61A Lecture 21

Monday, October 17

Space Consumption

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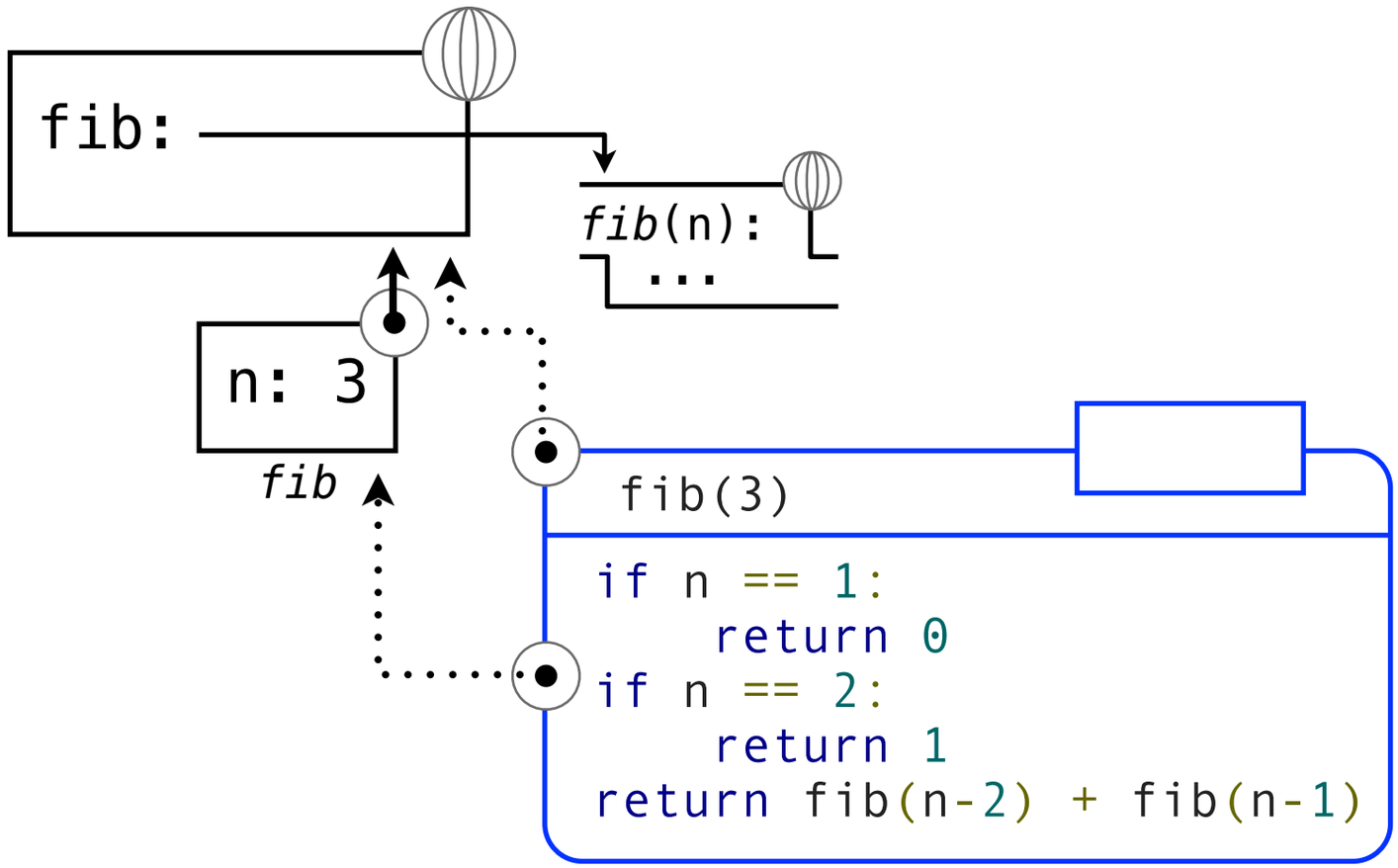
- The environment for the current expression being evaluated
- Environments for calls that depend upon the value of the current expression
- Environments associated with functions referenced by active environments

Fibonacci Environment Diagram



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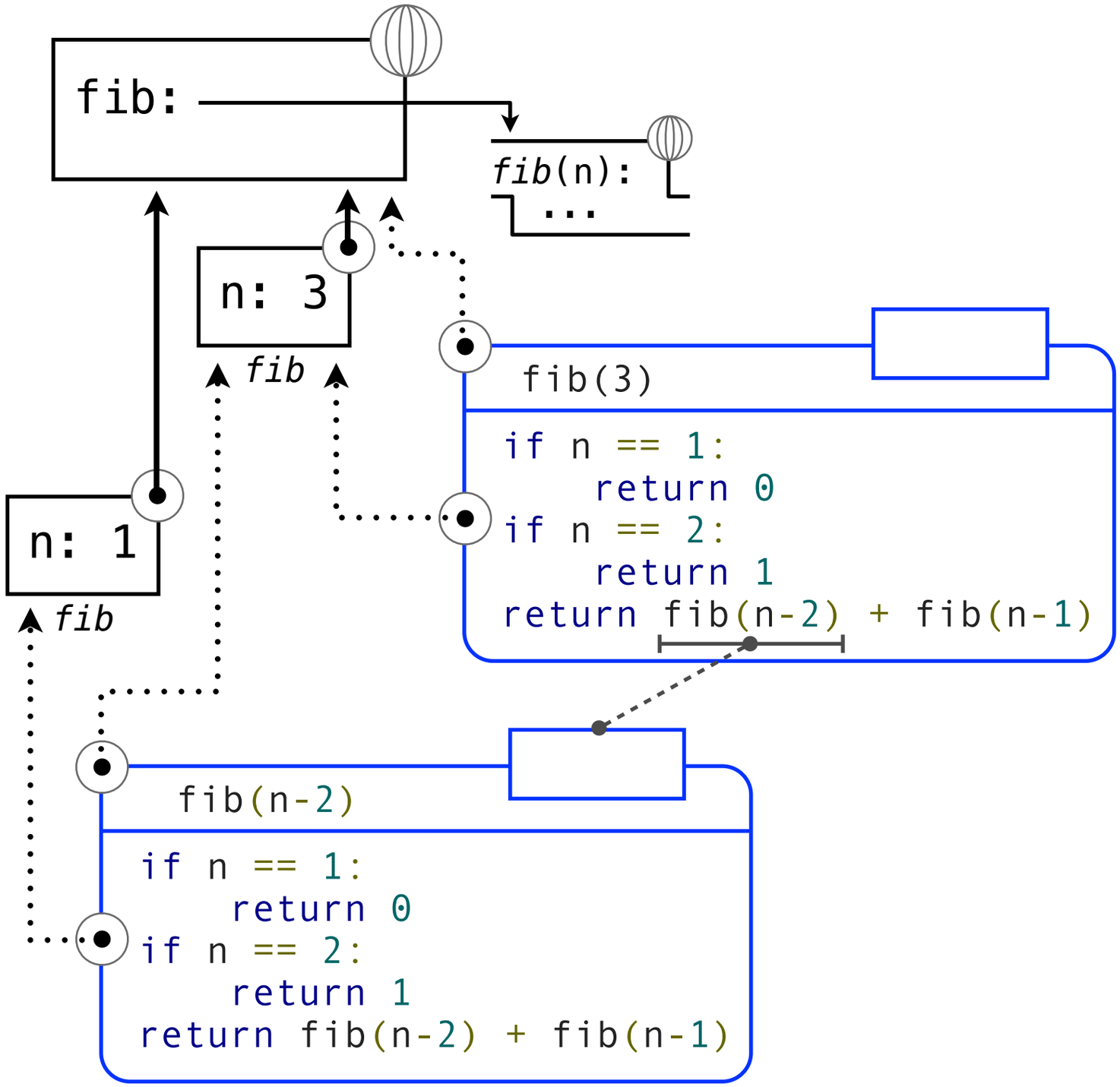

