

CS 61A Lecture 9

Friday, September 16

The Sequence Abstraction

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Length. A sequence has a finite length.

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The sequence abstraction is shared among several types.

Tuples are Sequences

(Demo)

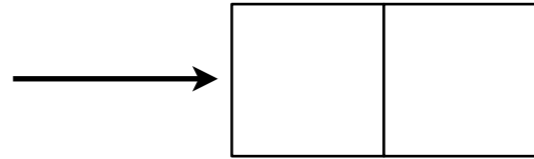
Box-and-Pointer Notation for Nested Pairs

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(1, 2)

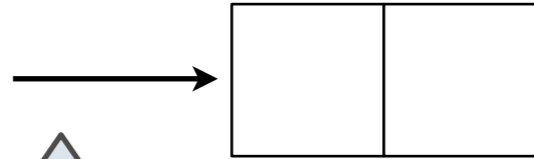
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Every object is
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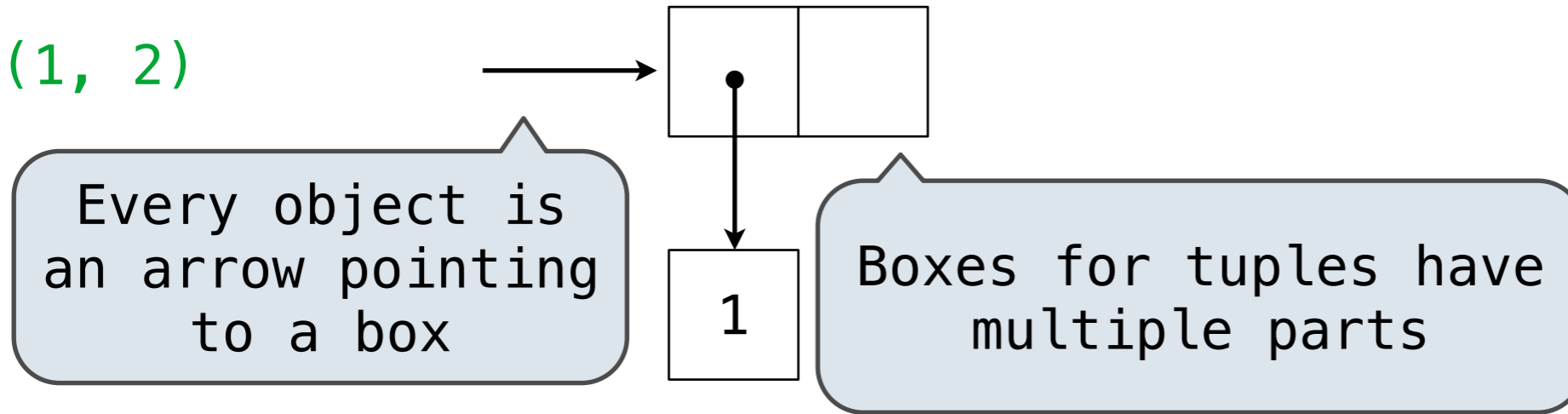


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Boxes for tuples have
multiple parts

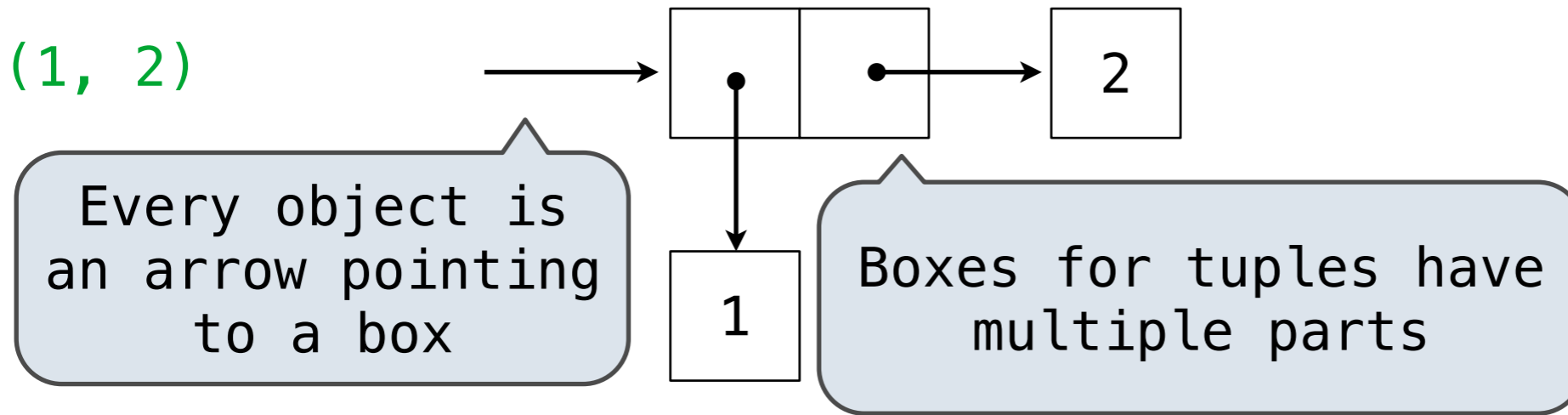
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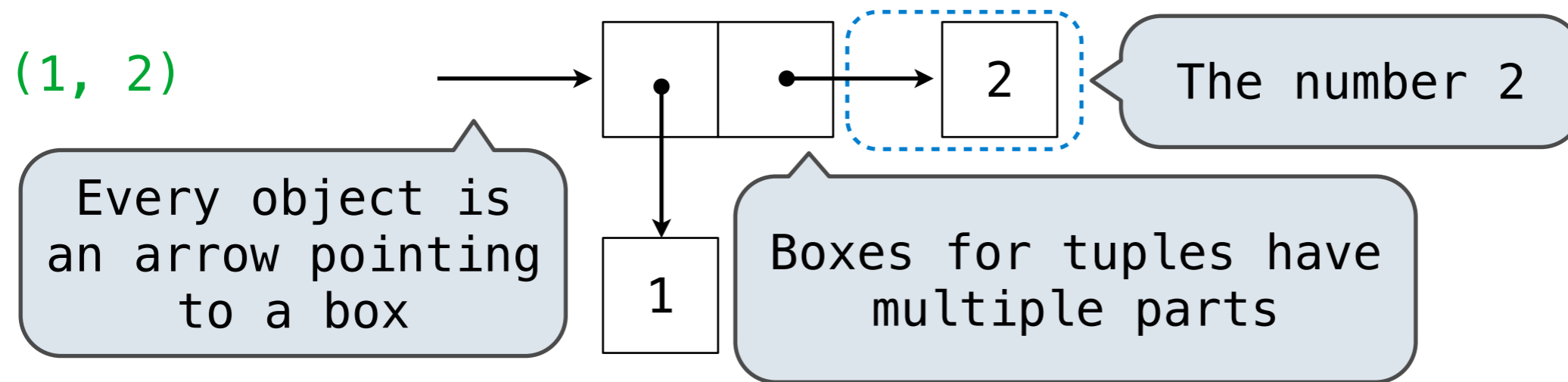


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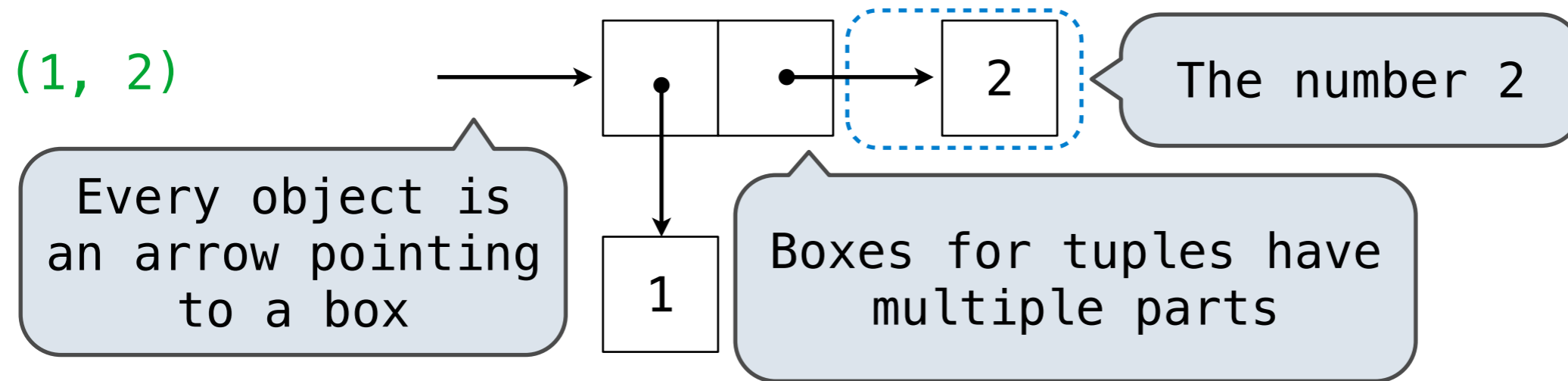
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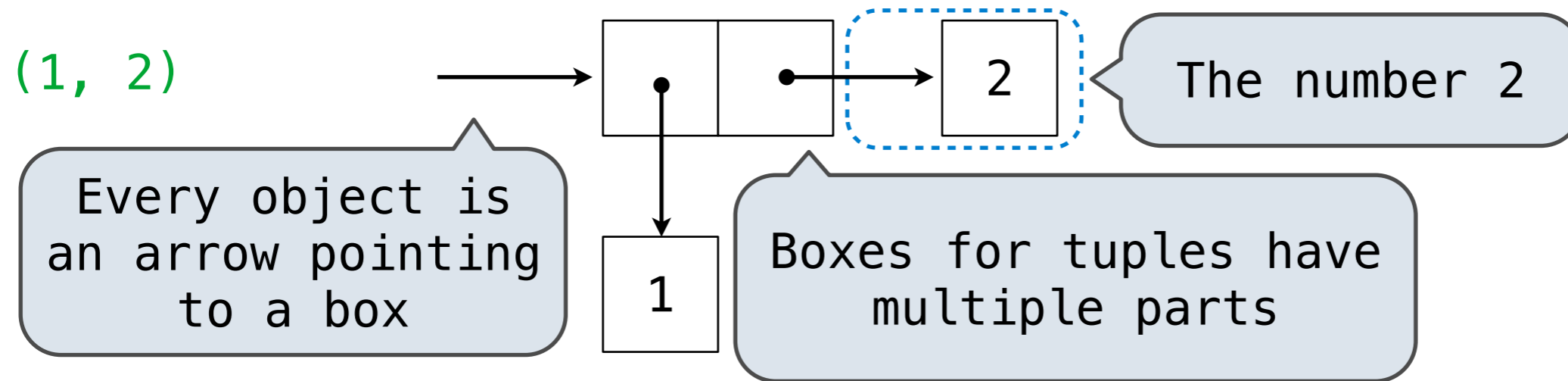


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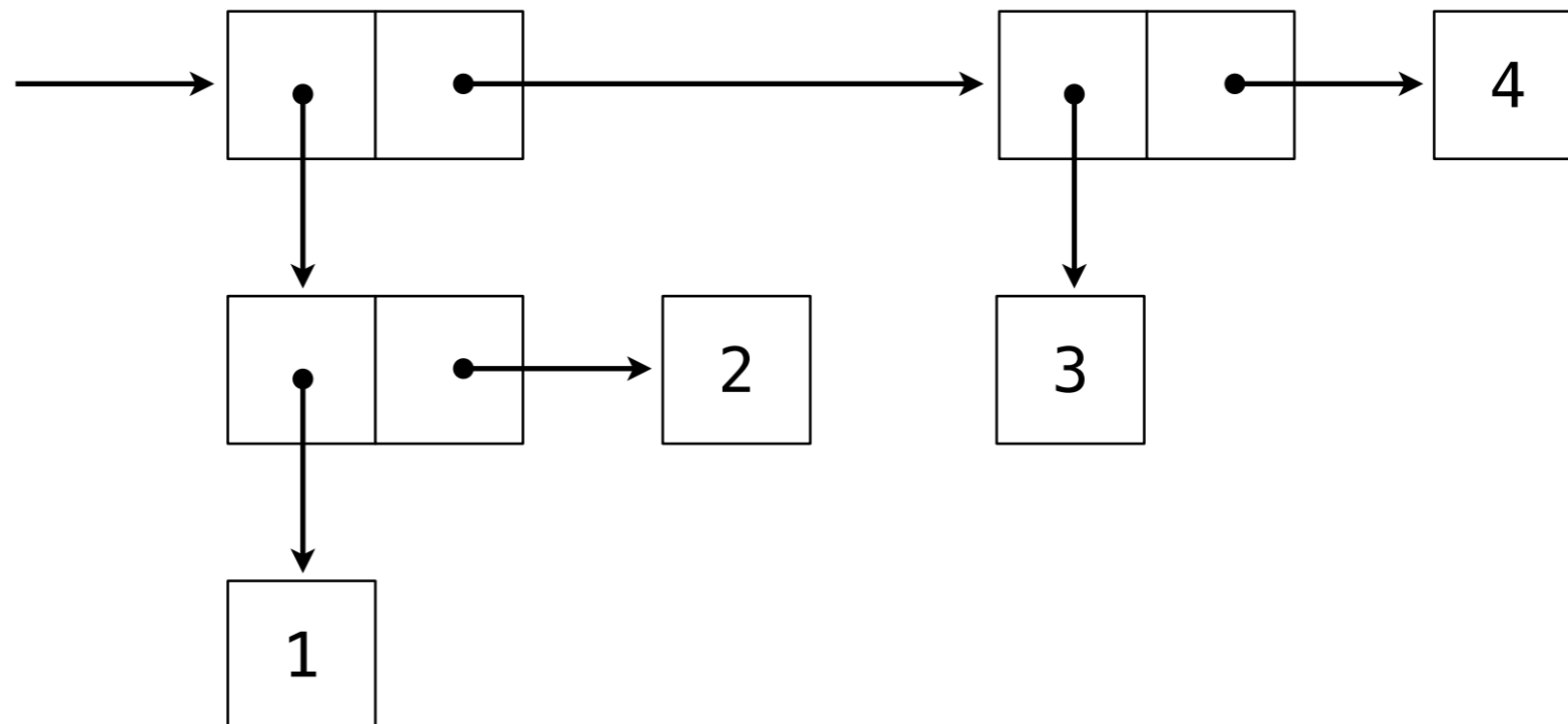


