61A Lecture 6

Friday, September 9

Lambda Expressions

Lambda Expressions

```
>>> ten = 10
```

Lambda Expressions

```
>>> ten = 10

>>> square = x * x
```

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An expression: this one evaluates to a number

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Also an expression: evaluates to a function

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A function

Lambda Expressions

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A function

with formal parameter x

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A function

with formal parameter x

and body "return x * x"

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Notice: no "return"

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Must be a single expression

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>>> square(4)
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Lambda expressions are rare in Python, but important in general

Lambda Expressions Versus Def Statements

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Currying: Transforming a multi-argument function into a single-argument, higher-order function.

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Fun Fact: Currying was discovered by Moses Schönfinkel and later re-discovered by Haskell Curry.

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Schönfinkeling?

Newton's Method Background

Finds approximations to zeroes of differentiable functions

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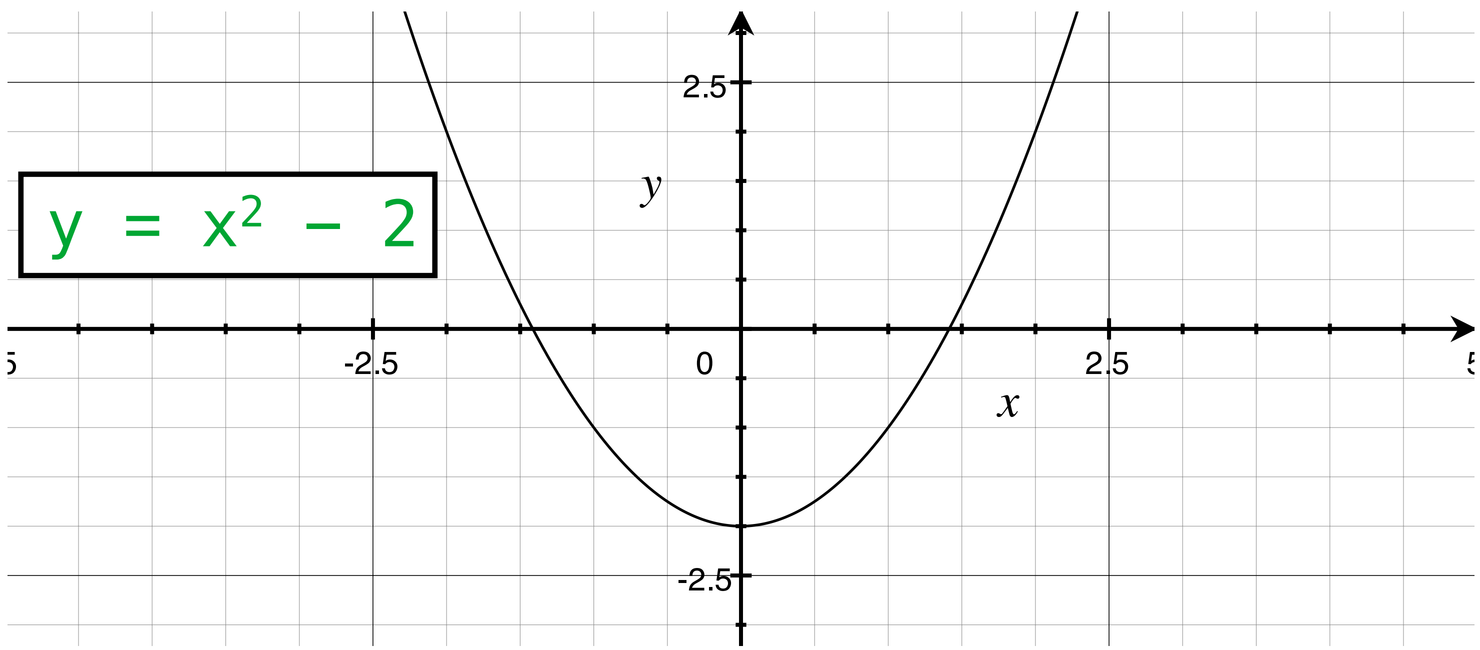
$$\boxed{y = x^2 - 2}$$