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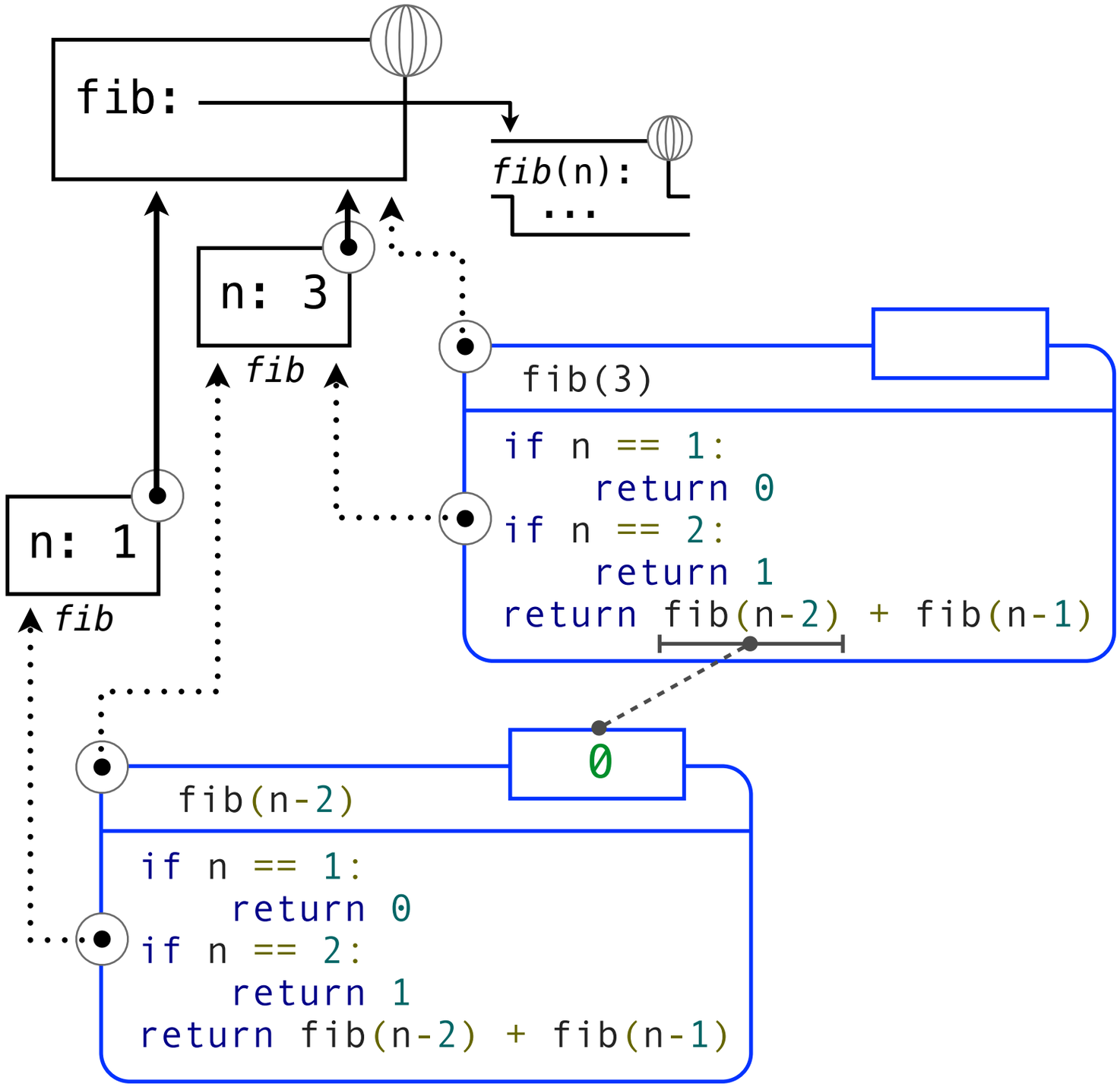

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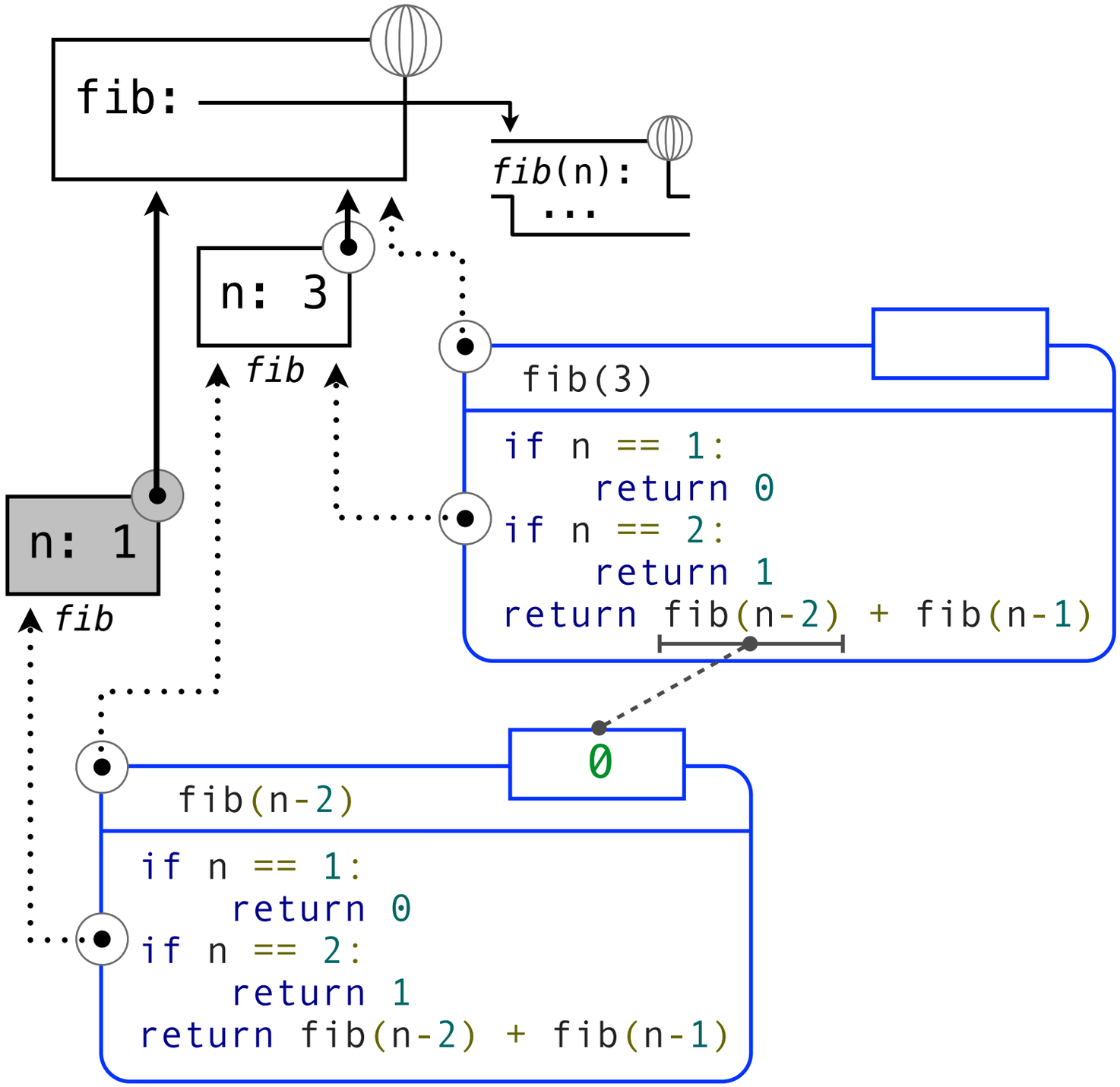