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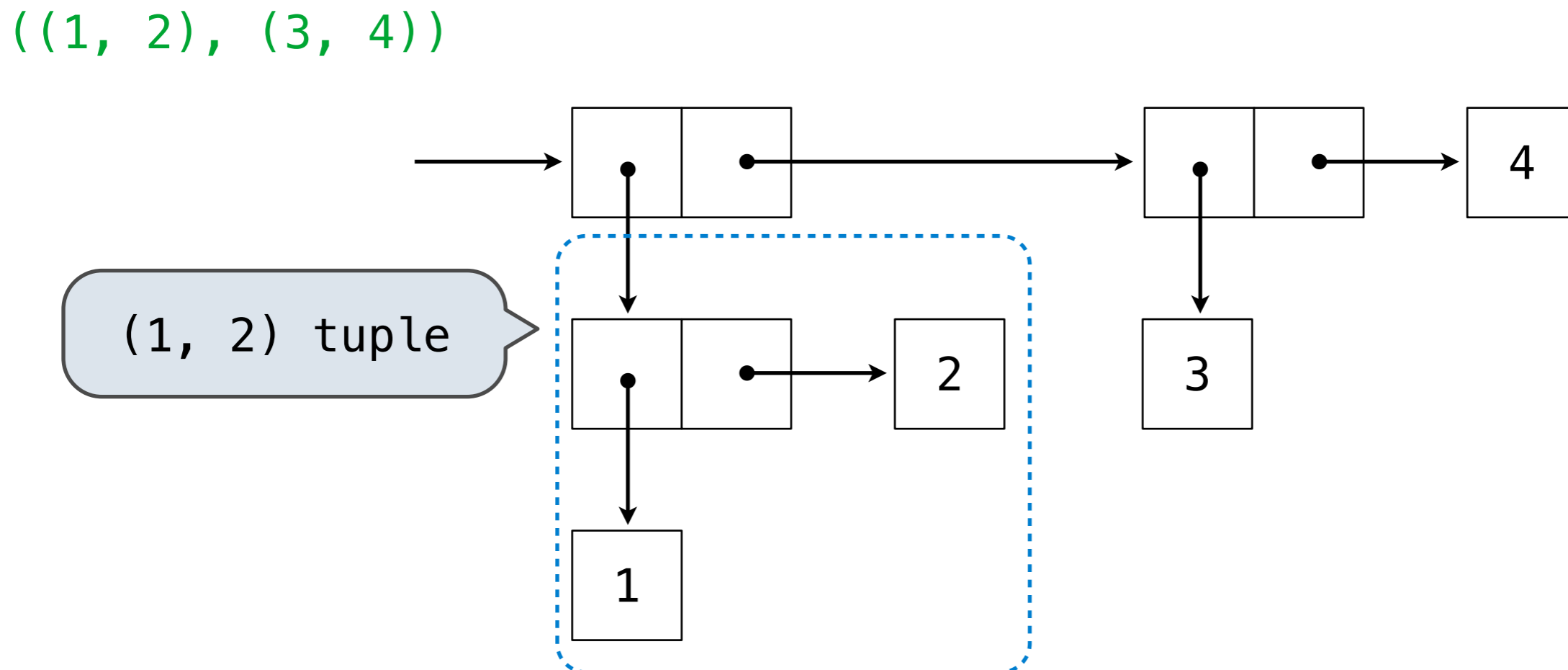
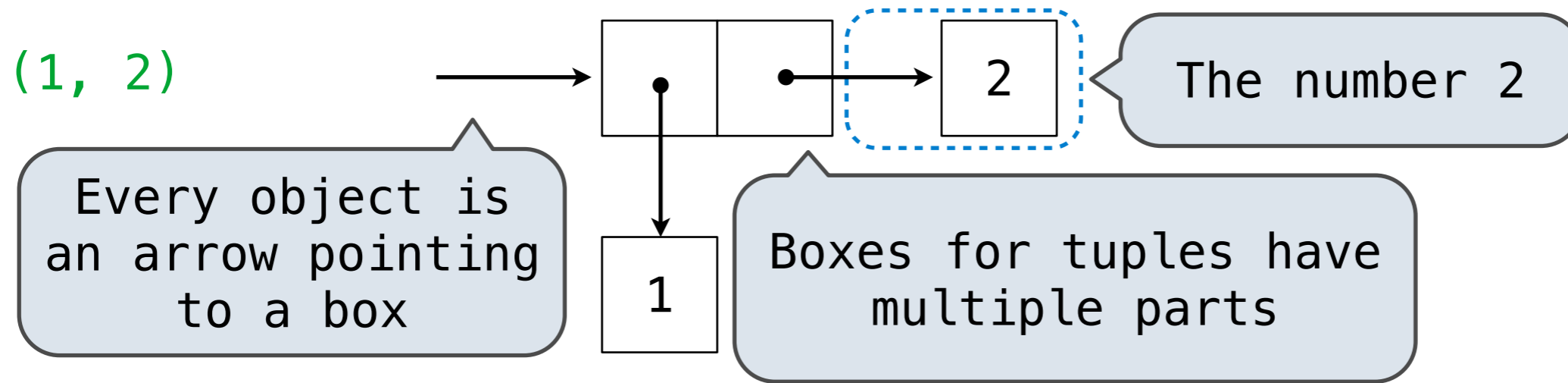
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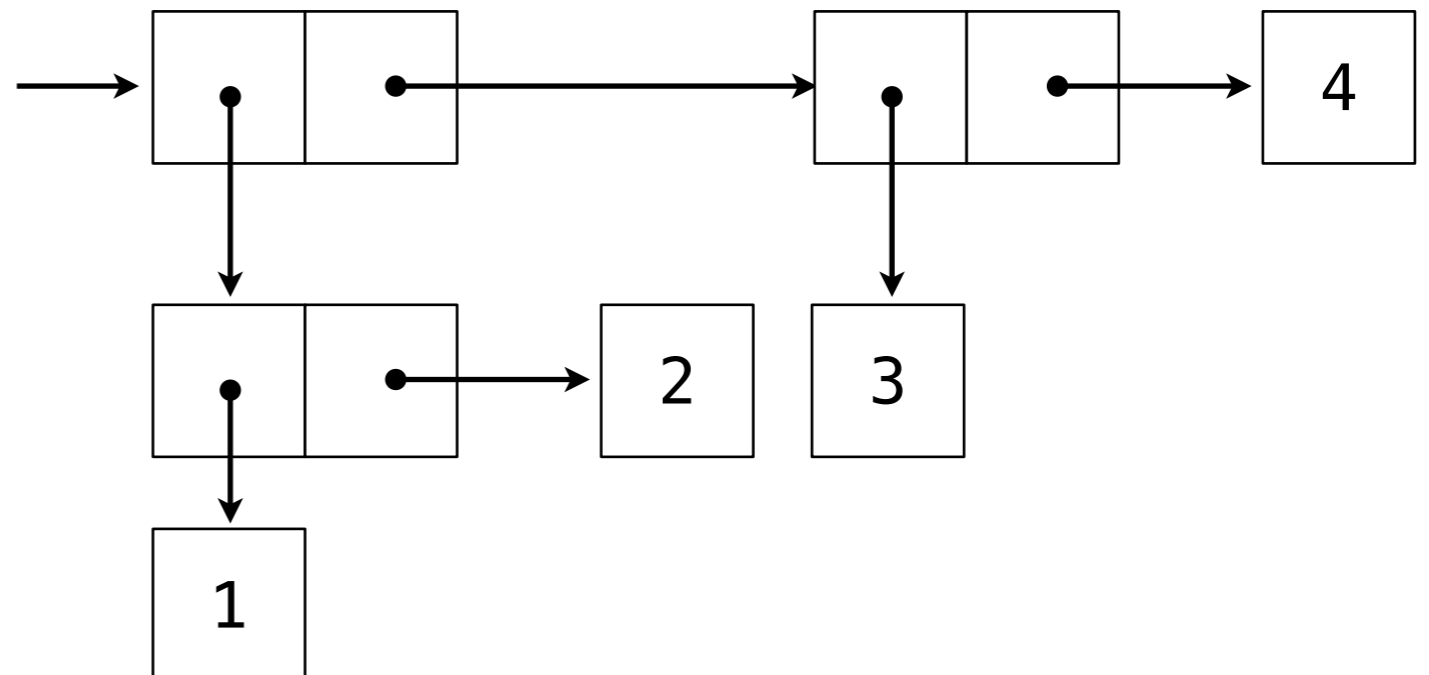


Box-and-Pointer Notation for Nested Pairs



Alternative Nested Structures

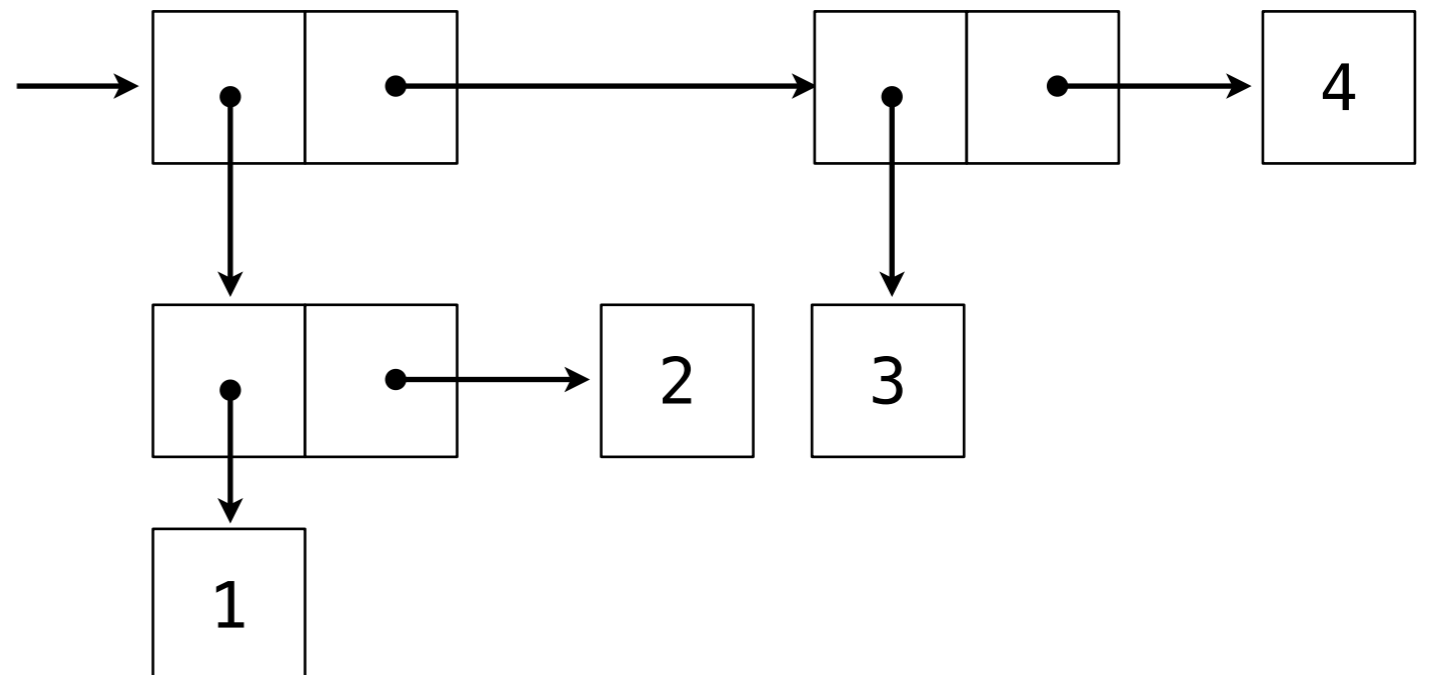
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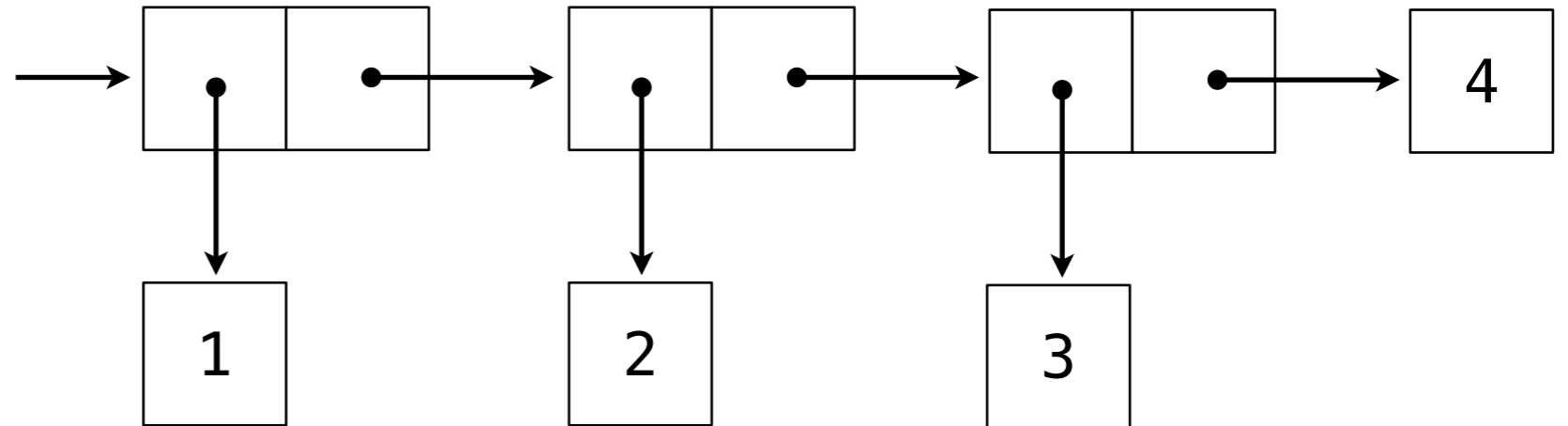
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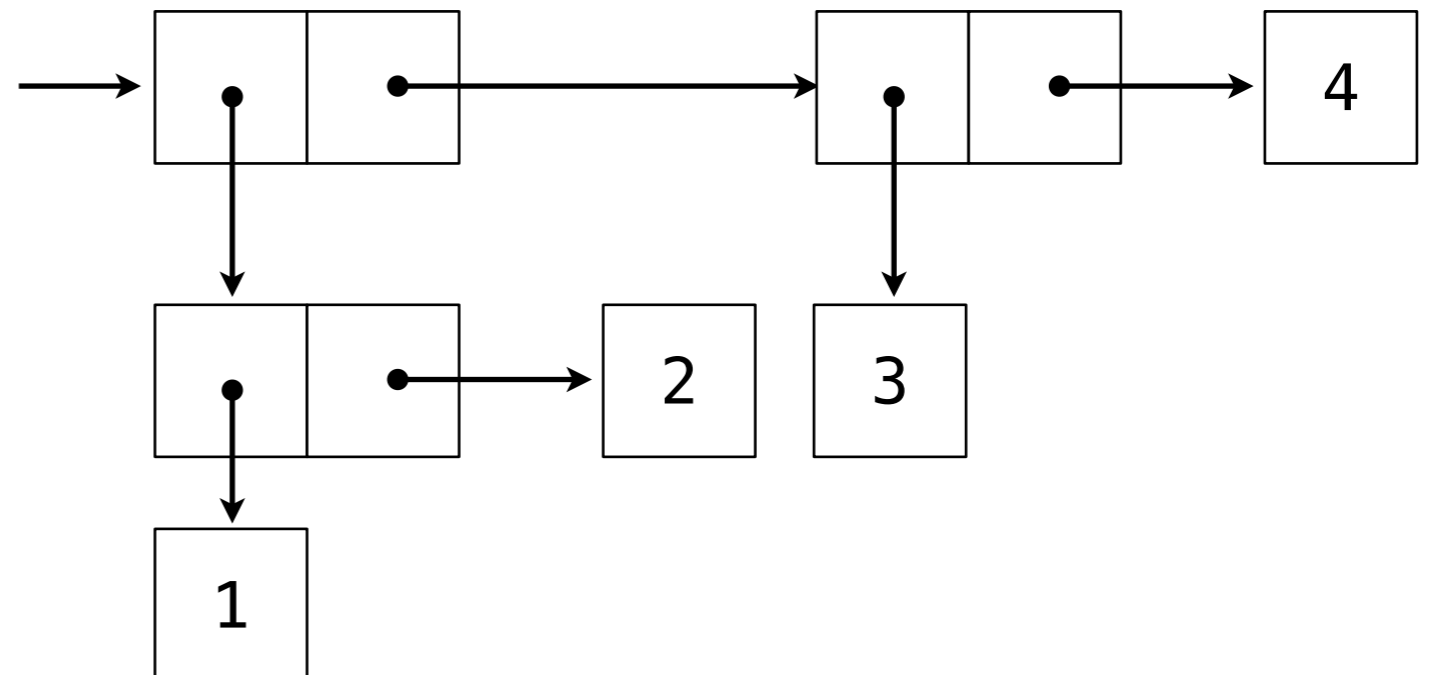