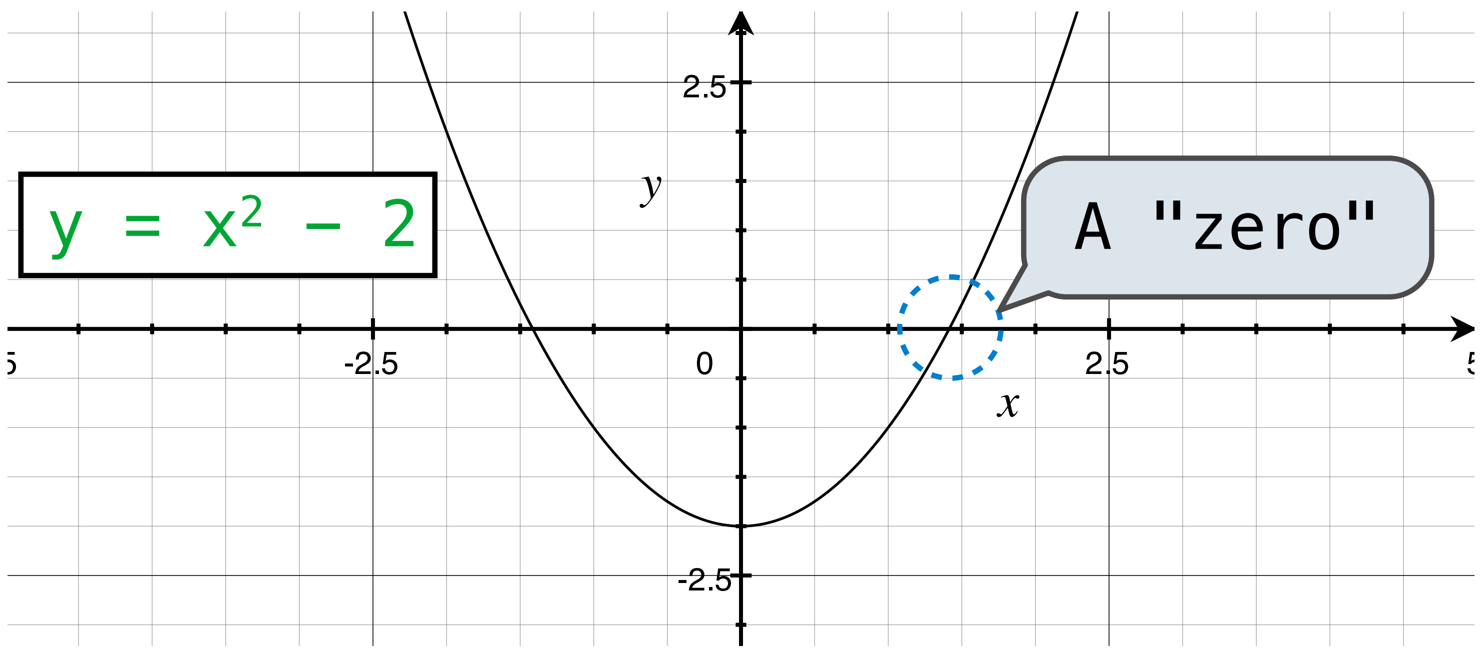
Newton's Method Background

Finds approximations to zeroes of differentiable functions



Newton's Method Background

Finds approximations to zeroes of differentiable functions



Newton's Method Background

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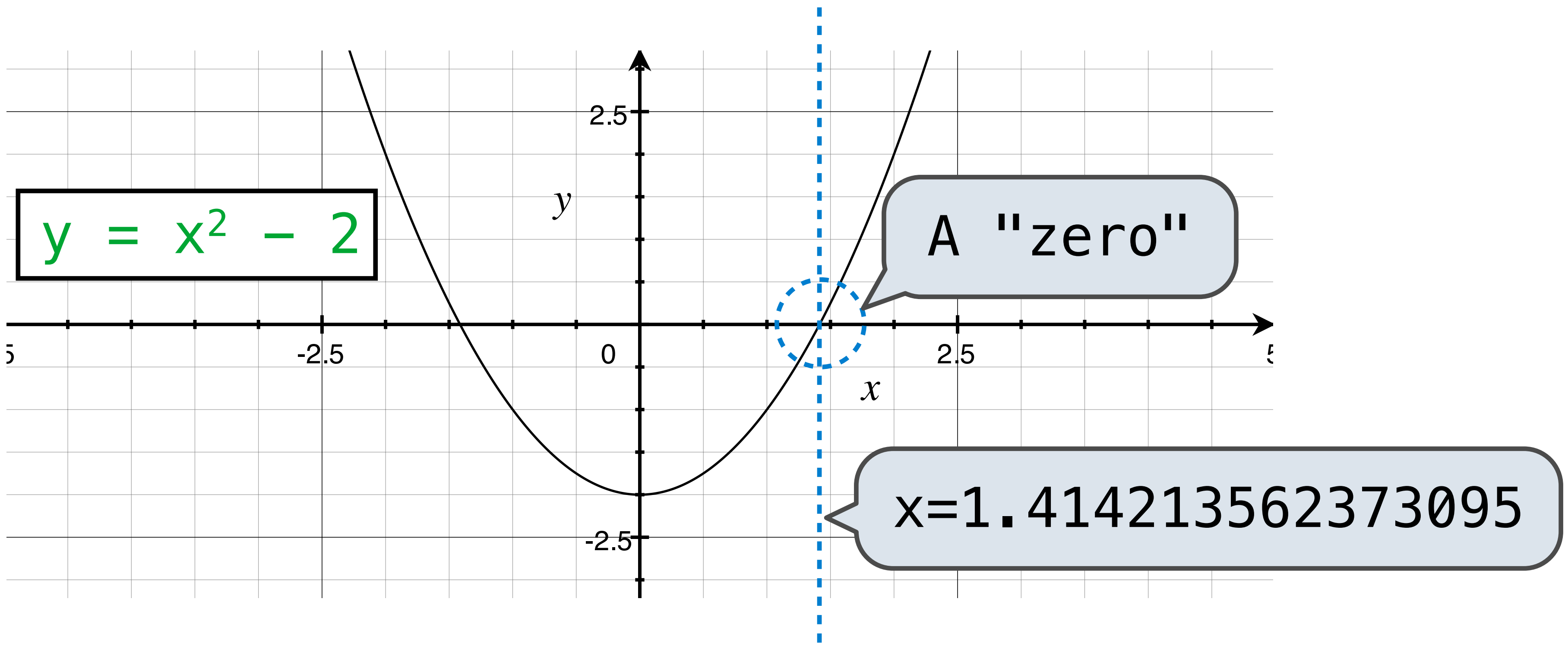