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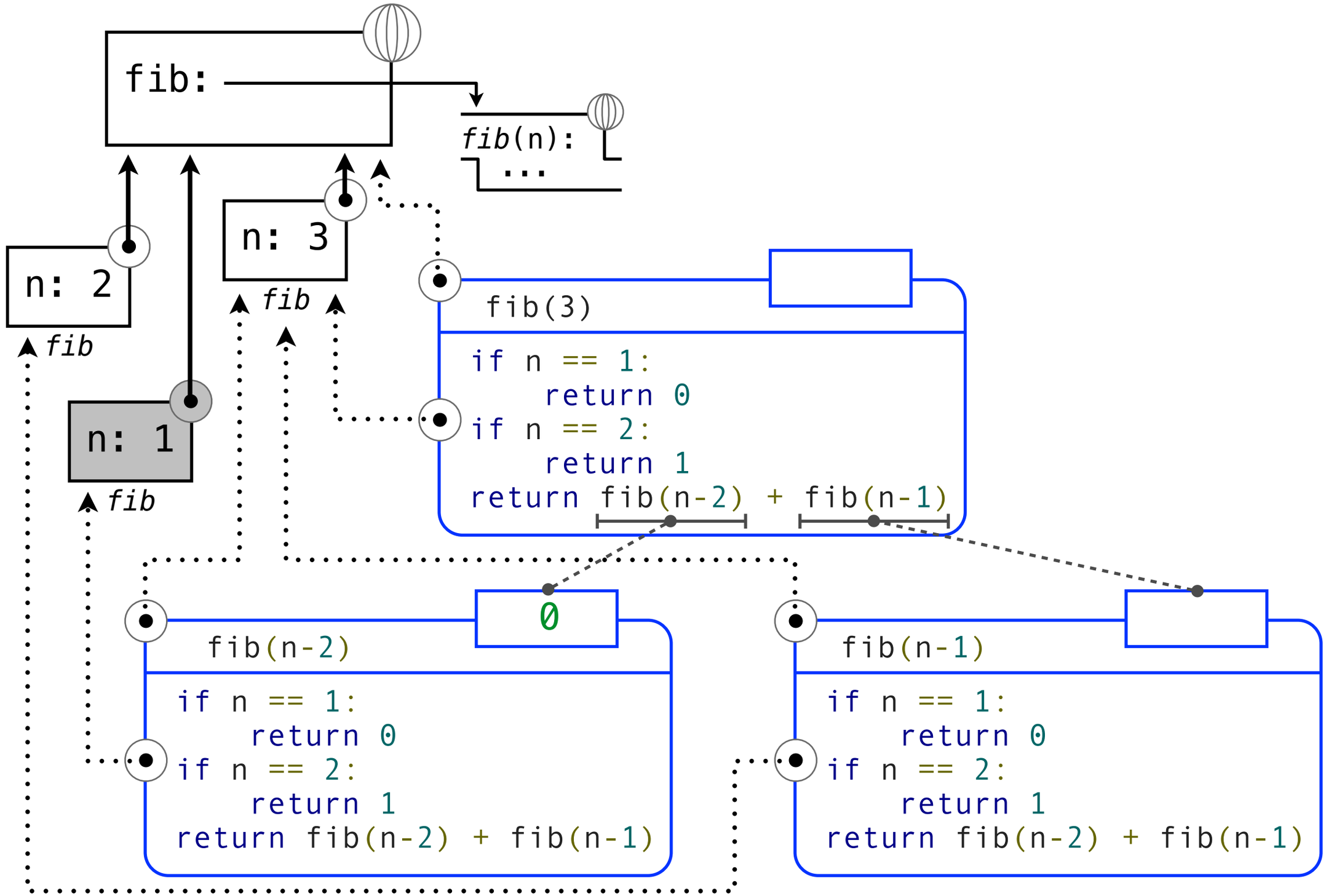
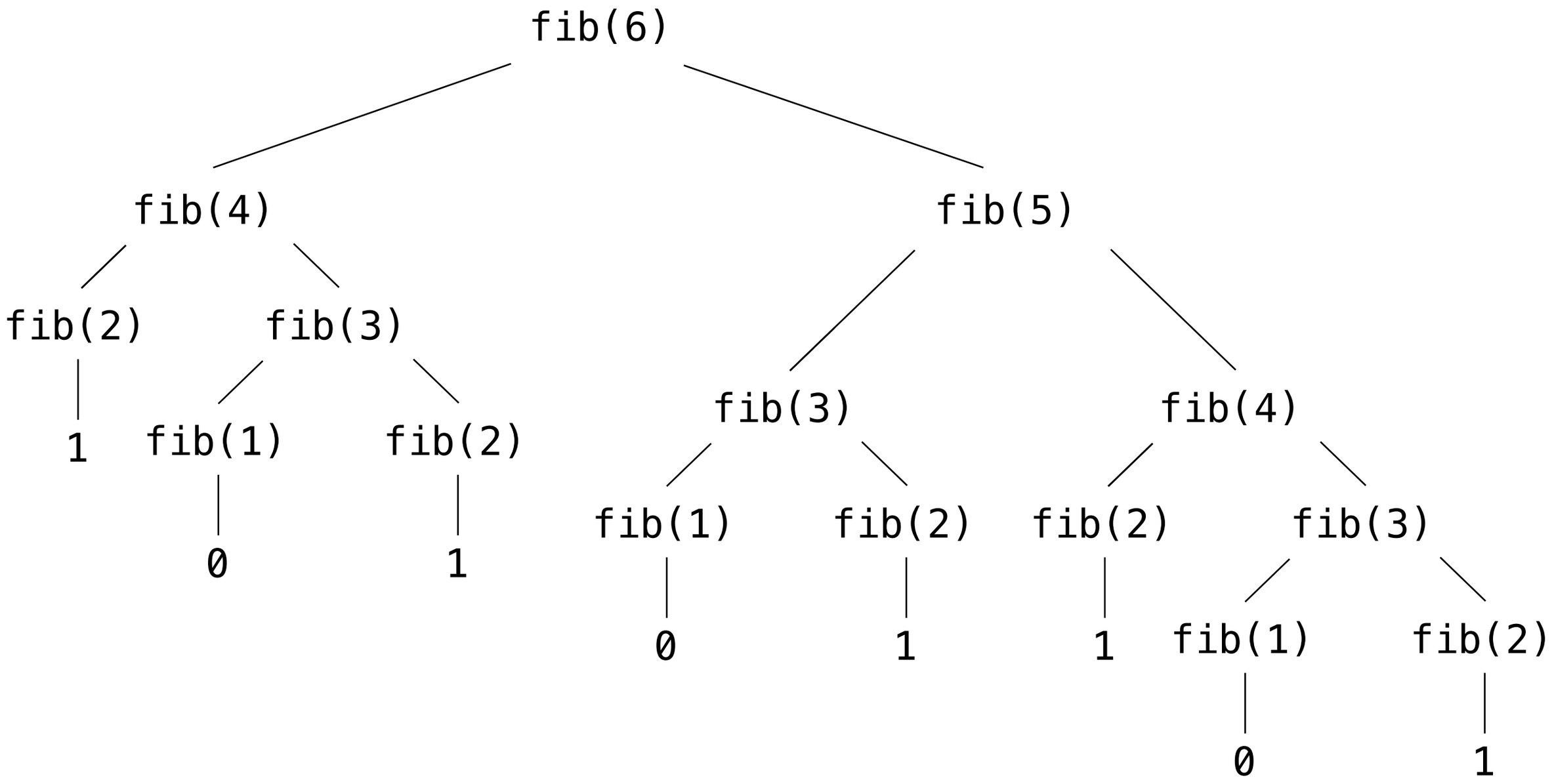


Fibonacci Memory Consumption



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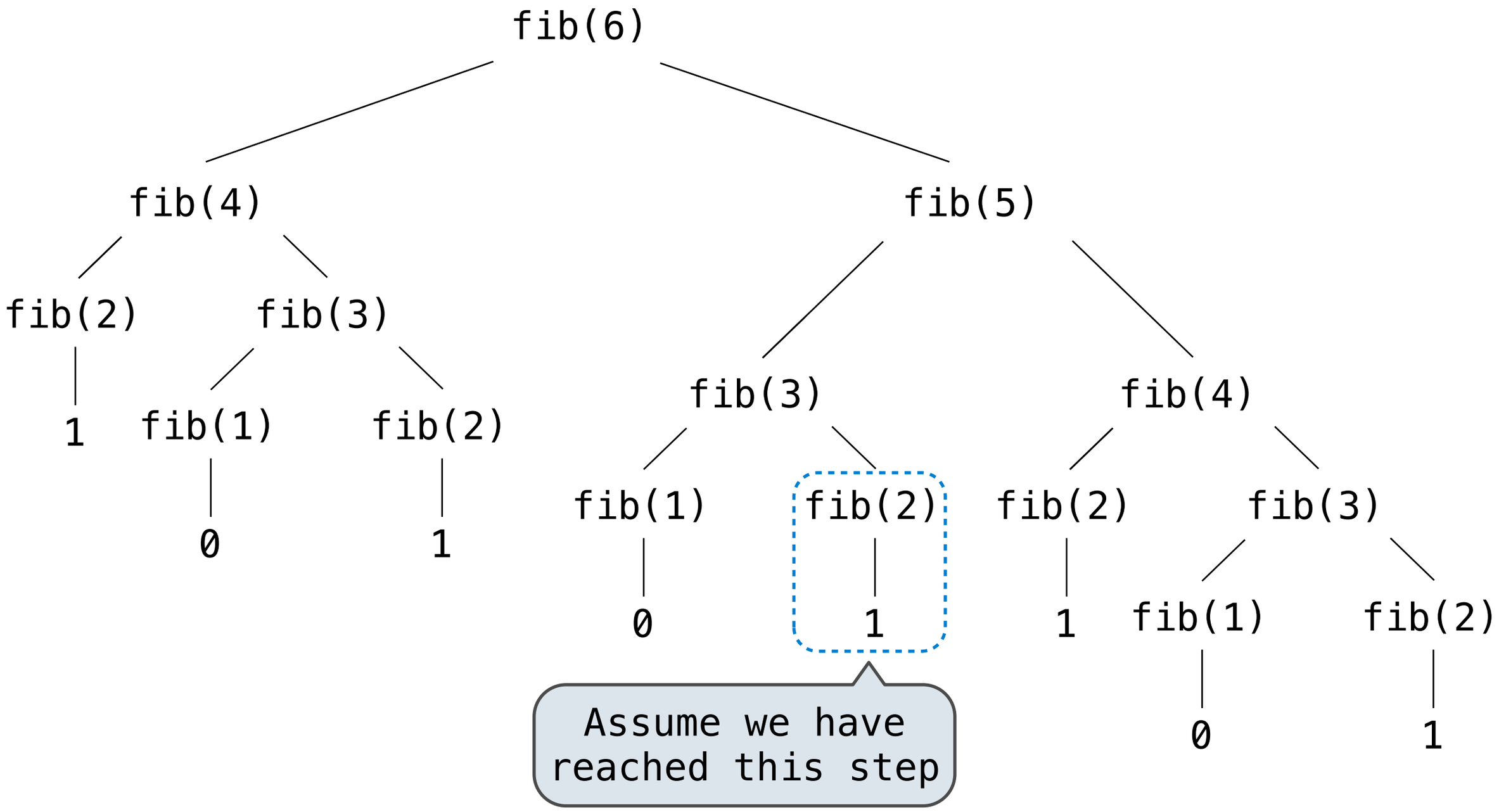

