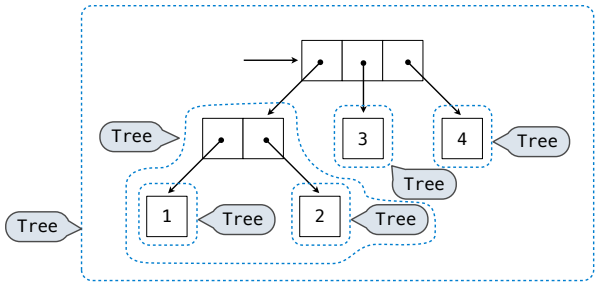
```
>>> def map_rlist(s, fn):
        if s is Rlist.empty:
            return s
        return Rlist(fn(s.first), map_rlist(s.rest, fn))


>>> def filter_rlist(s, fn):
        if s is Rlist.empty:
            return s
        rest = filter_rlist(s.rest, fn)
        if fn(s.first):
            return Rlist(s.first, rest)
        return rest
```


Tree Structured Data

Nested Sequences are Hierarchical Structures.

```
>>> ((1, 2), 3, 4)
((1, 2), 3, 4)
```



Recursive Tree Processing

Tree operations typically make recursive calls on branches

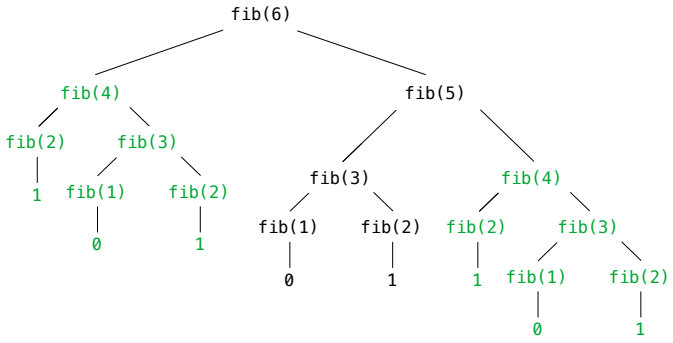
```
def count_leaves(tree):
    if type(tree) != tuple:
        return 1
    return sum(map(count_leaves, tree))


def map_tree(tree, fn):
    if type(tree) != tuple:
        return fn(tree)
    return tuple(map_tree(branch, fn) for branch in tree)
```

Demo

Trees with Internal Node Values

Trees need not only have values at their leaves.



Trees with Internal Node Values

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
    right = fib_tree(n-1)
    return Tree(left.entry + right.entry, left, right)
```

Demo

Sets

One more built-in Python container type

- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}


>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Demo