Newton's Method Background

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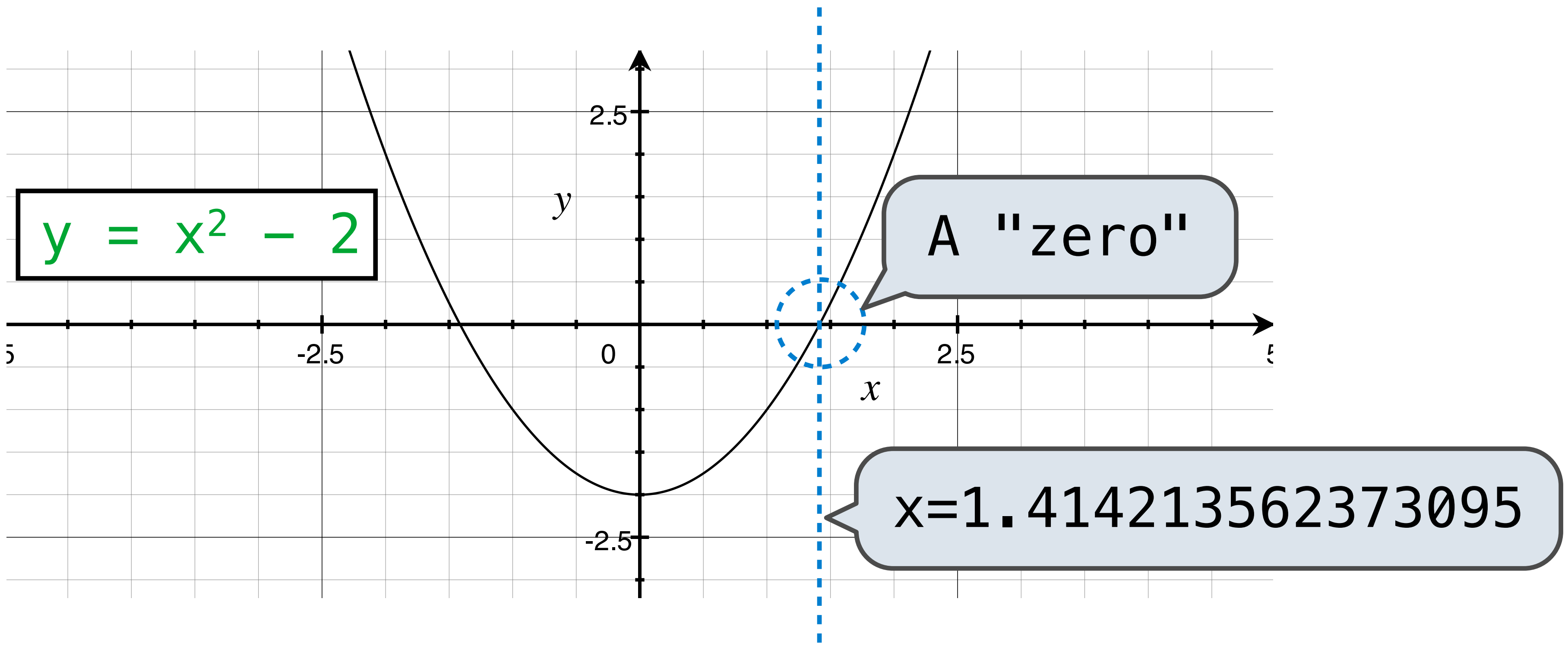

Application: a method for (approximately) computing square roots, using only basic arithmetic.

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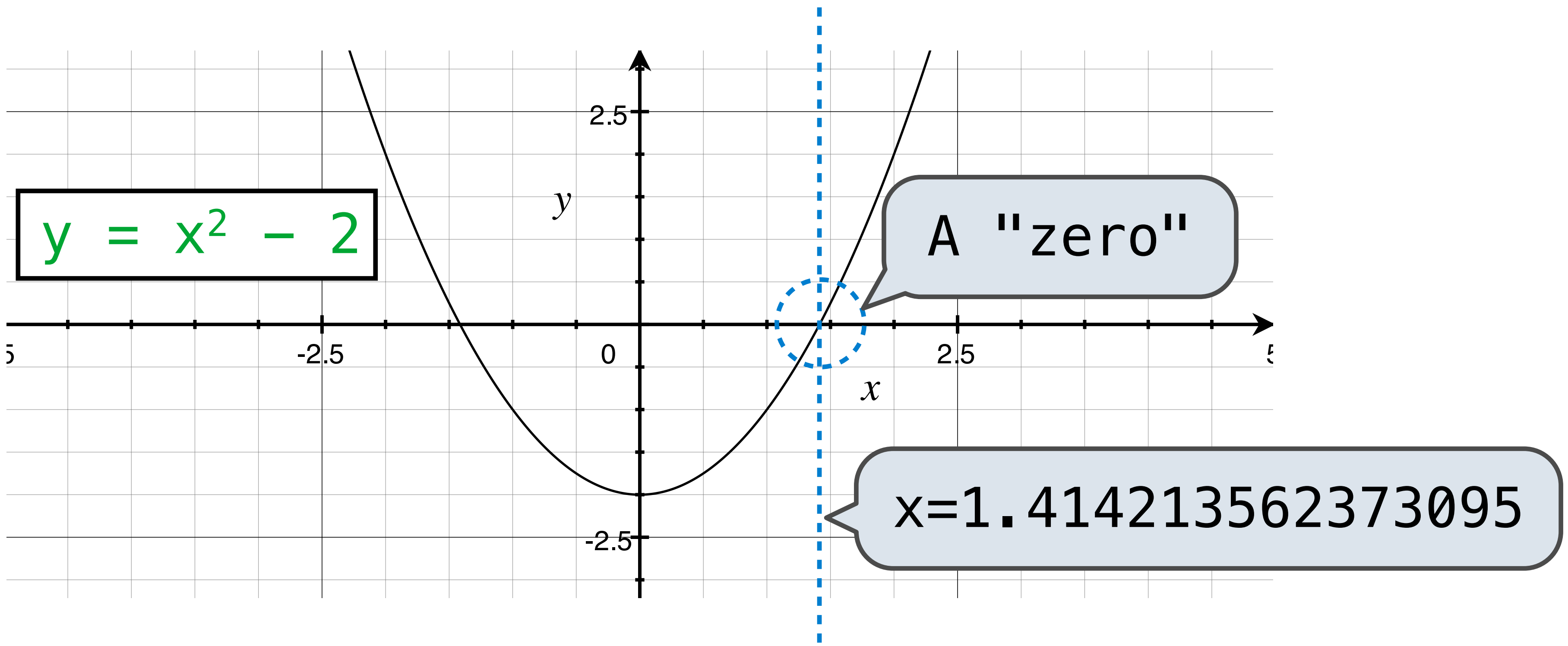