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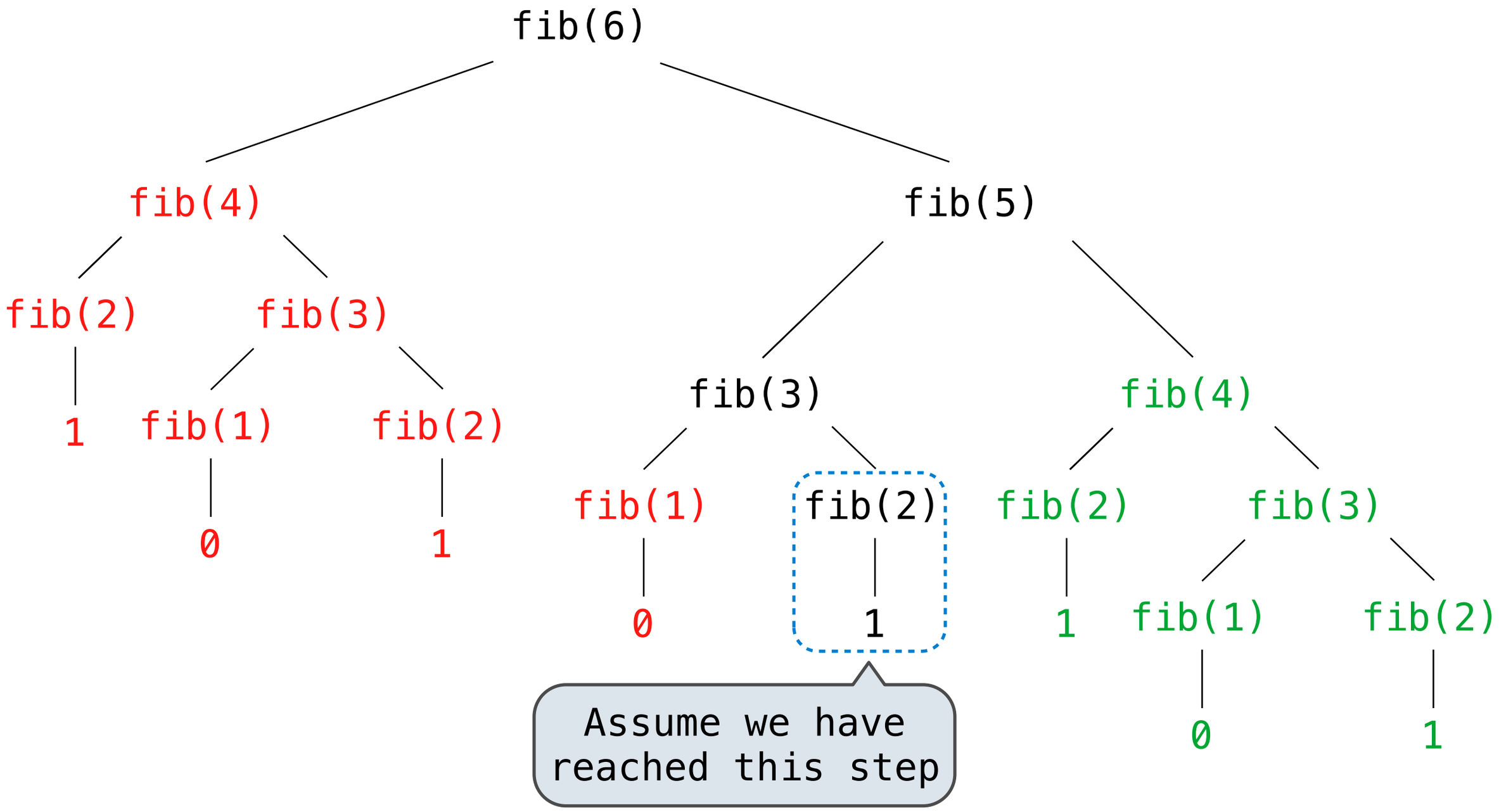

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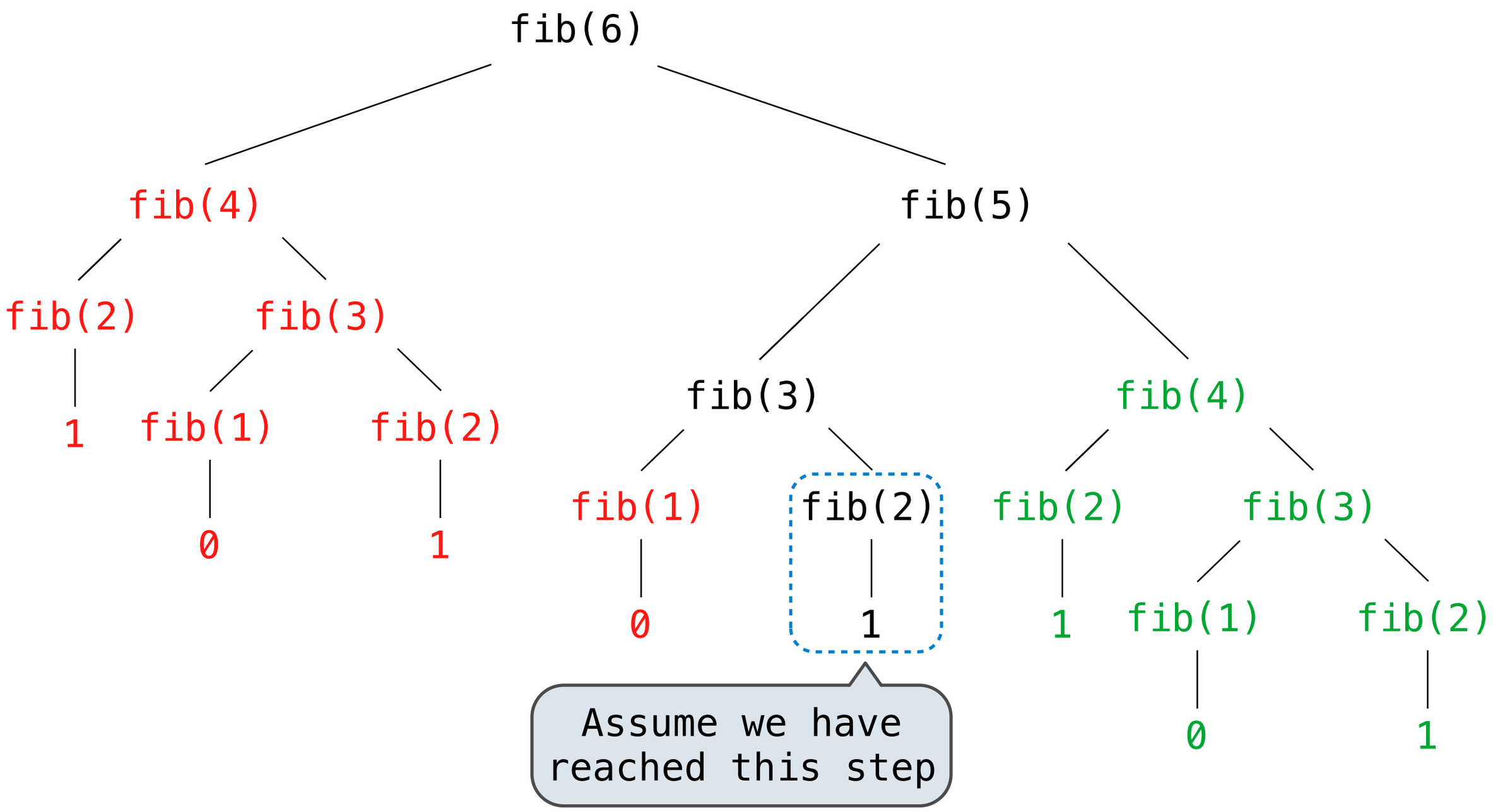