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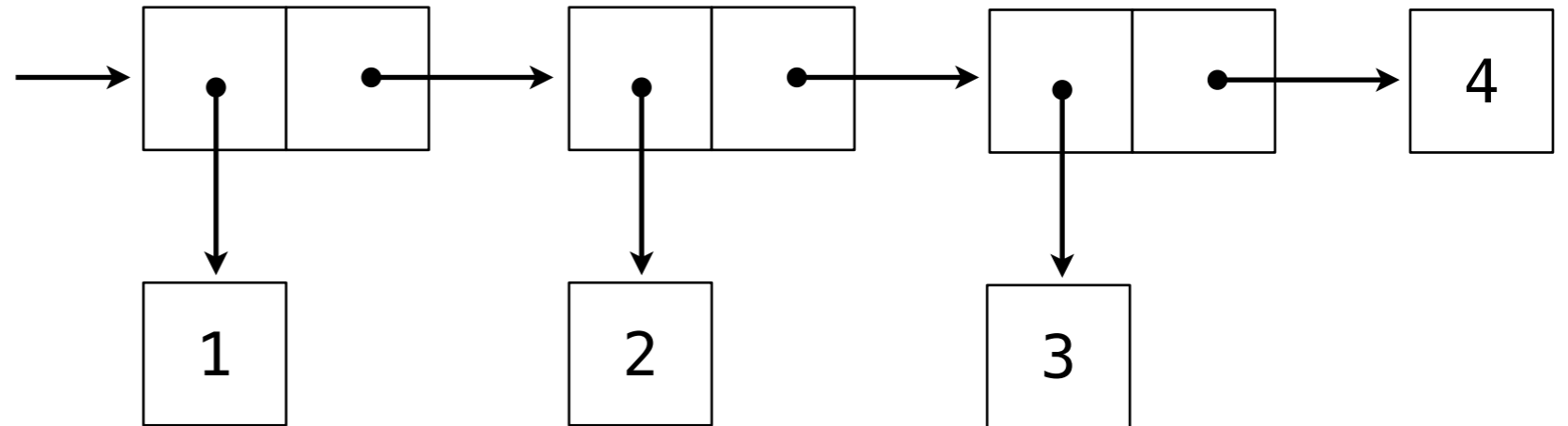


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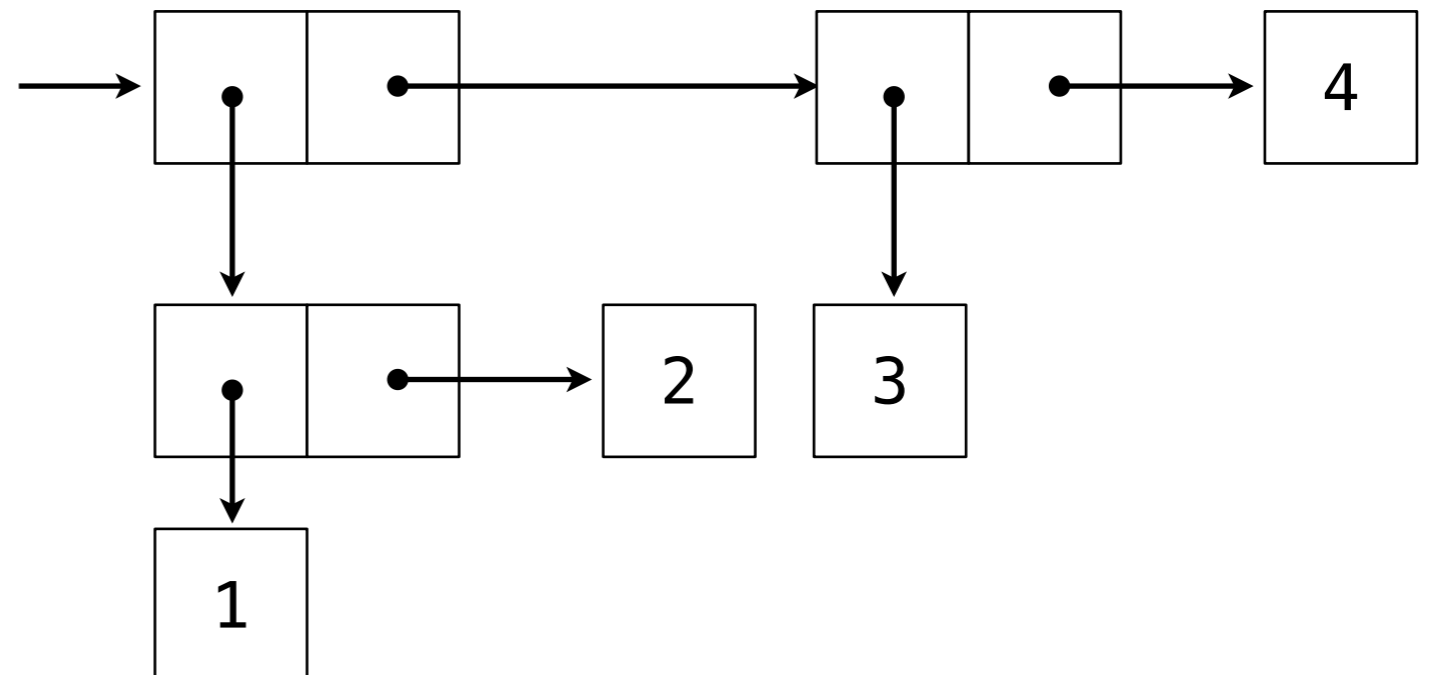
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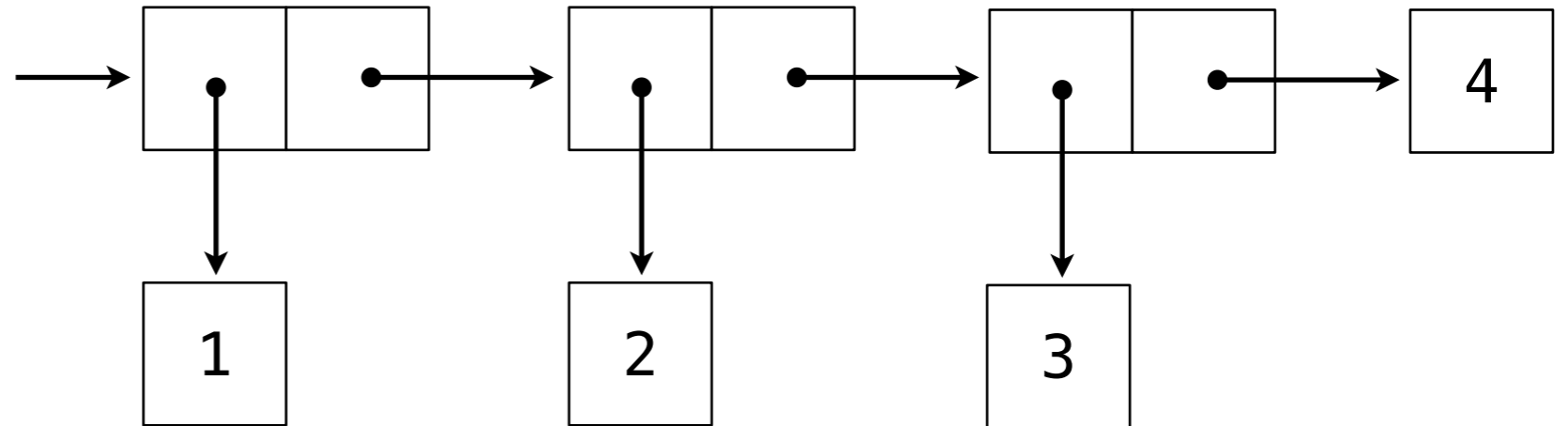
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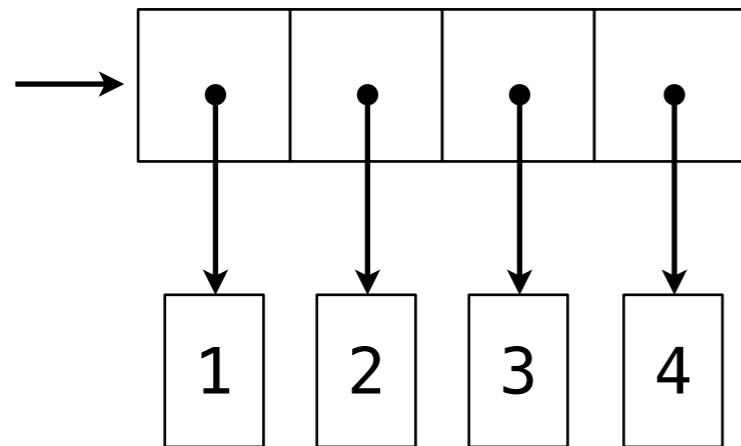


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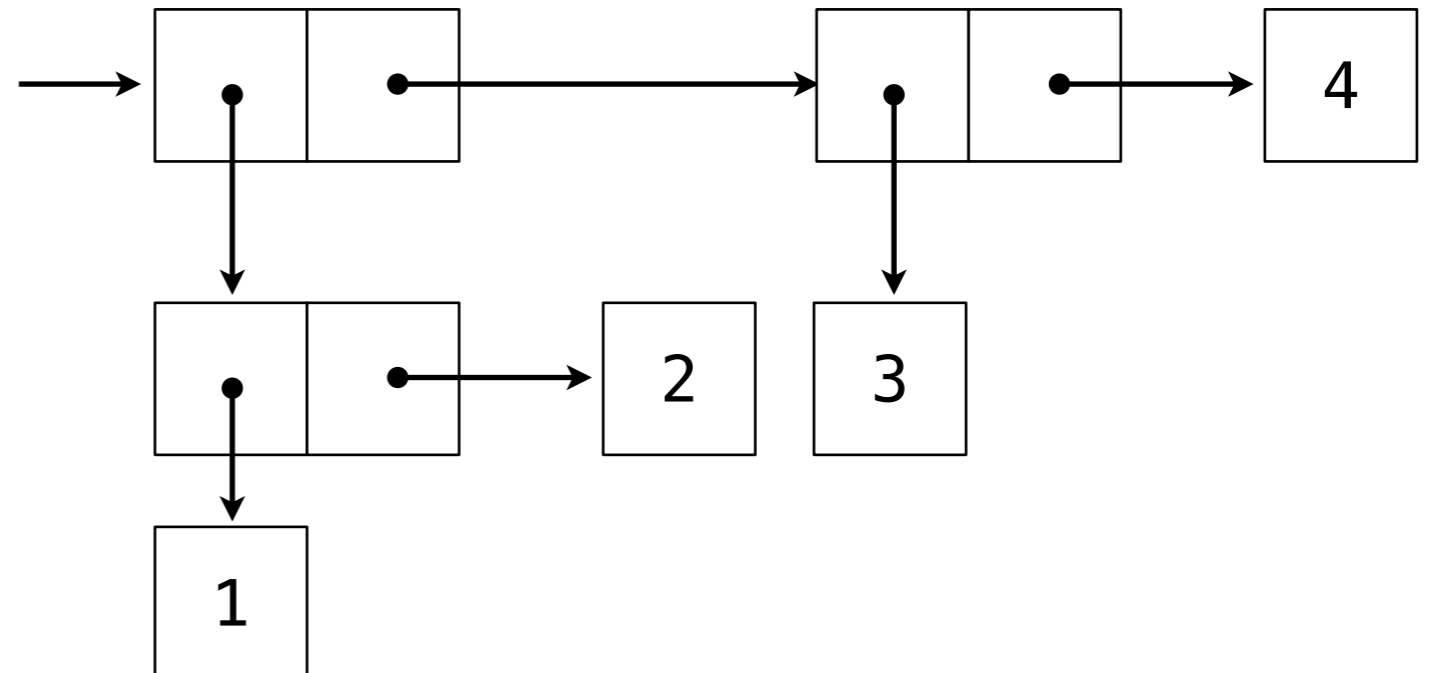
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Tuples can contain tuples as elements