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The positive zero of $y = x^2 - a$ is

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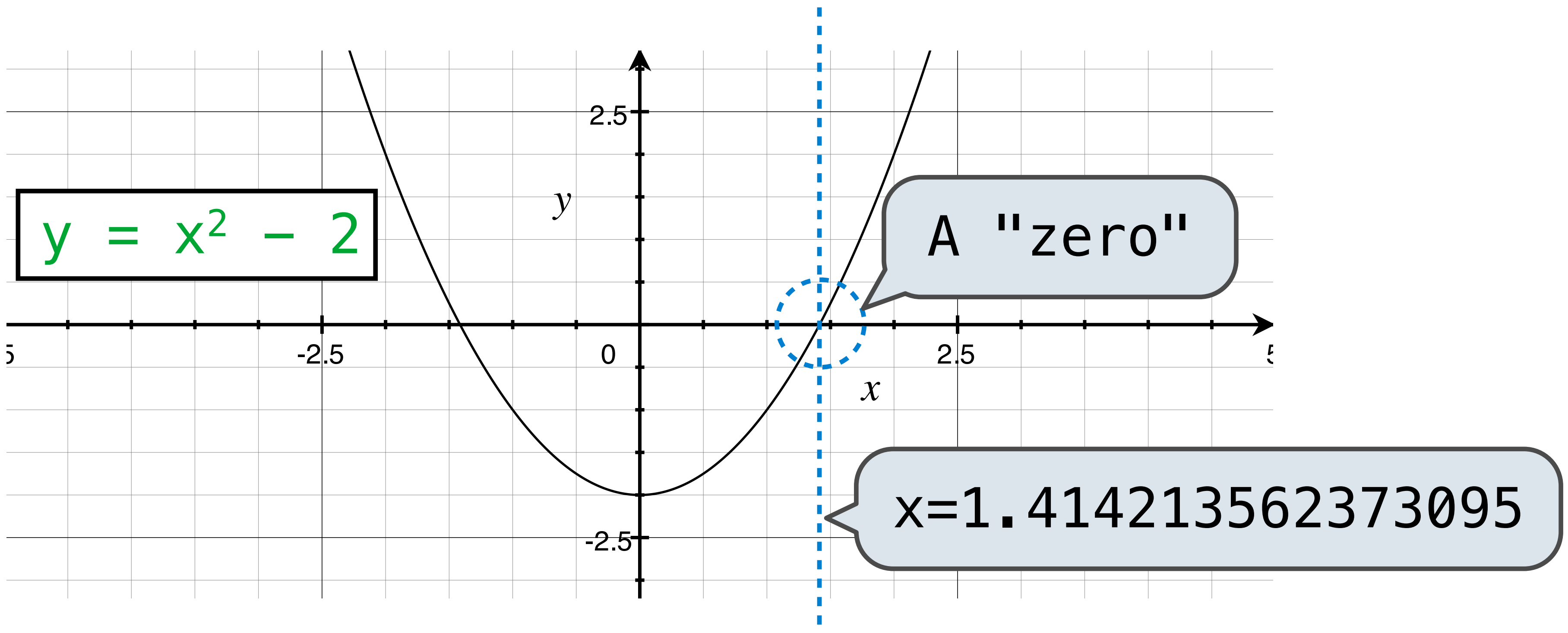