Fibonacci Memory Consumption

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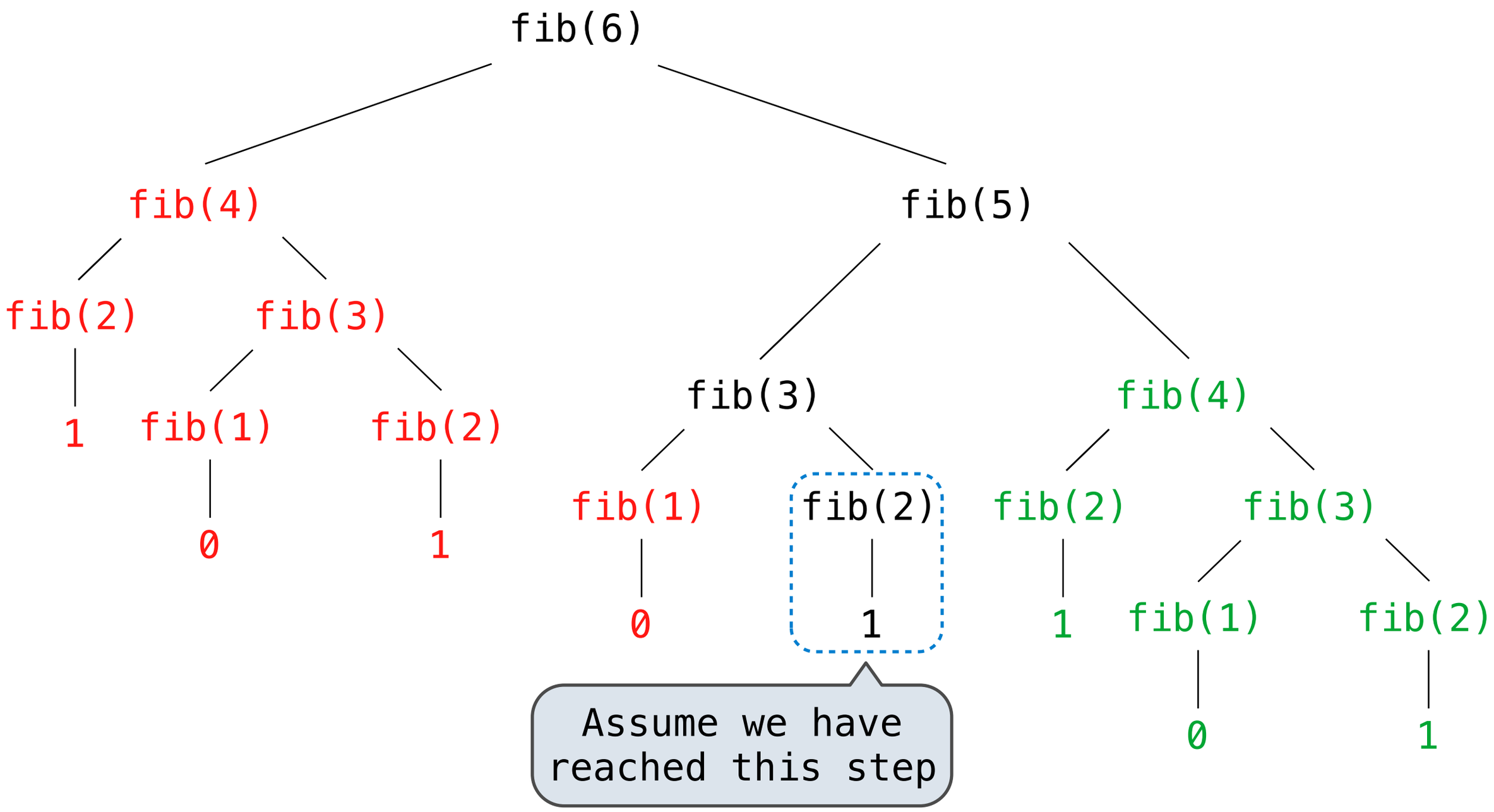
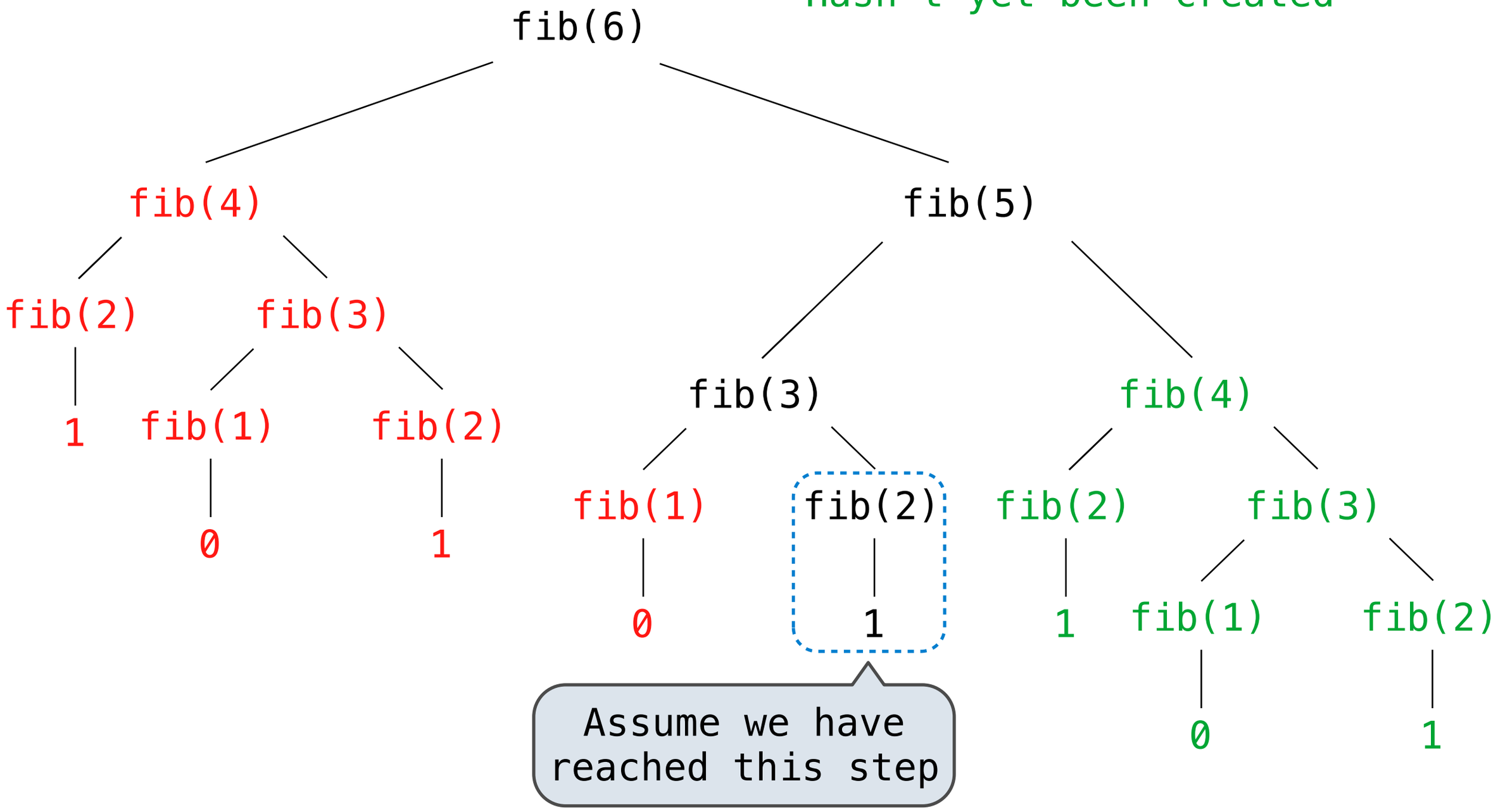