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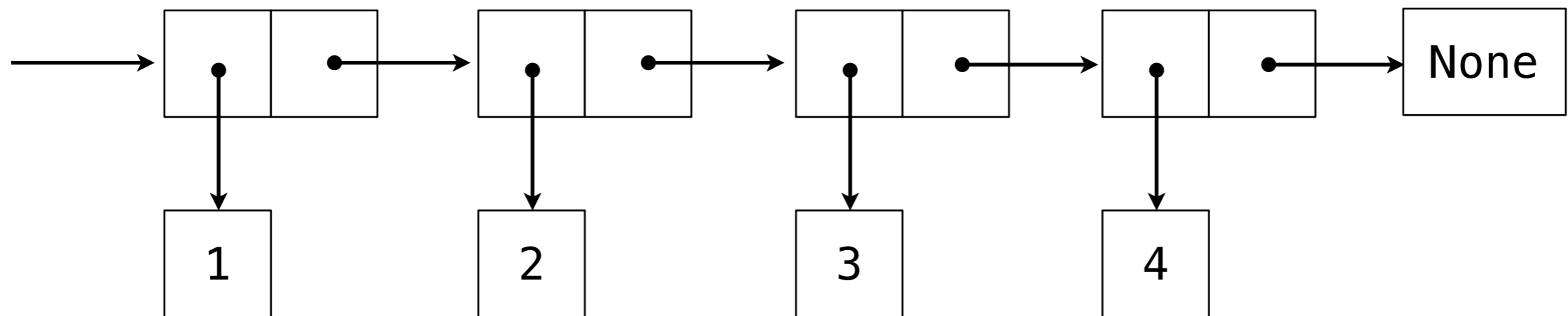
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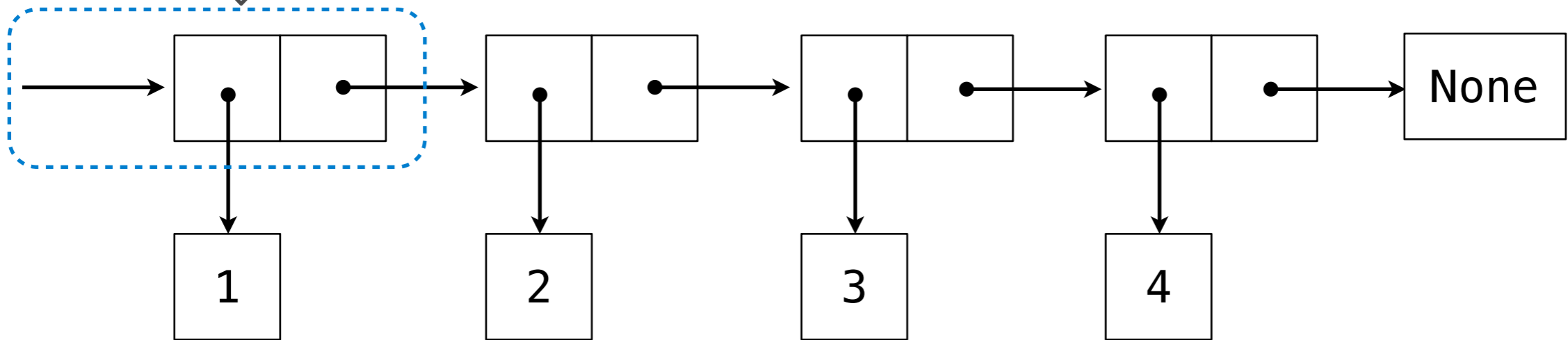
- `first(s)` returns f , and
- `rest(s)` returns r , which is a recursive list.

Implementing Recursive Lists with Pairs



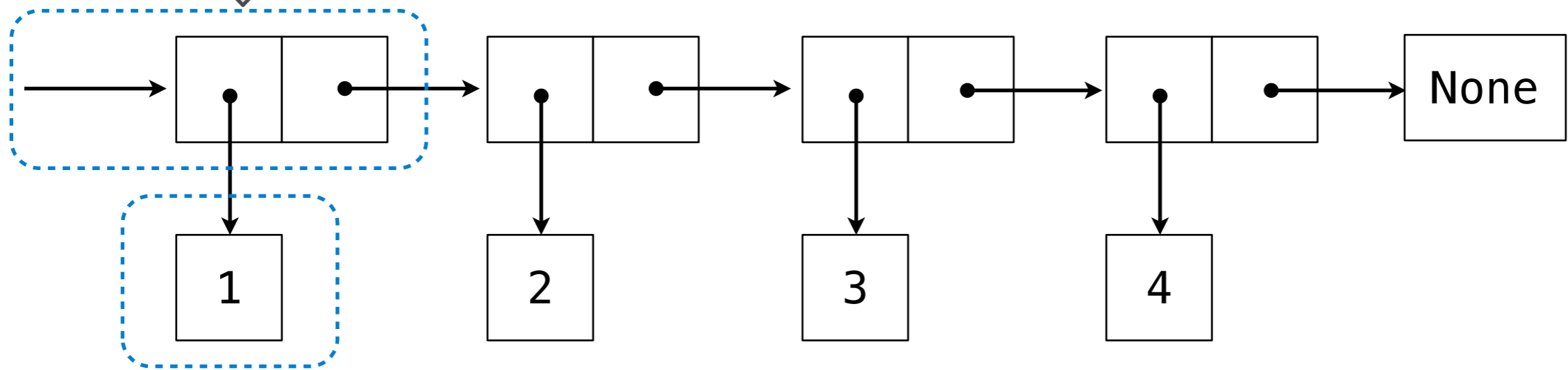
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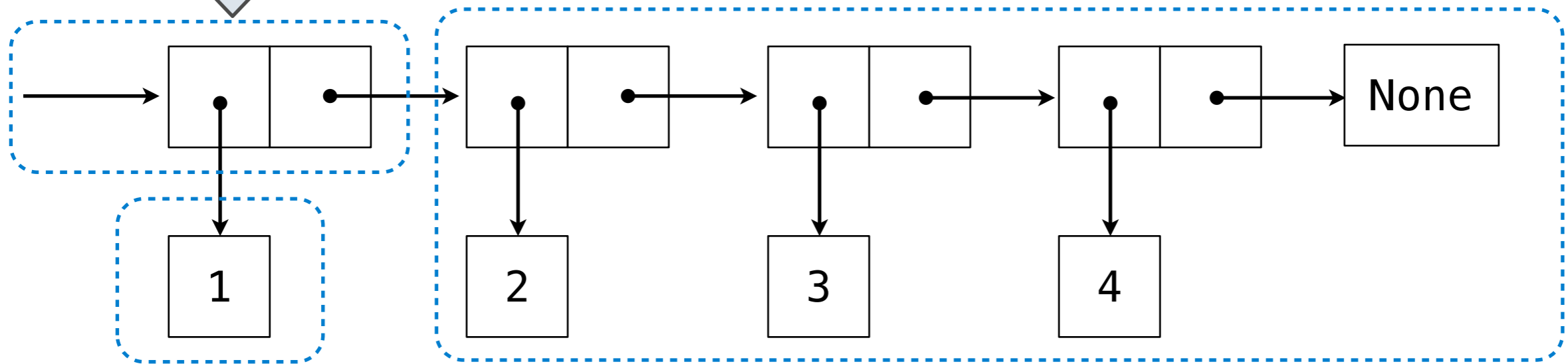
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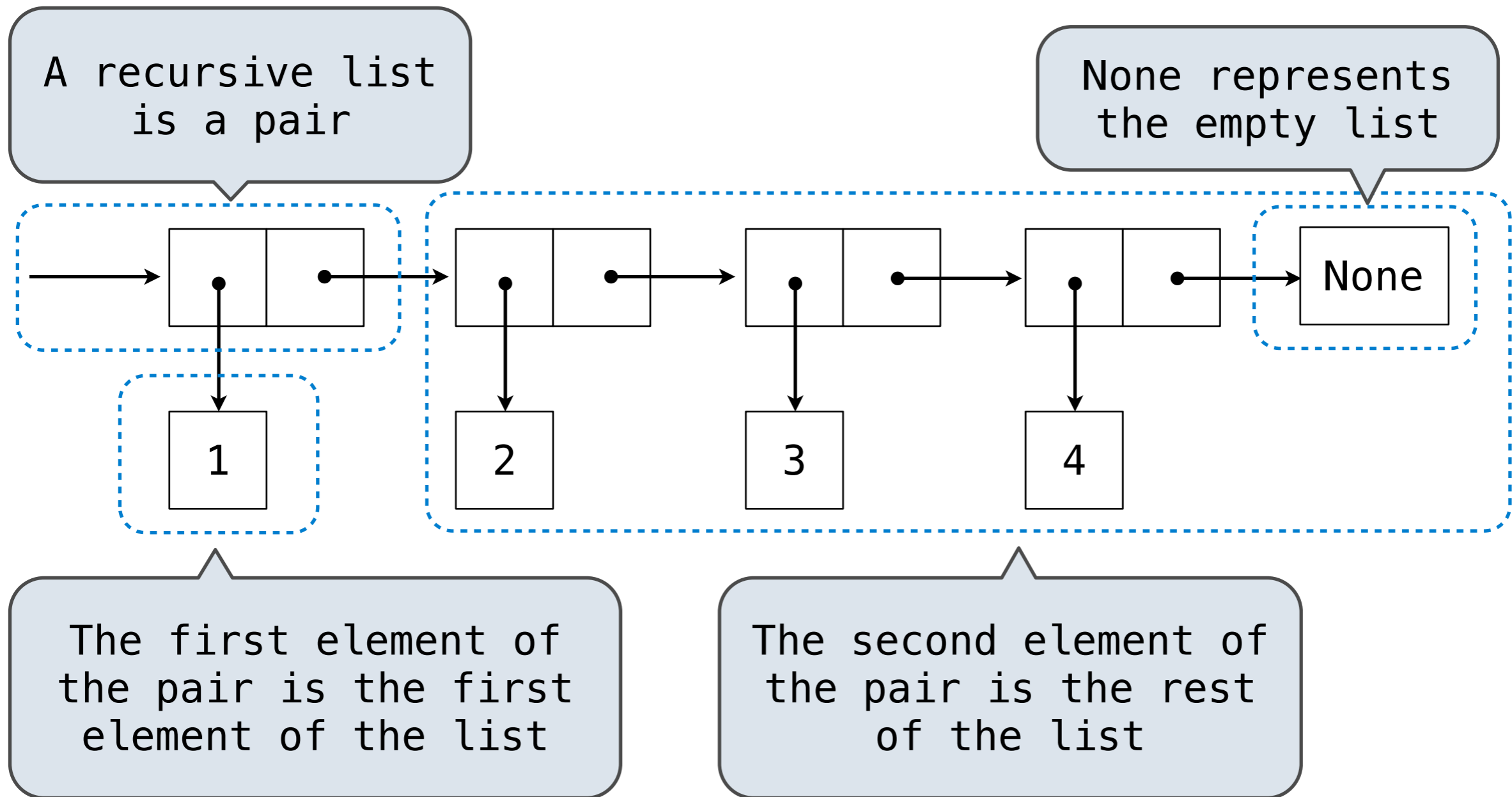
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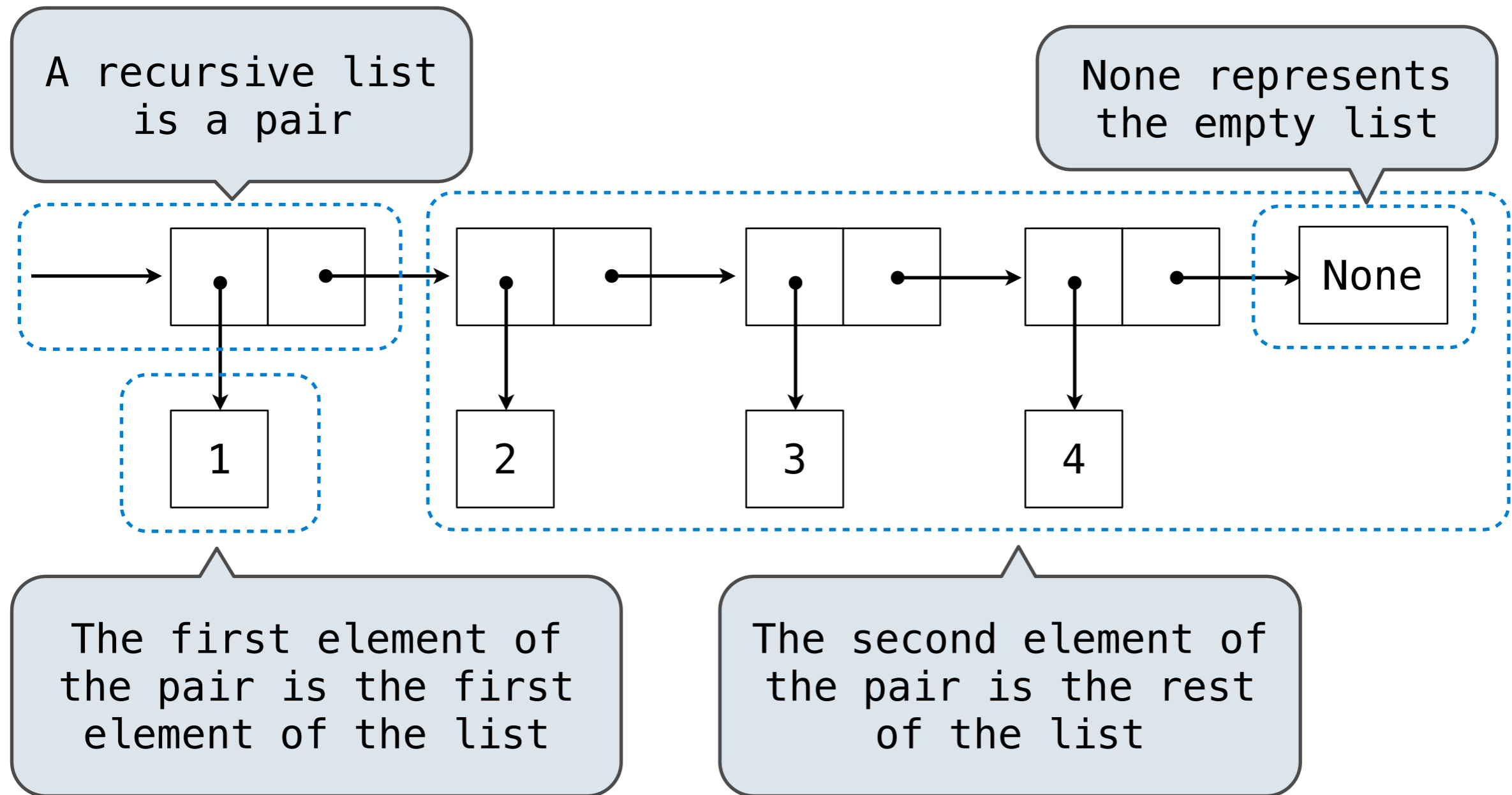
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The second element of the pair is the rest of the list

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(Demo)