Application: a method for (approximately) computing square roots, using only basic arithmetic.

The positive zero of $y = x^2 - a$ is $\sqrt{a}$

Newton's Method

`Begin with a function f and`

`an initial guess x`

$$x - \frac{f(x)}{f'(x)}$$

Newton's Method

Begin with a function f and

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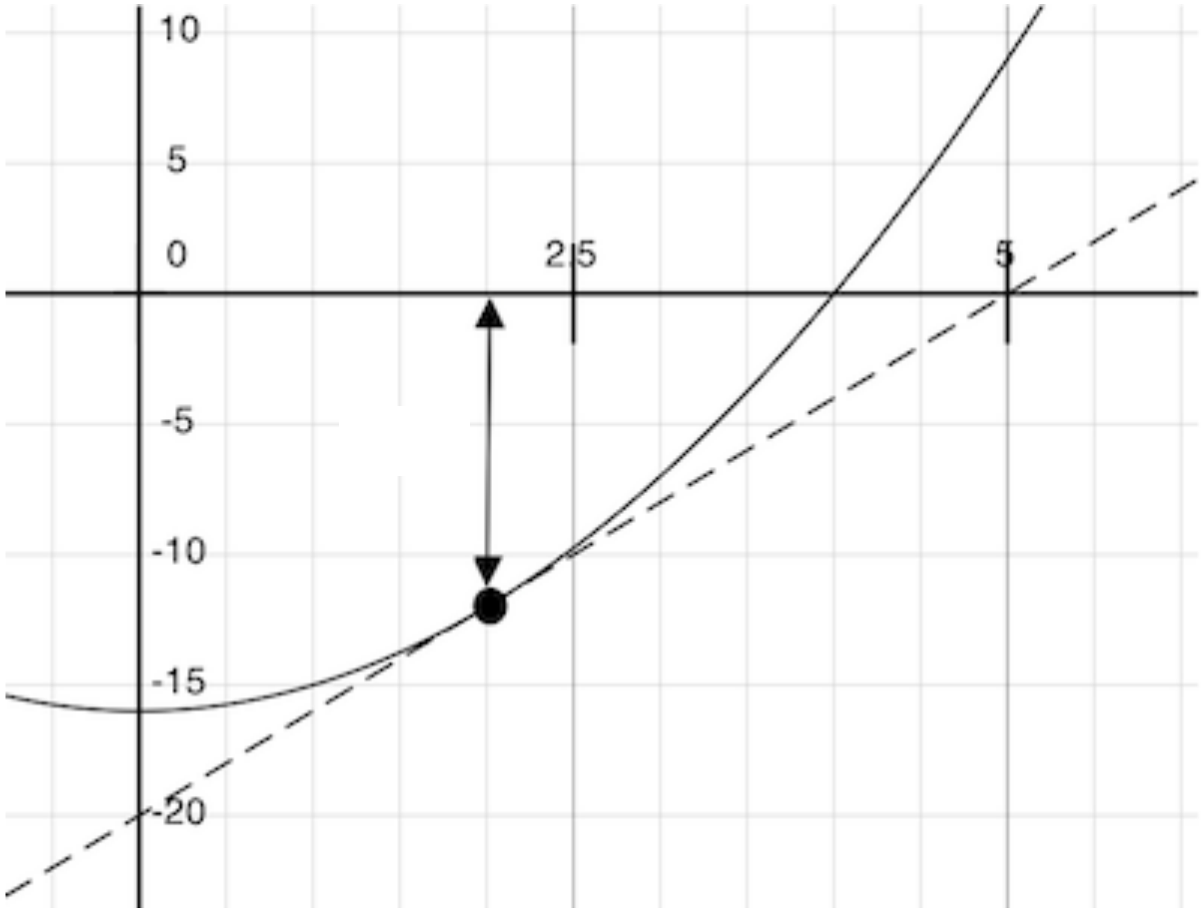