Fibonacci Memory Consumption

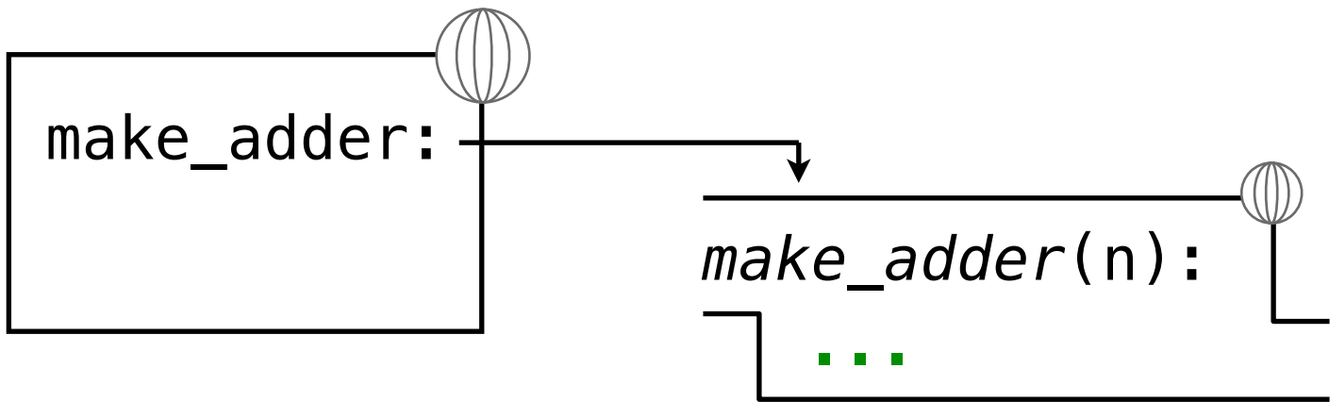
Has an active environment

Can be reclaimed

Hasn't yet been created



Active Environments for Returned Functions

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add1 = make_adder(1)
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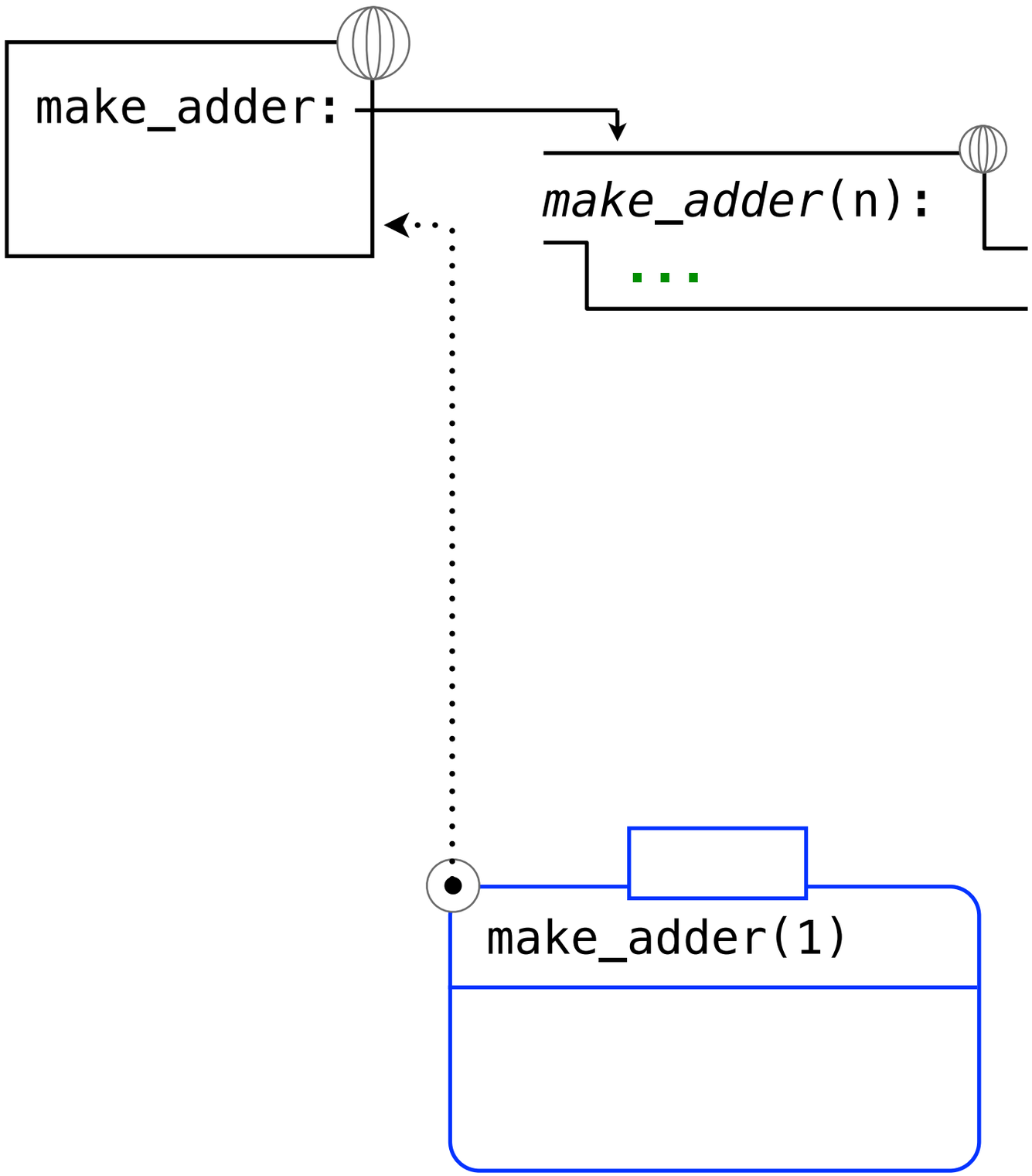


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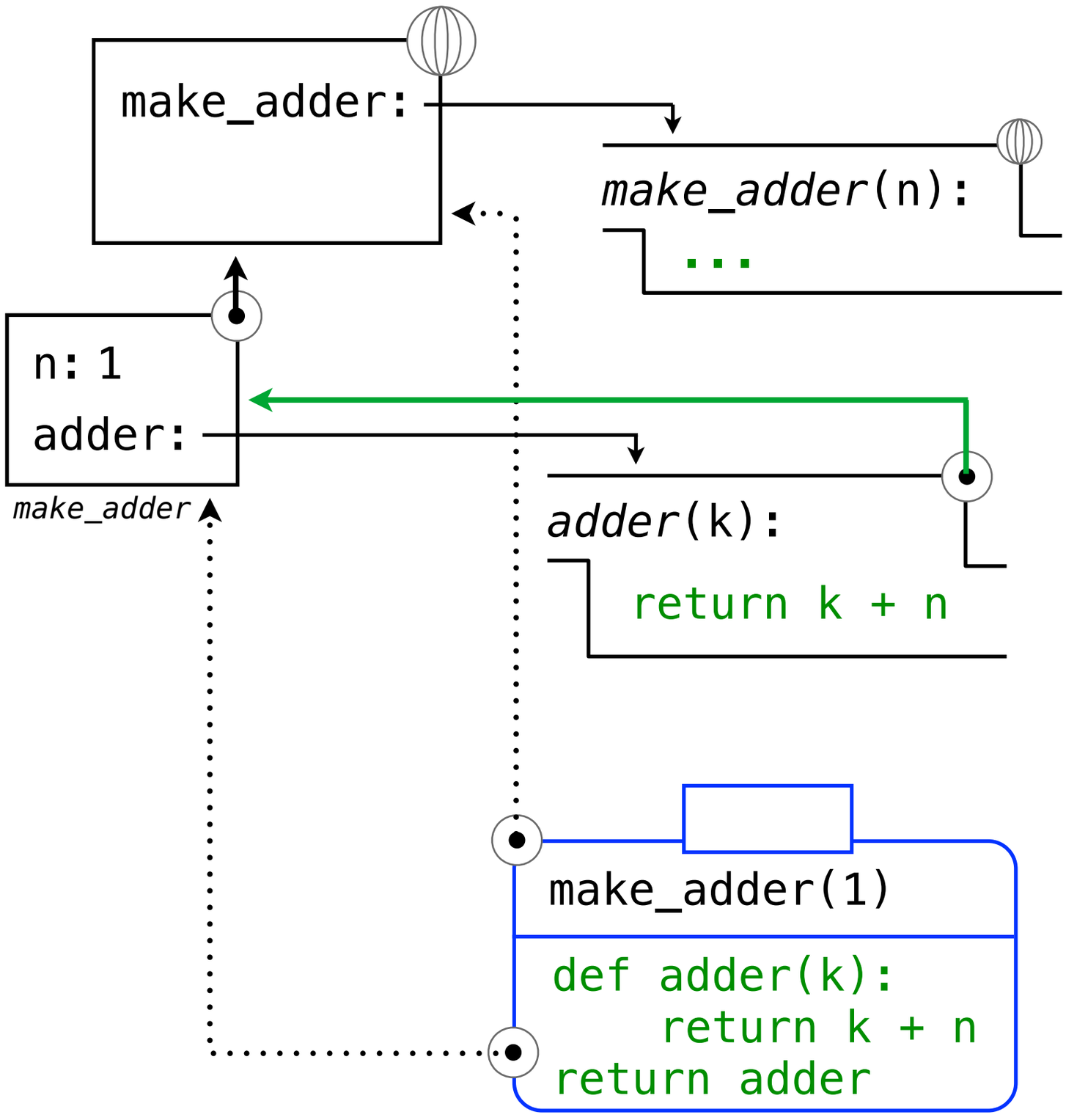