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```
def len_rlist(s):  
    """Return the length of recursive list s."""  
    length = 0  
    while s != empty_rlist:  
        s, length = rest(s), length + 1  
    return length
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def getitem_rlist(s, i):
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    while i > 0:
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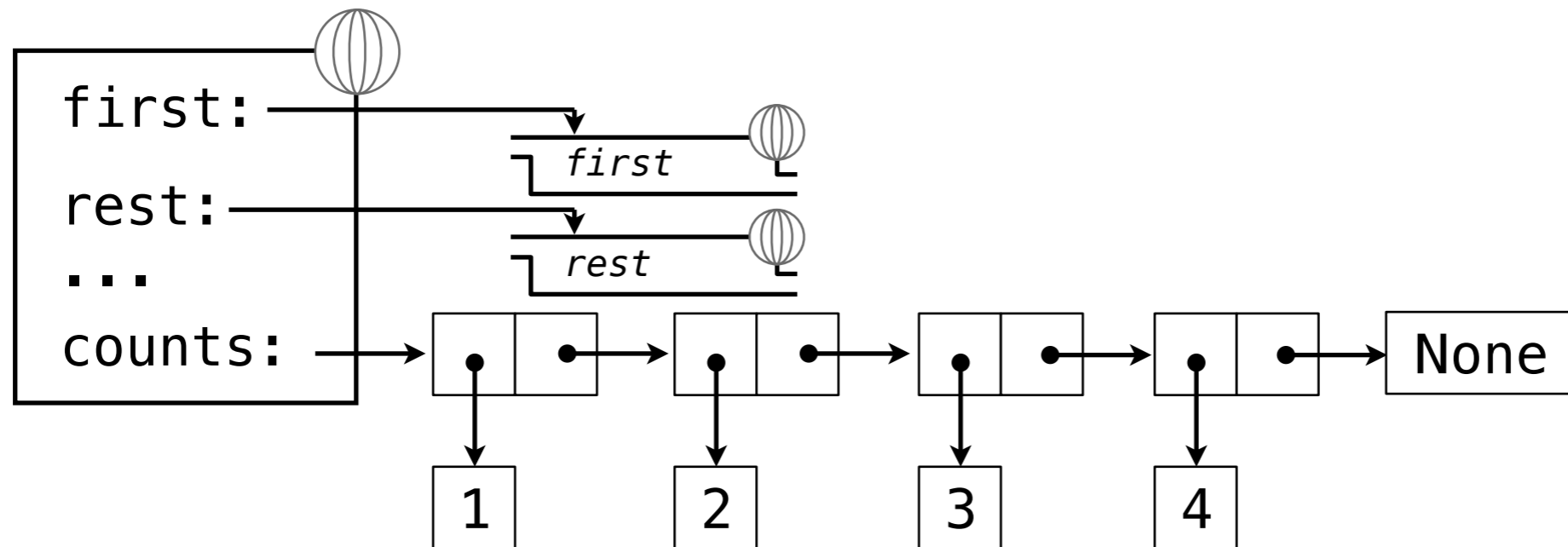
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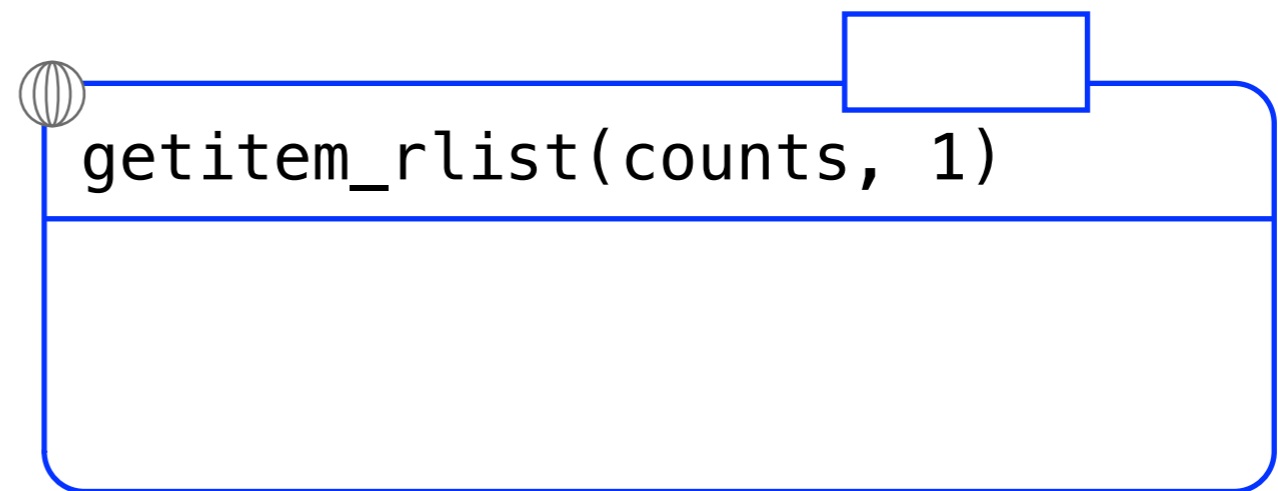
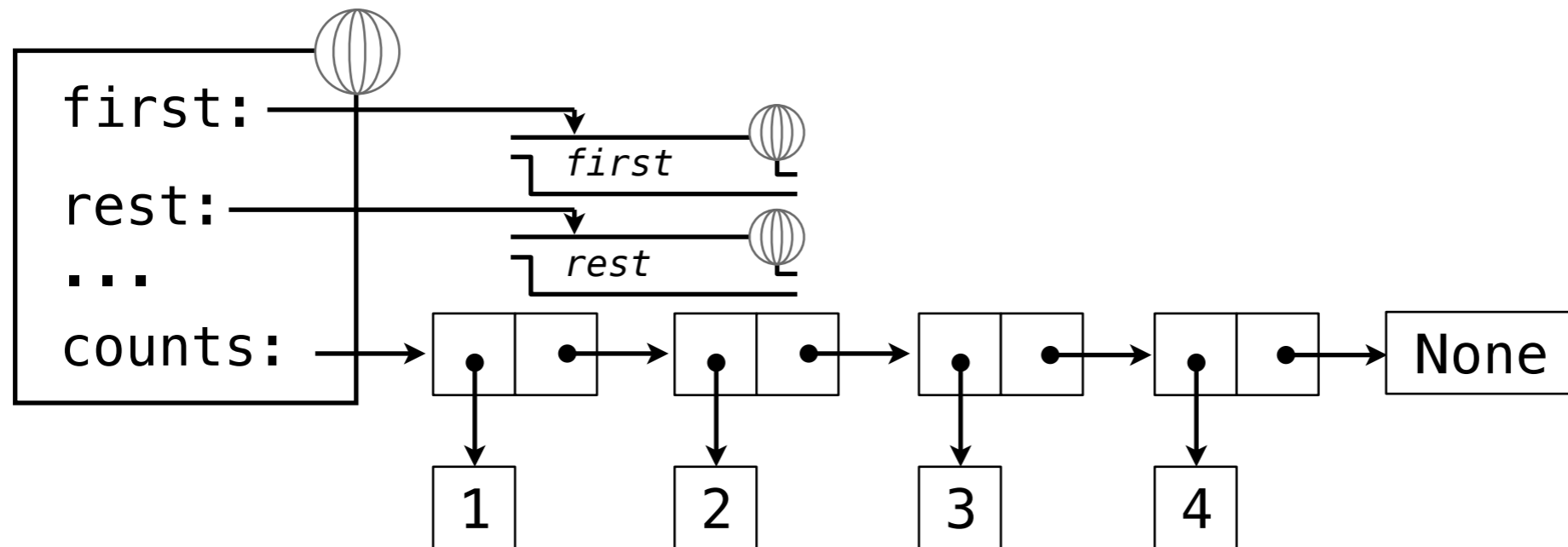
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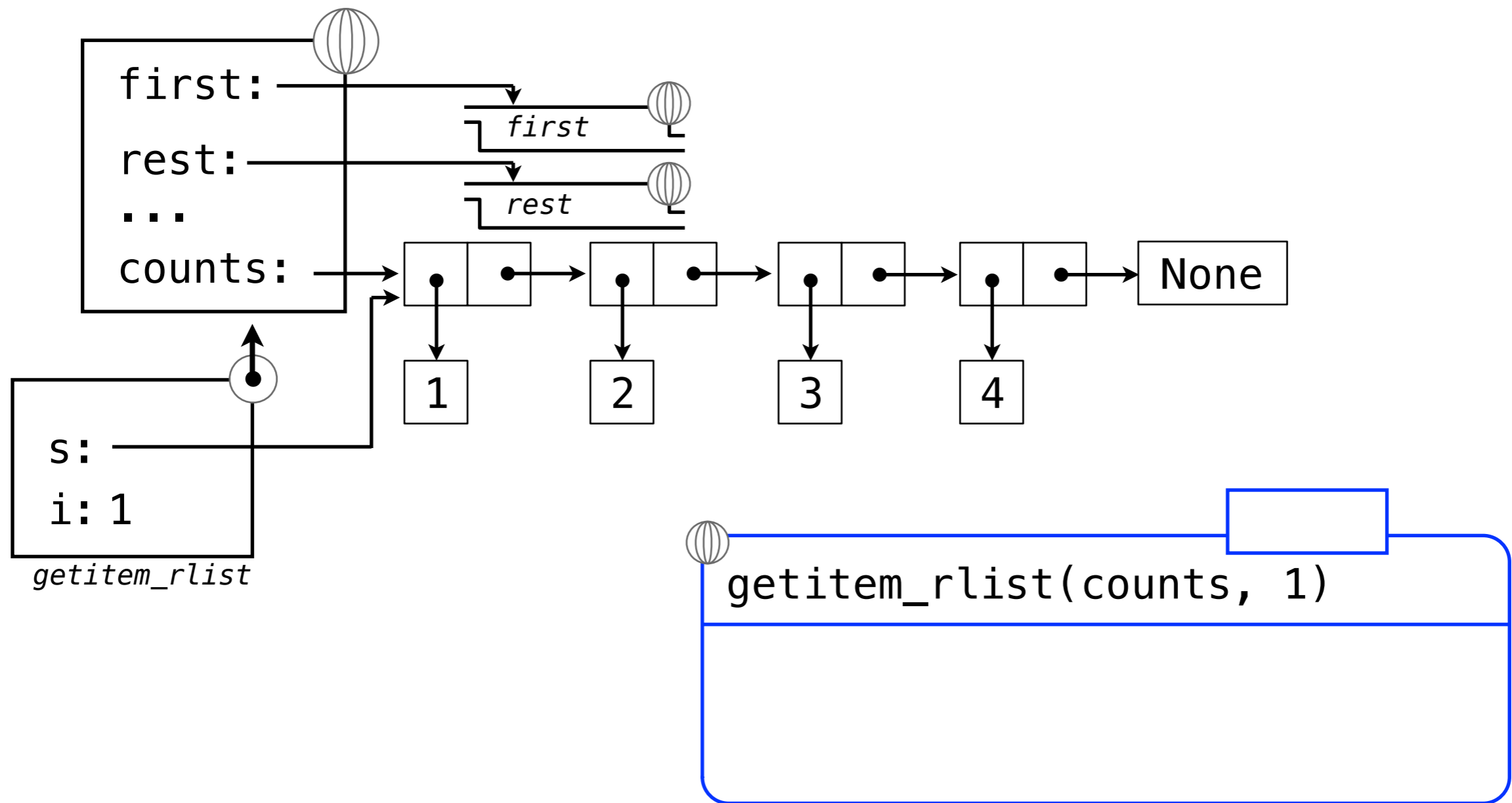
Environment Diagram for `getitem_rlist`