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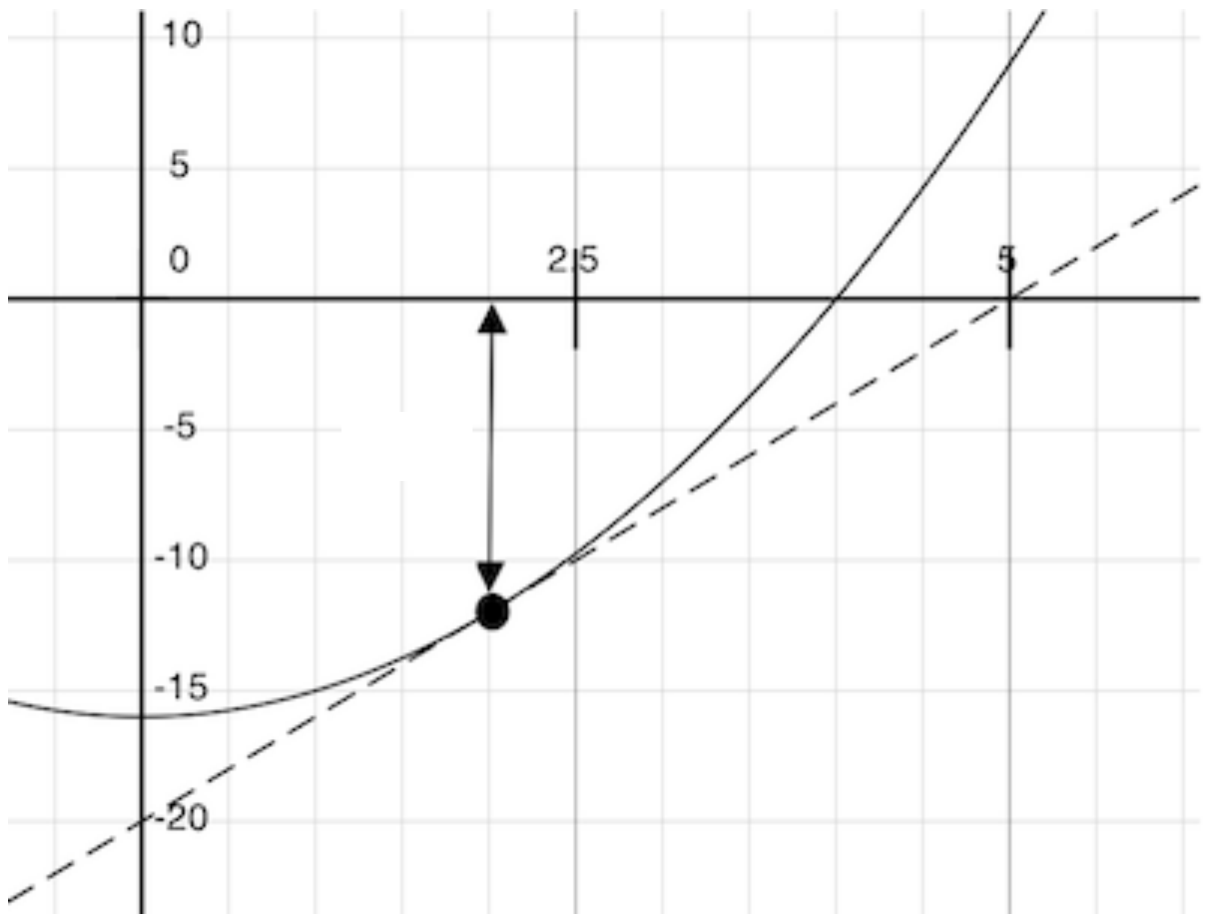