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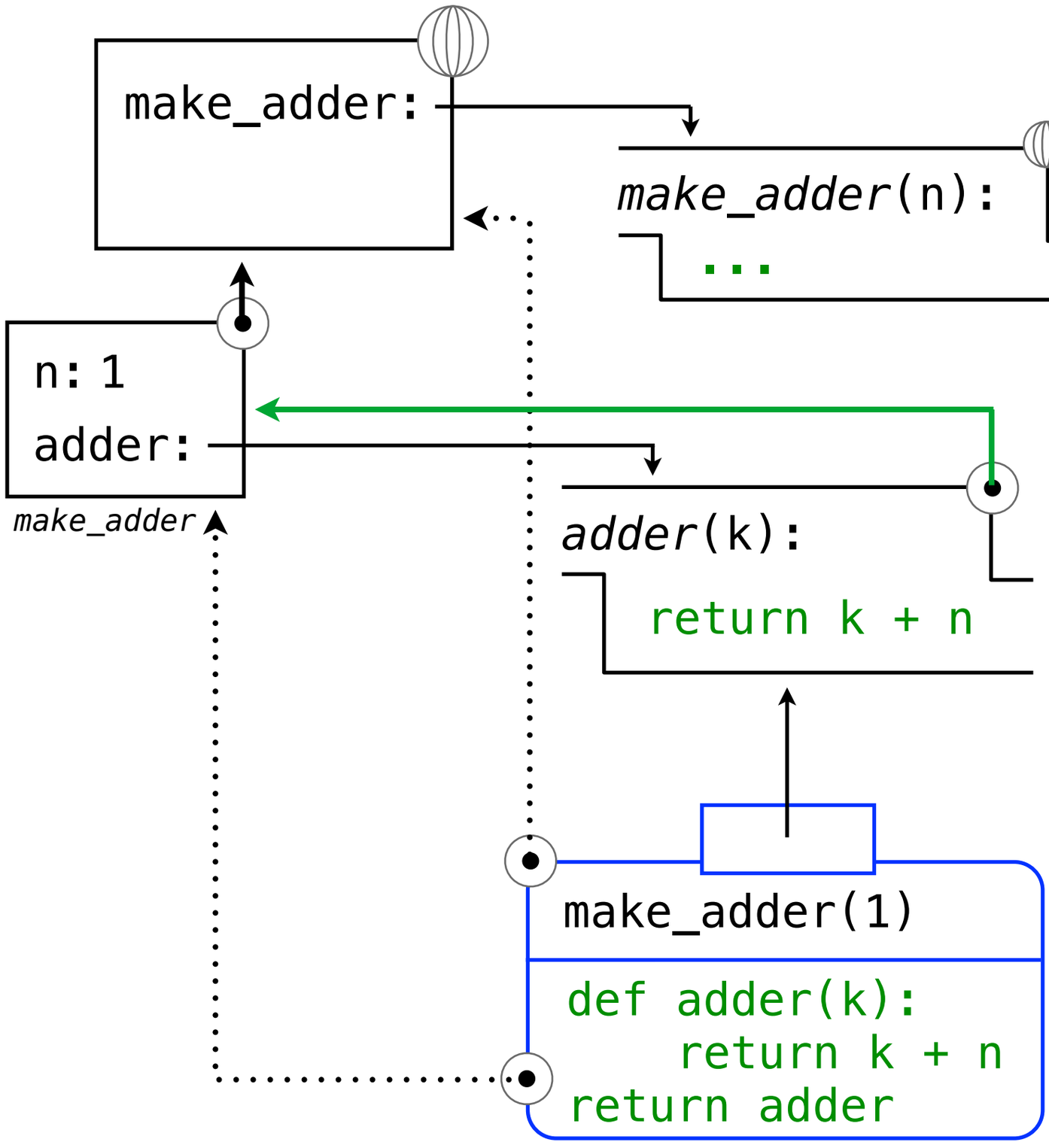


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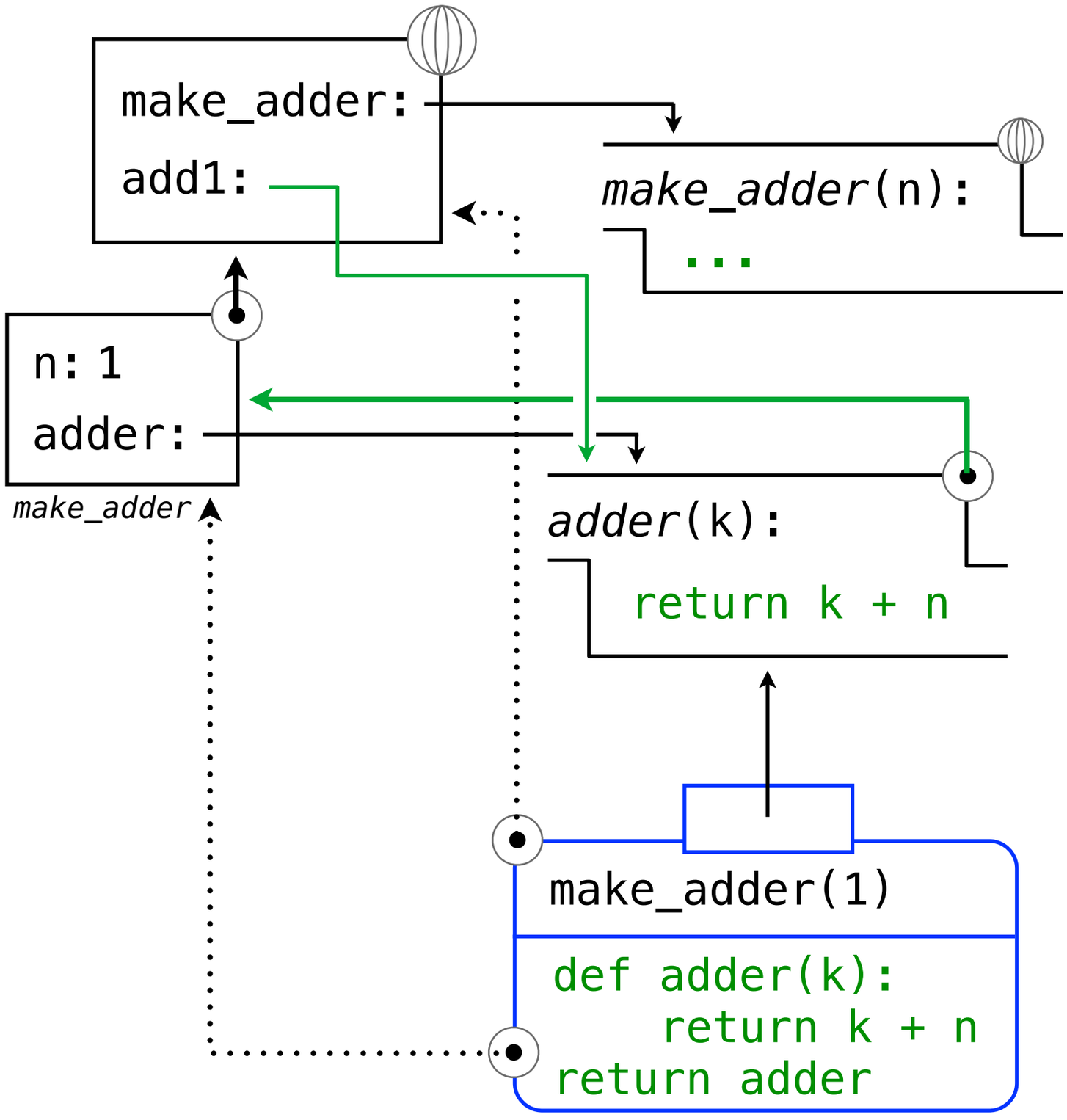