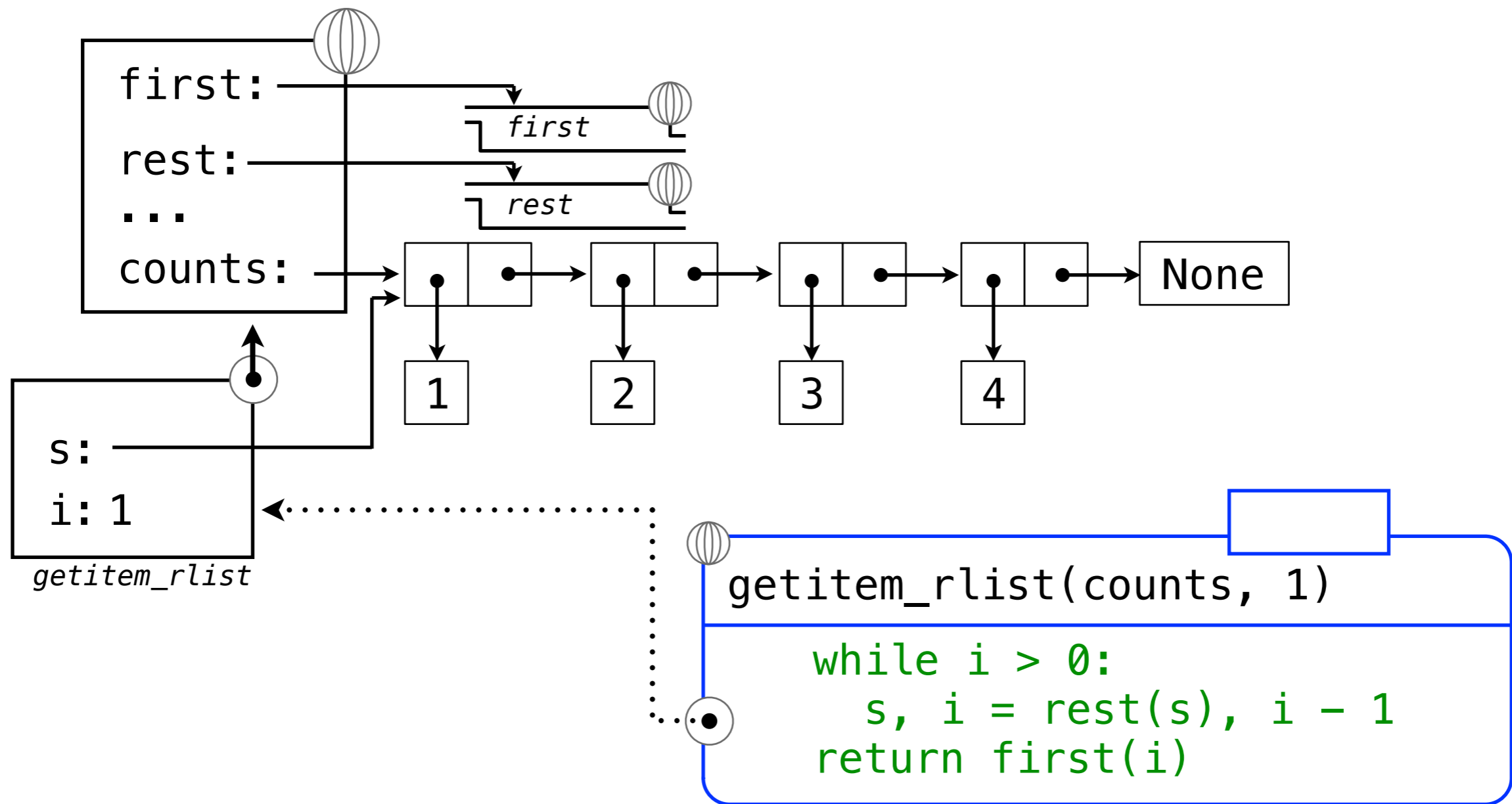
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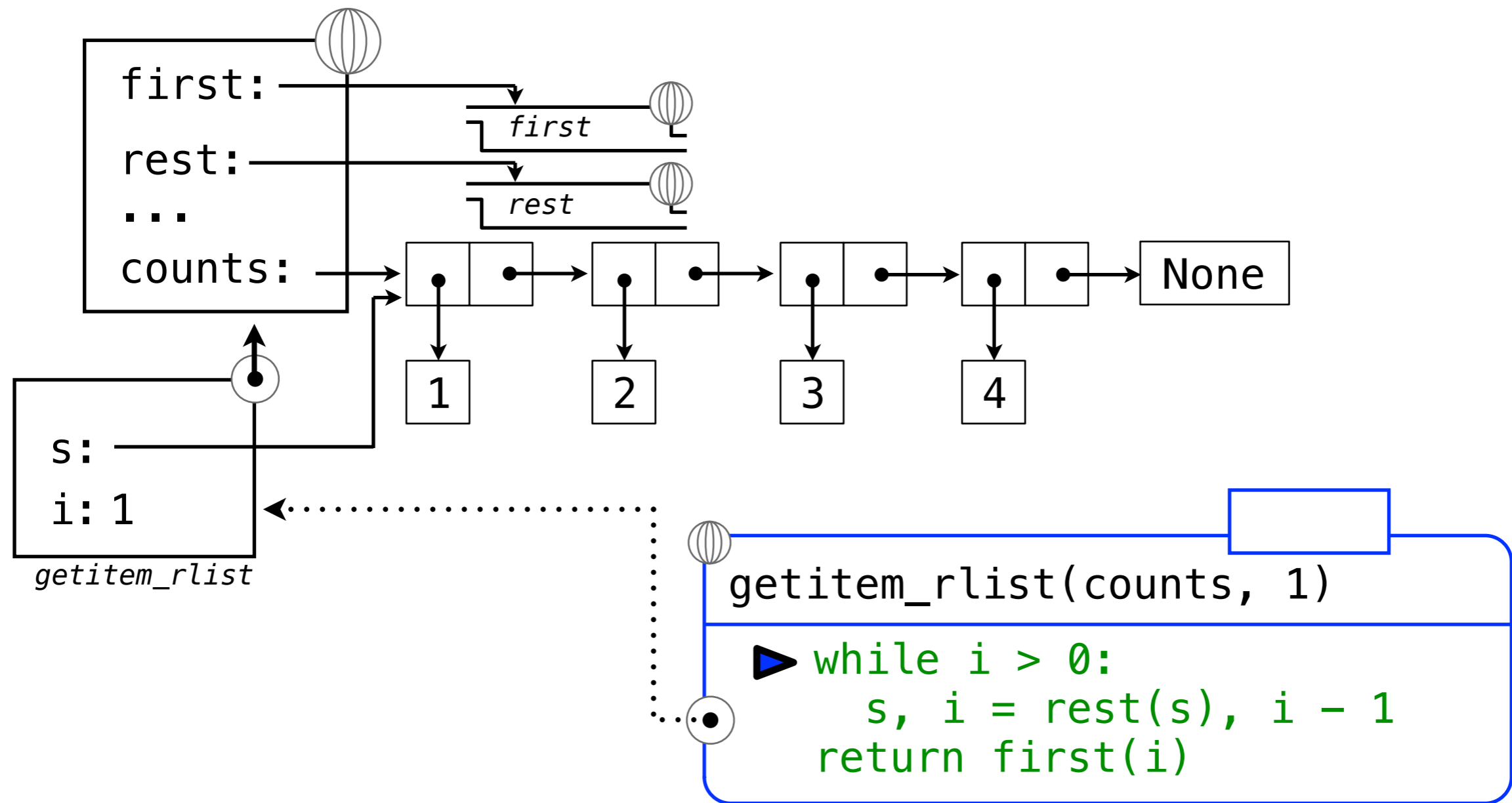
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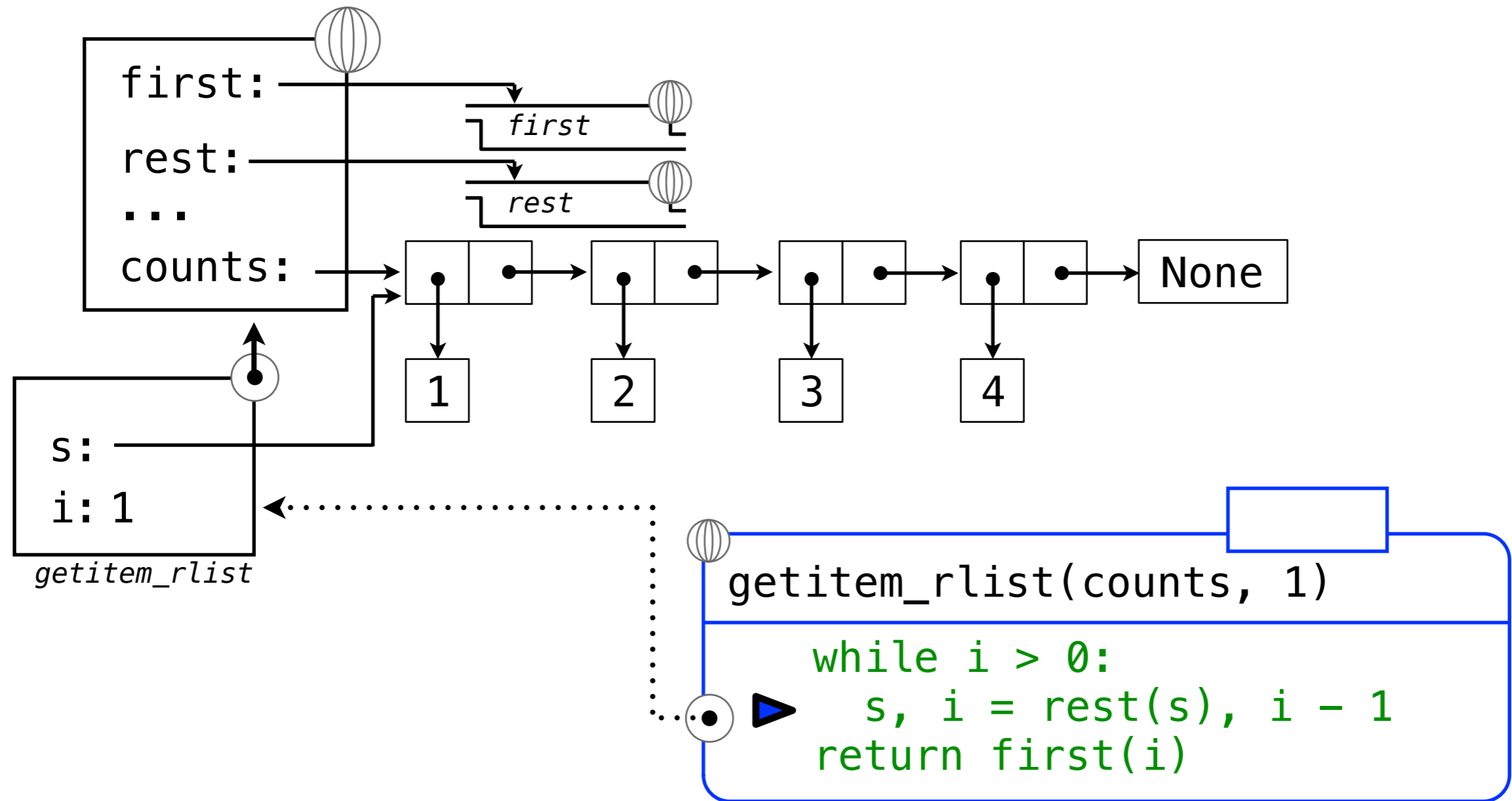
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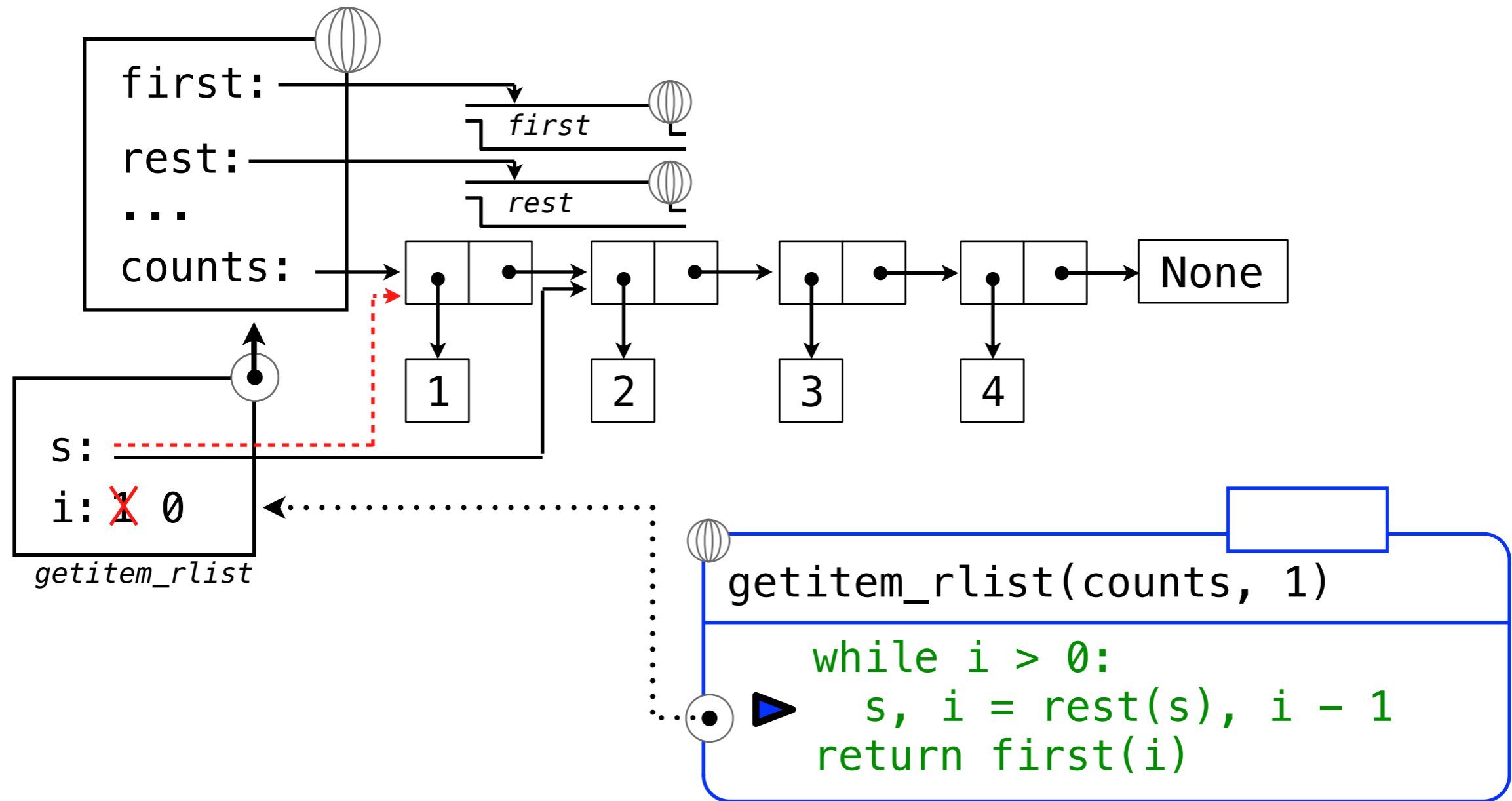
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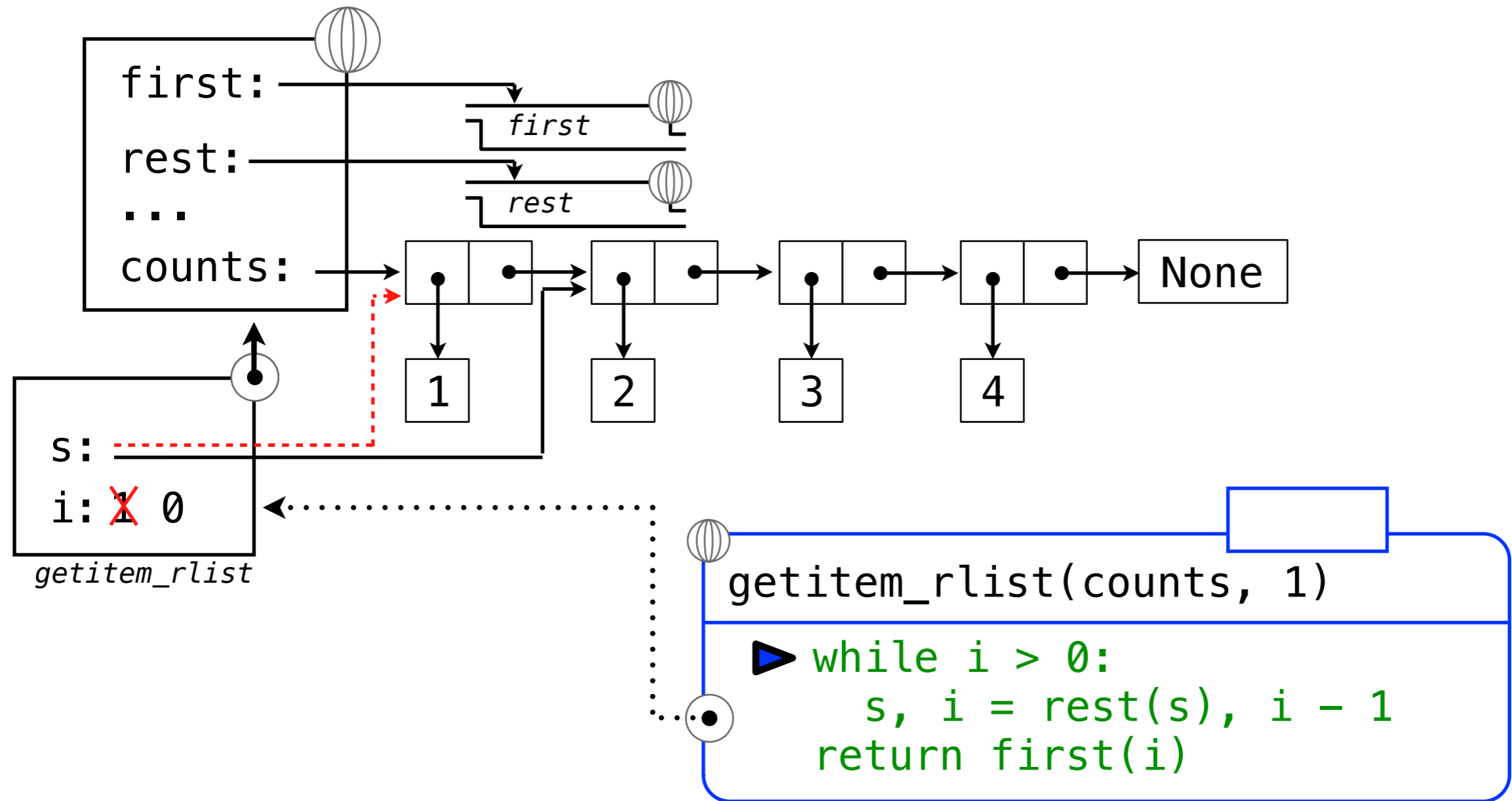
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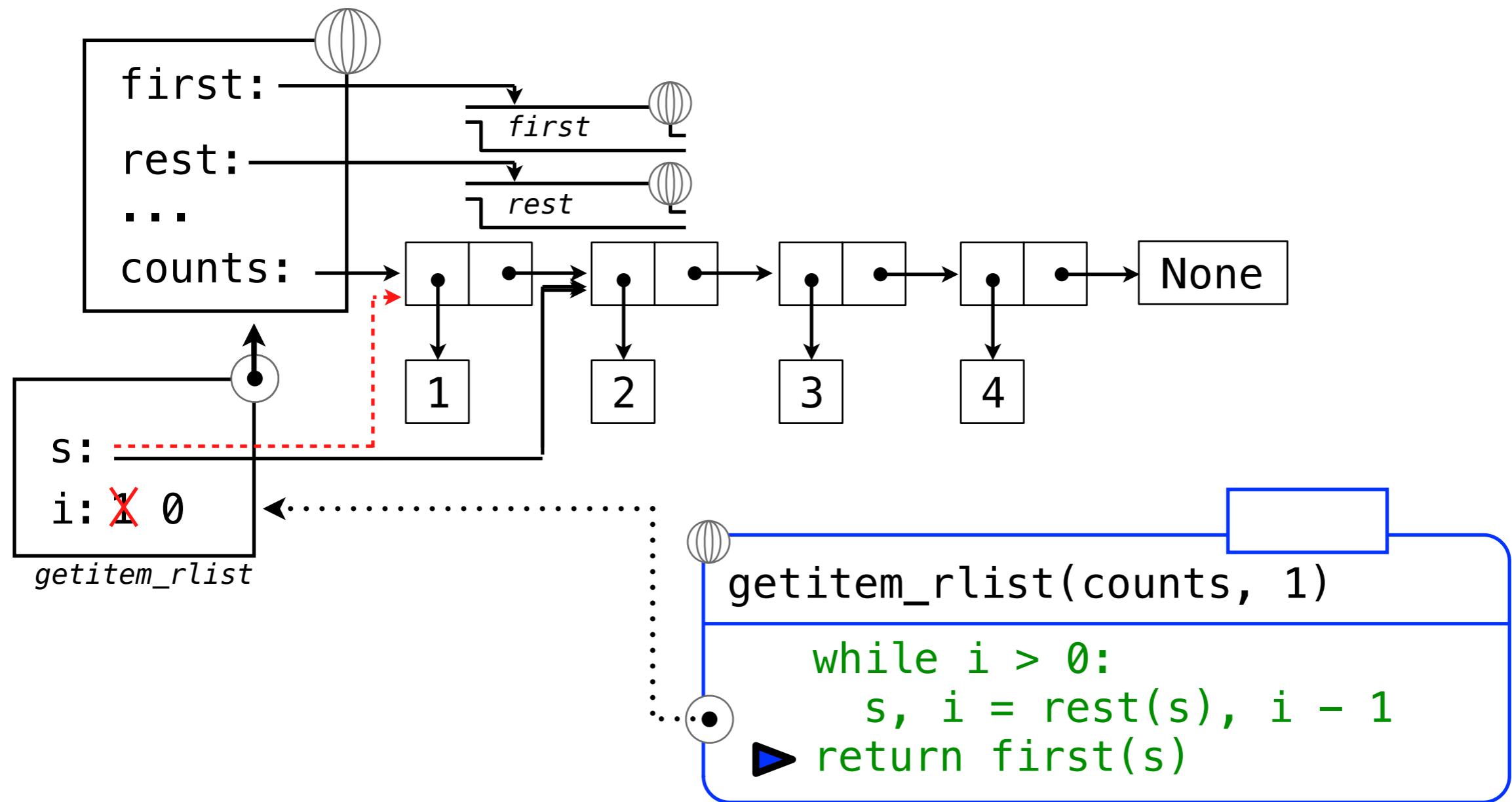
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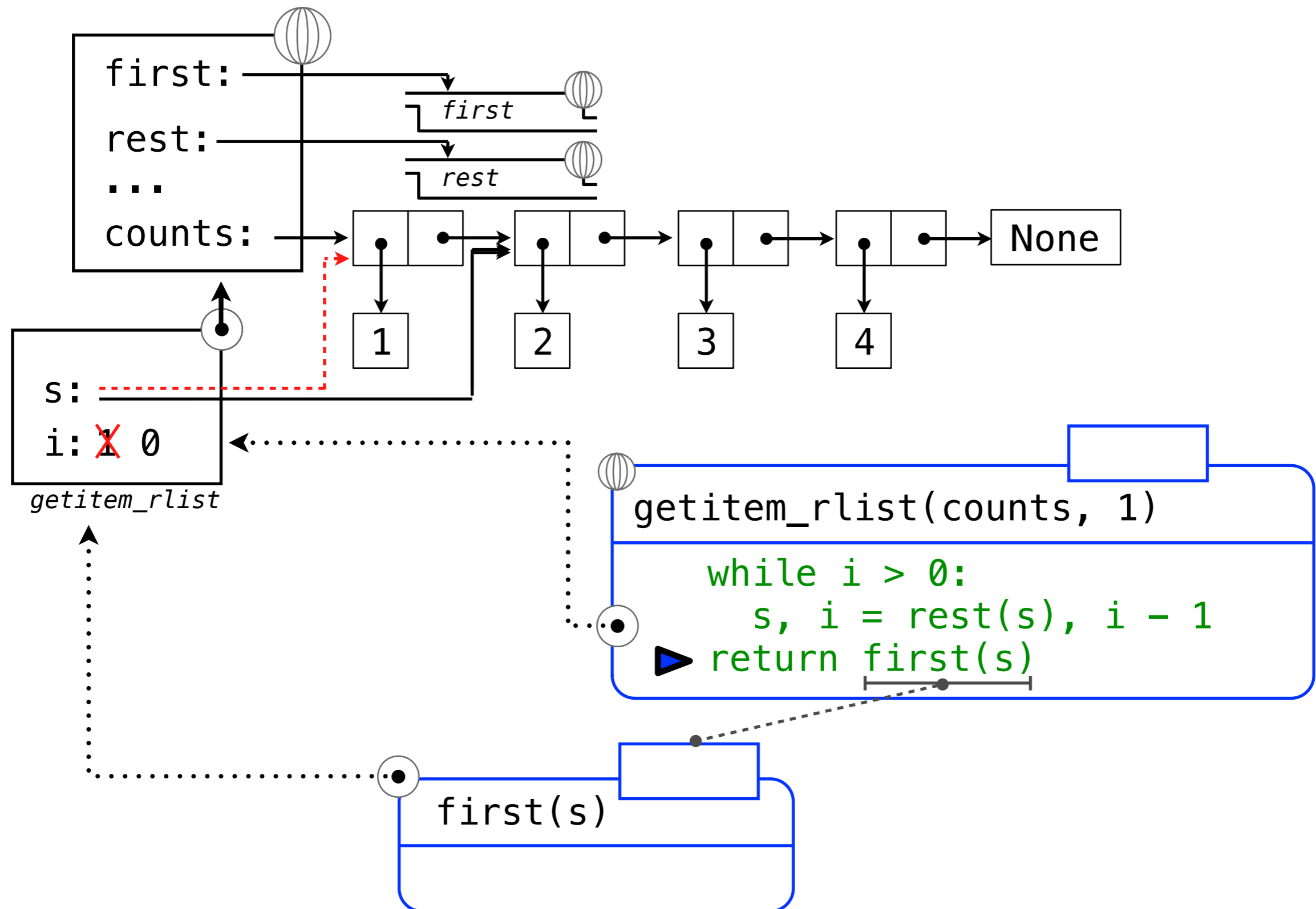
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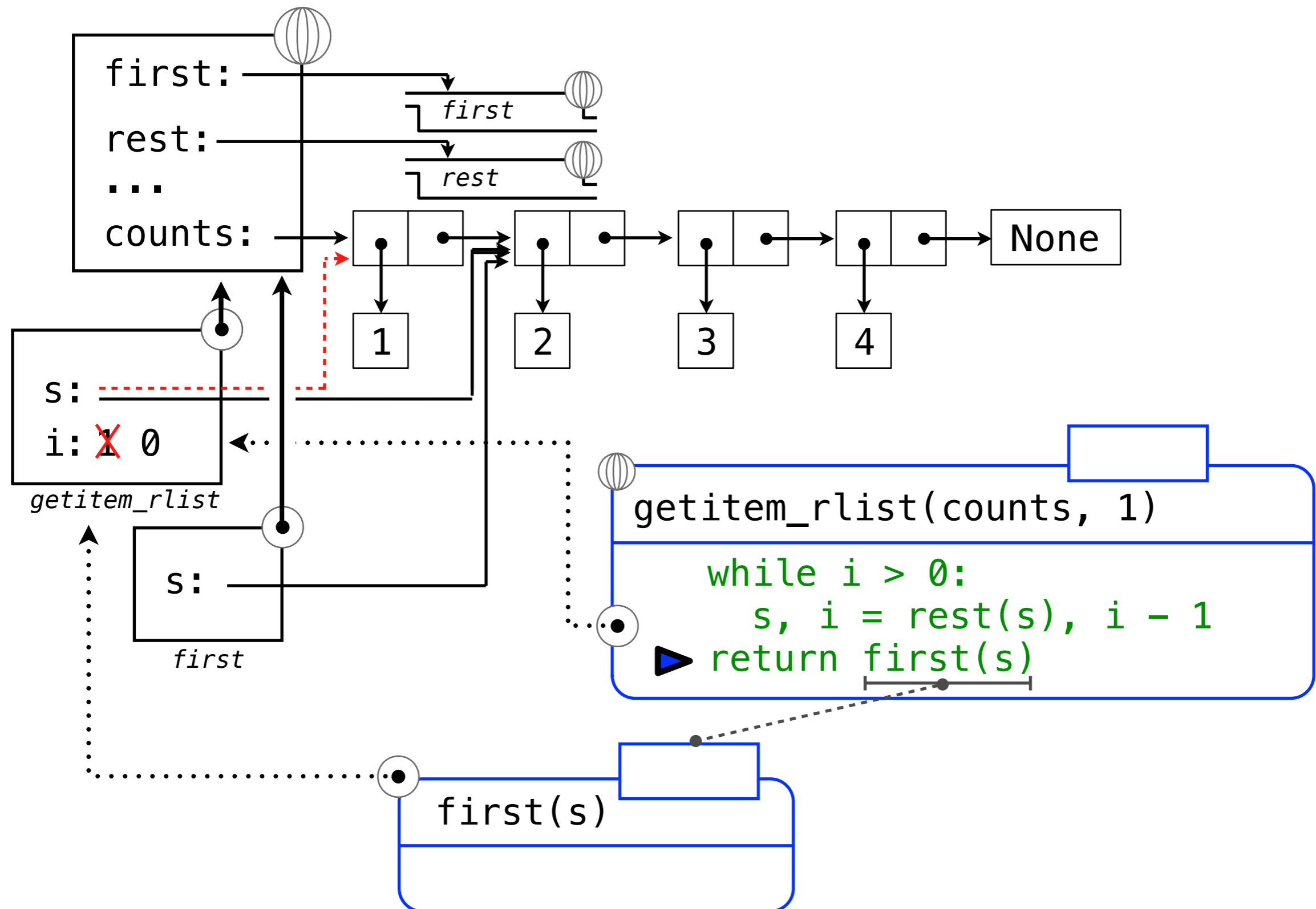
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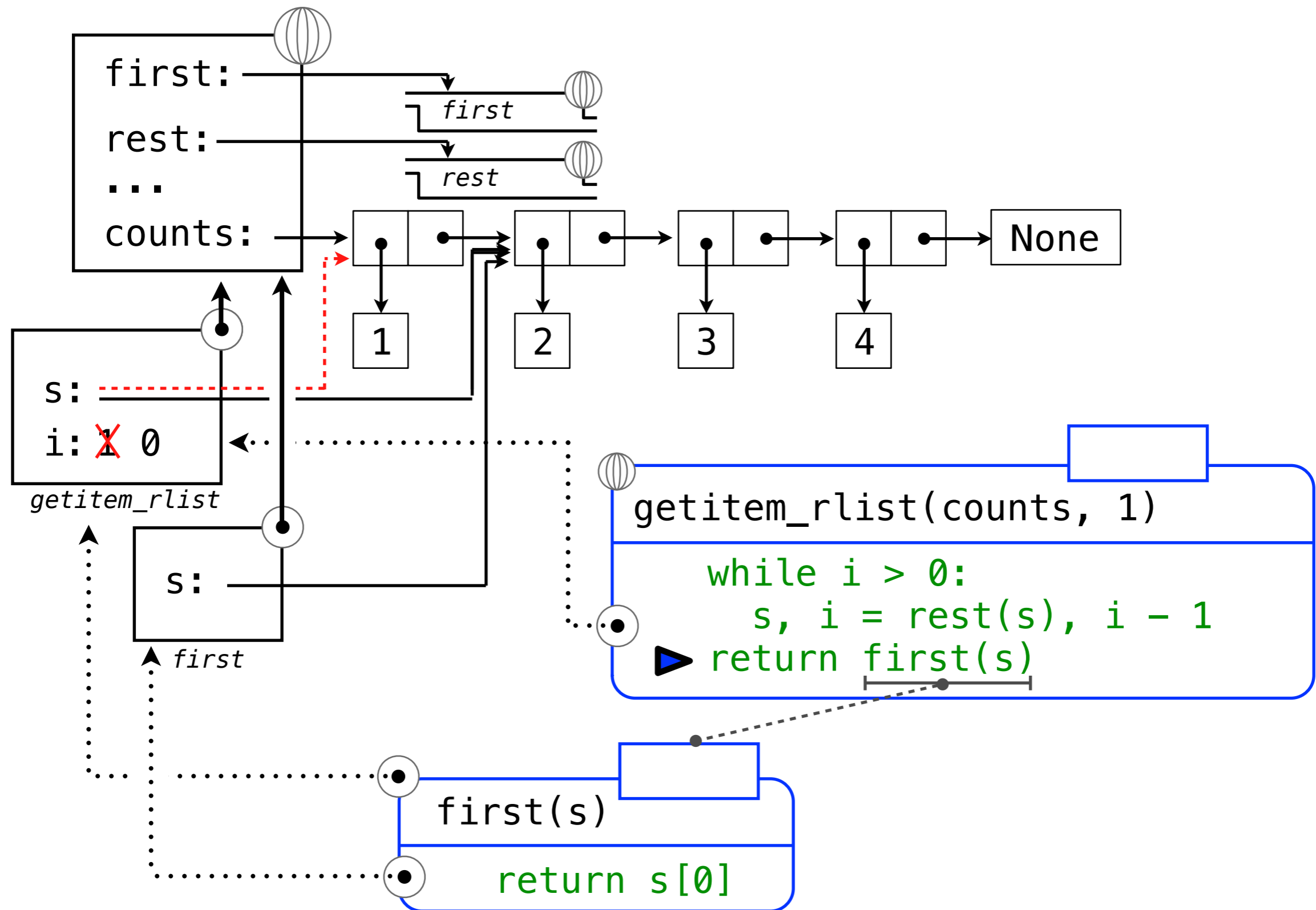
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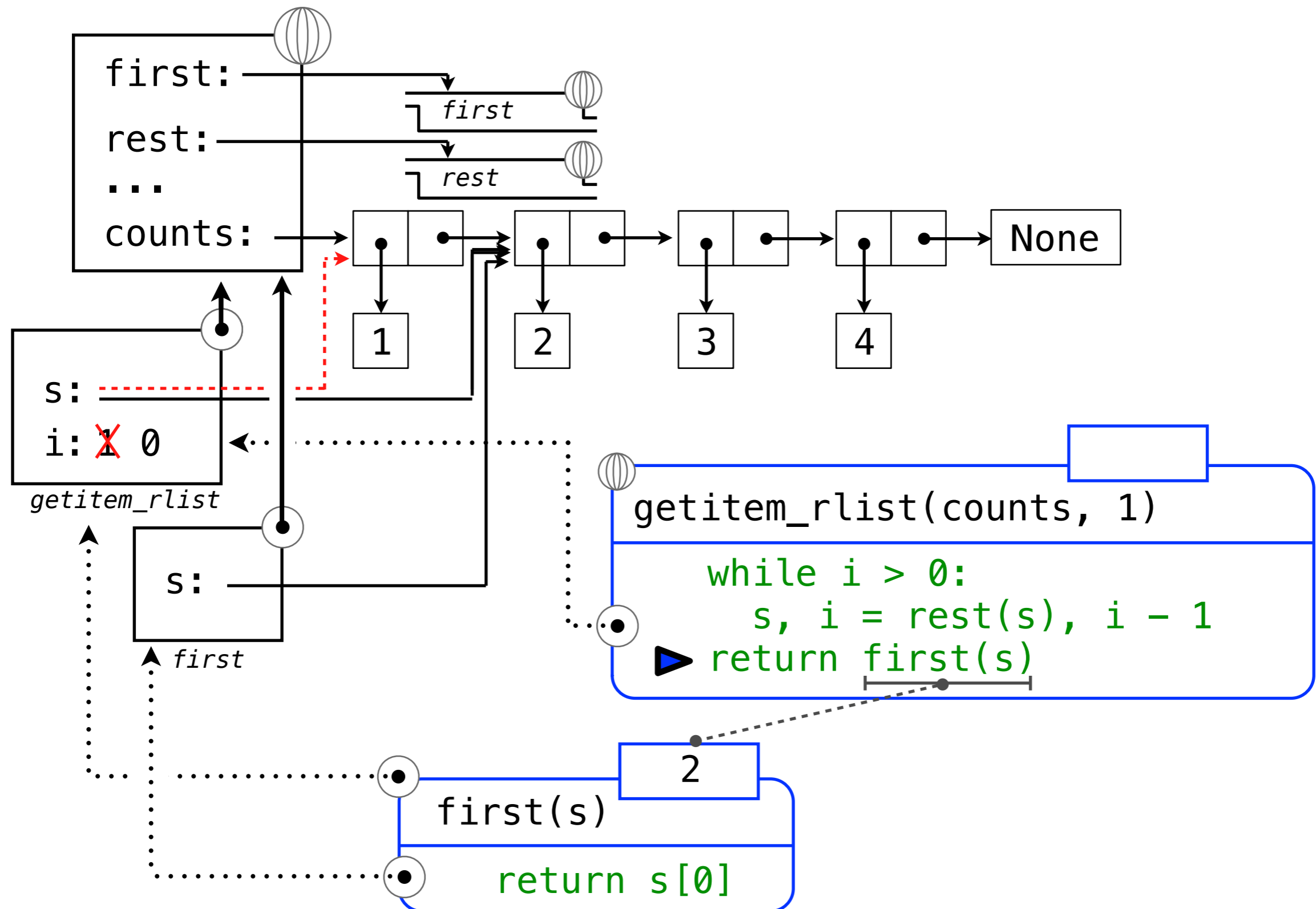
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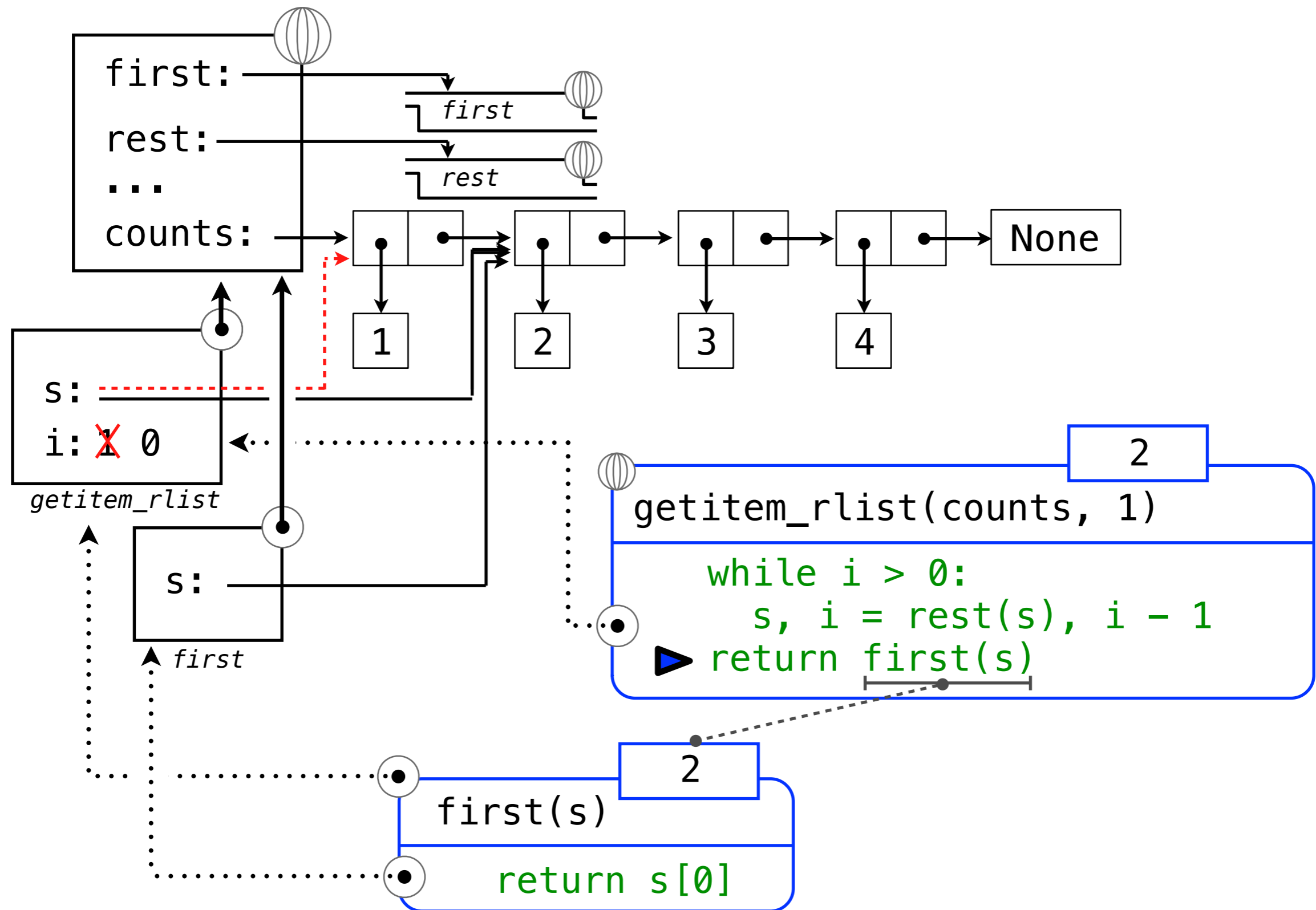
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Sequence Iteration