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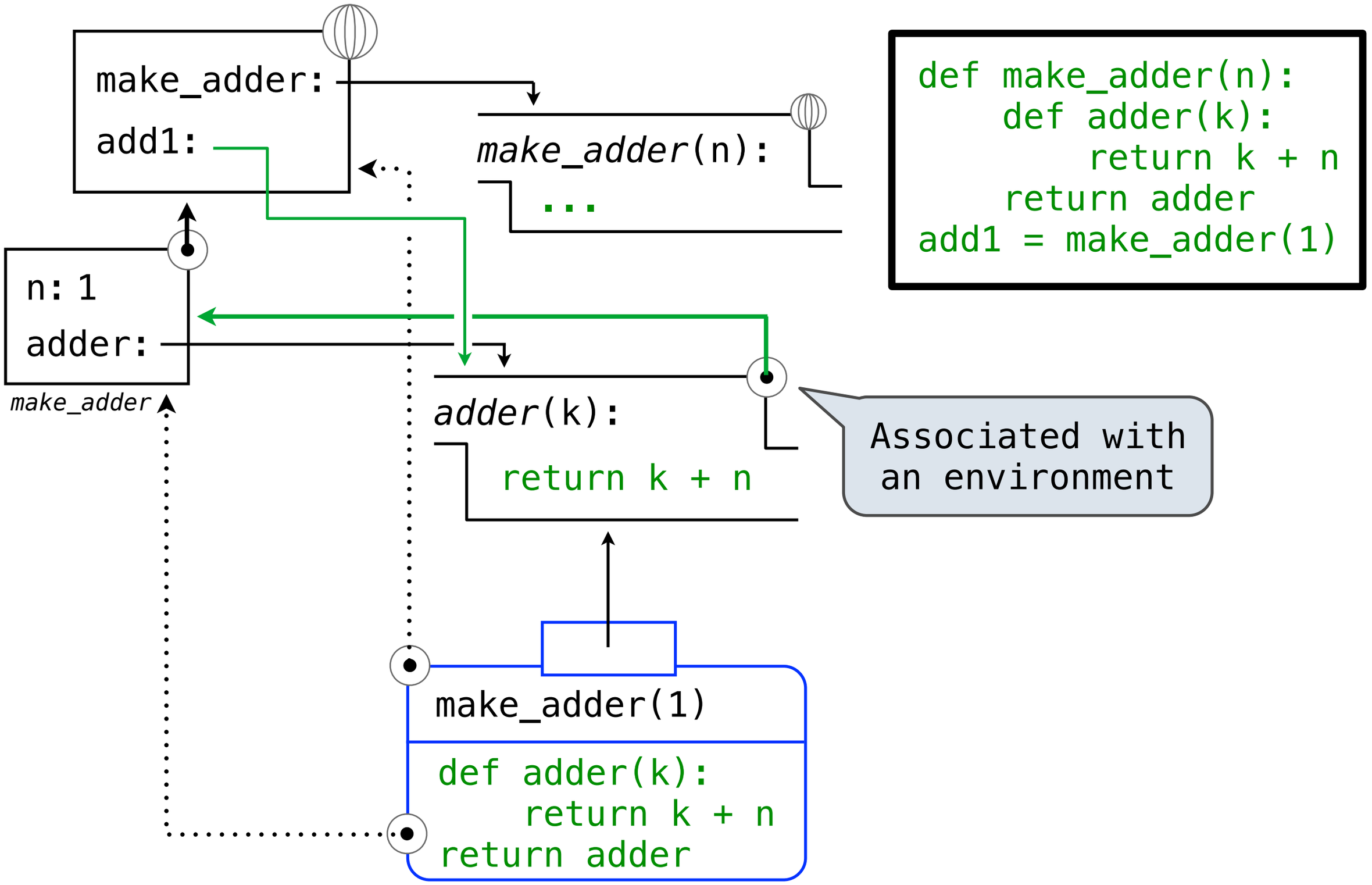


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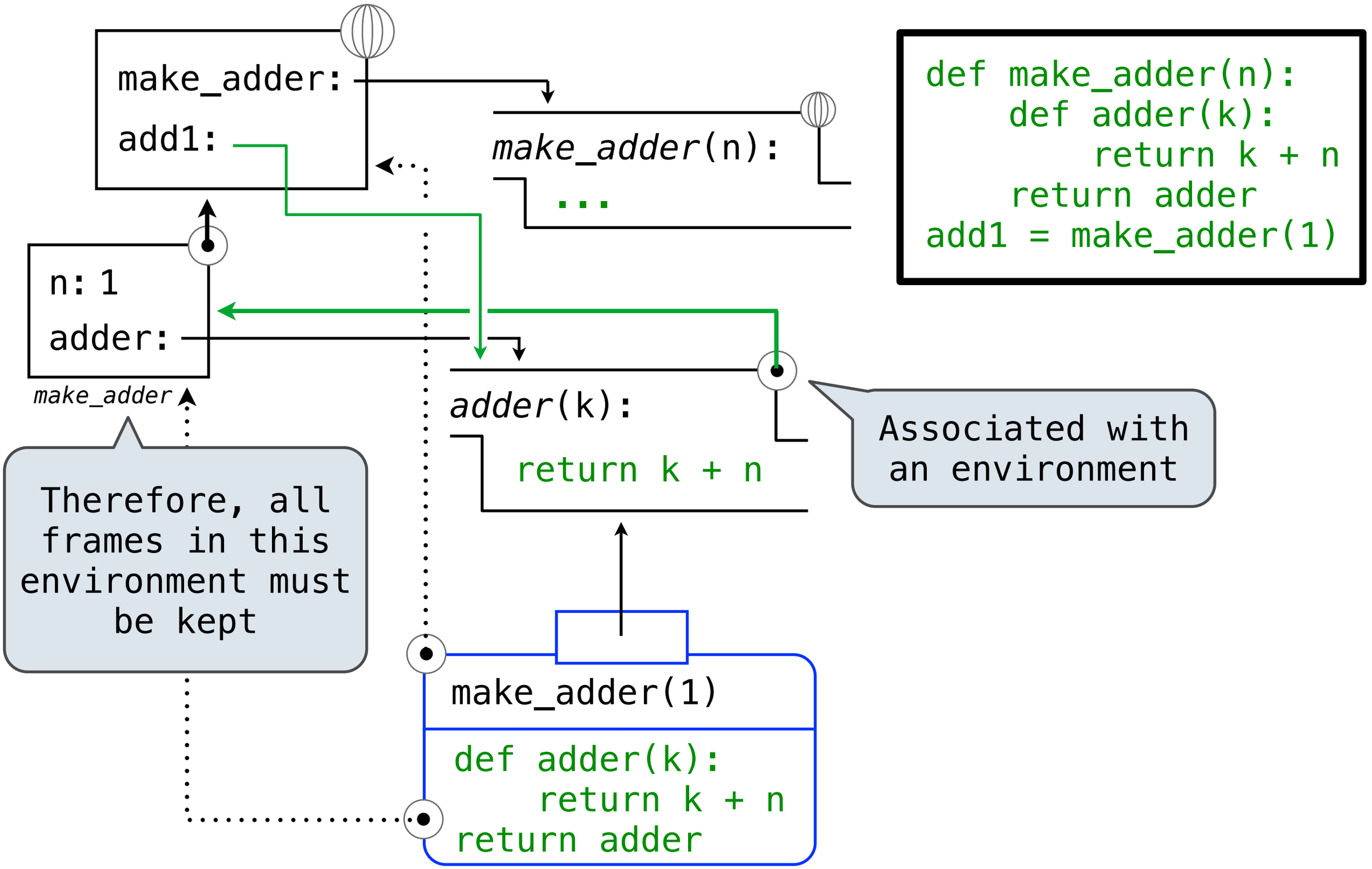

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for sufficiently large values of ***n*.**

Iteration vs Memoized Tree Recursion

Iterative and memoized implementations are not the same.

	Time	Space

```
def fib_iter(n):
    prev, curr = 1, 0
    for _ in range(n-1):
        prev, curr = curr, prev + curr
    return curr
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@memo
def fib(n):
    if n == 1:
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$\Theta(1)$ Constant. The problem size doesn't matter.

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