(Demo)

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```
def count(s, value):  
    total = 0  
    for elem in s:
```

Name bound in the first frame
of the current environment

```
        if elem == value:  
            total = total + 1  
    return total
```

For Statement Execution Procedure

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```
for <name> in <expression>:  
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 - B. Execute the <suite>.

Sequence Unpacking in For Statements

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```
>>> pairs = ((1, 2), (2, 2), (2, 3), (4, 4))
```

```
>>> same_count = 0
```

Sequence Unpacking in For Statements

A sequence of
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A name for each element in
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Each name is bound to a value,
as in multiple assignment

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A sequence of consecutive integers is a range.*

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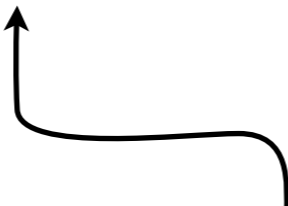
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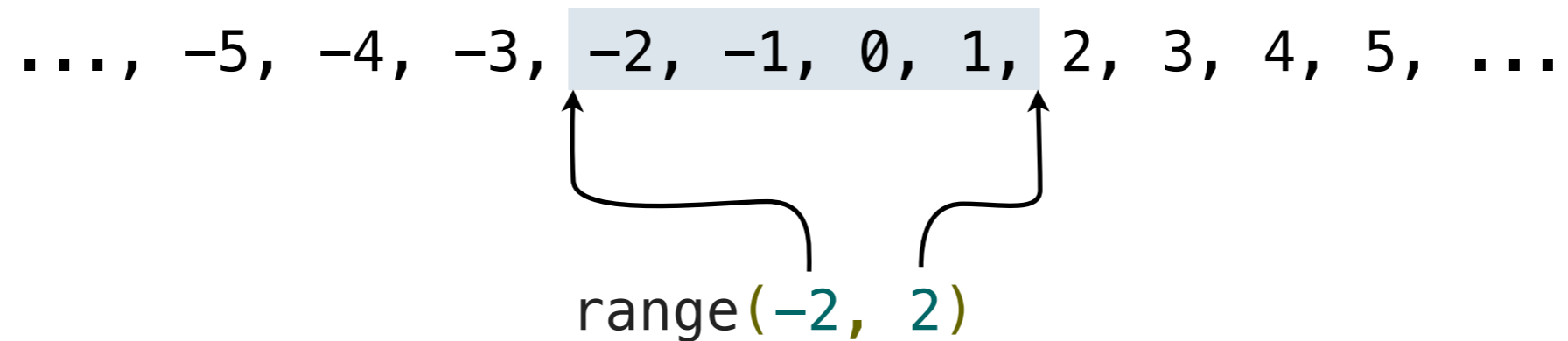


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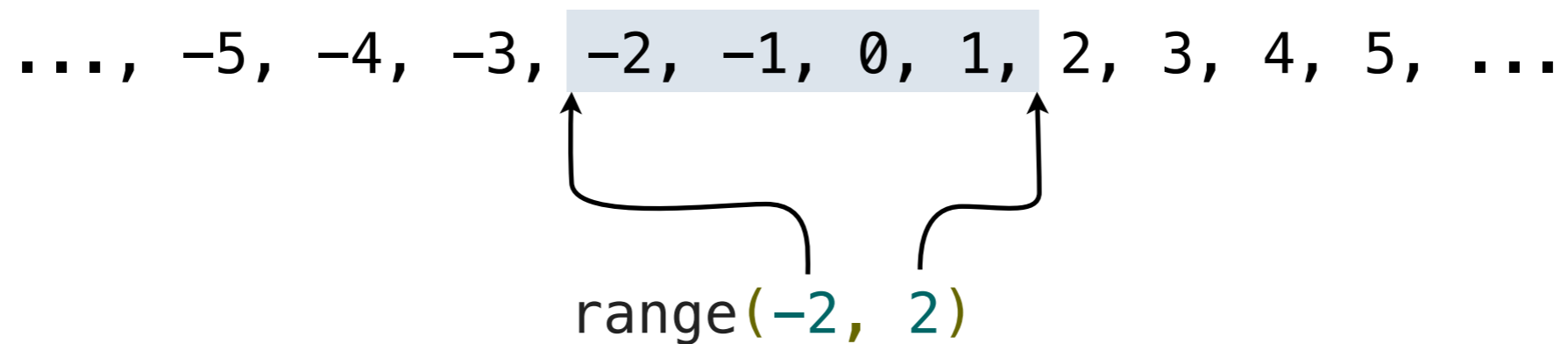
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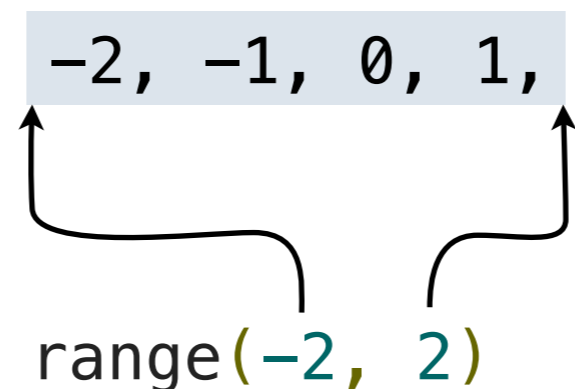
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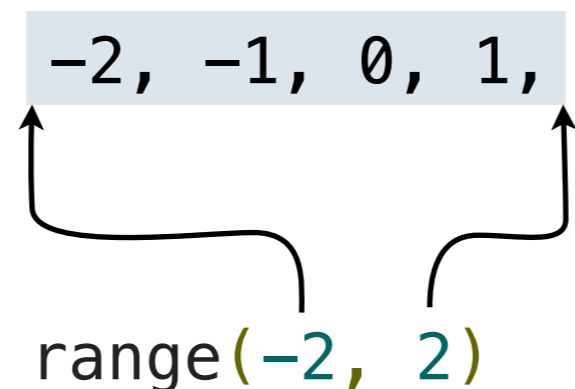
Element selection: starting value + index

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Element selection: starting value + index

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```

```
>>> tuple(range(4))  
(0, 1, 2, 3)
```

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>>> digits[0:2]
(1, 8)
>>> digits[1:]
(8, 2, 8)
```