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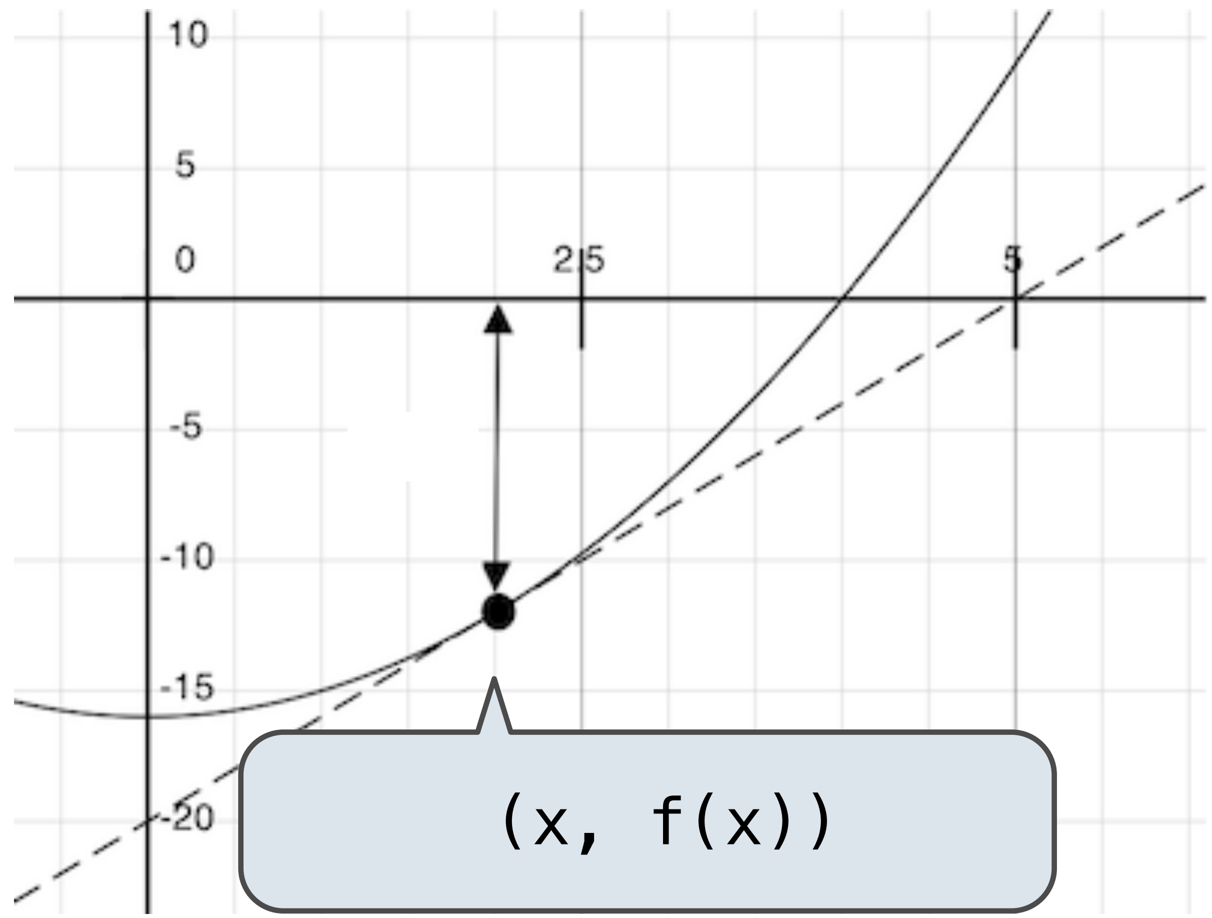

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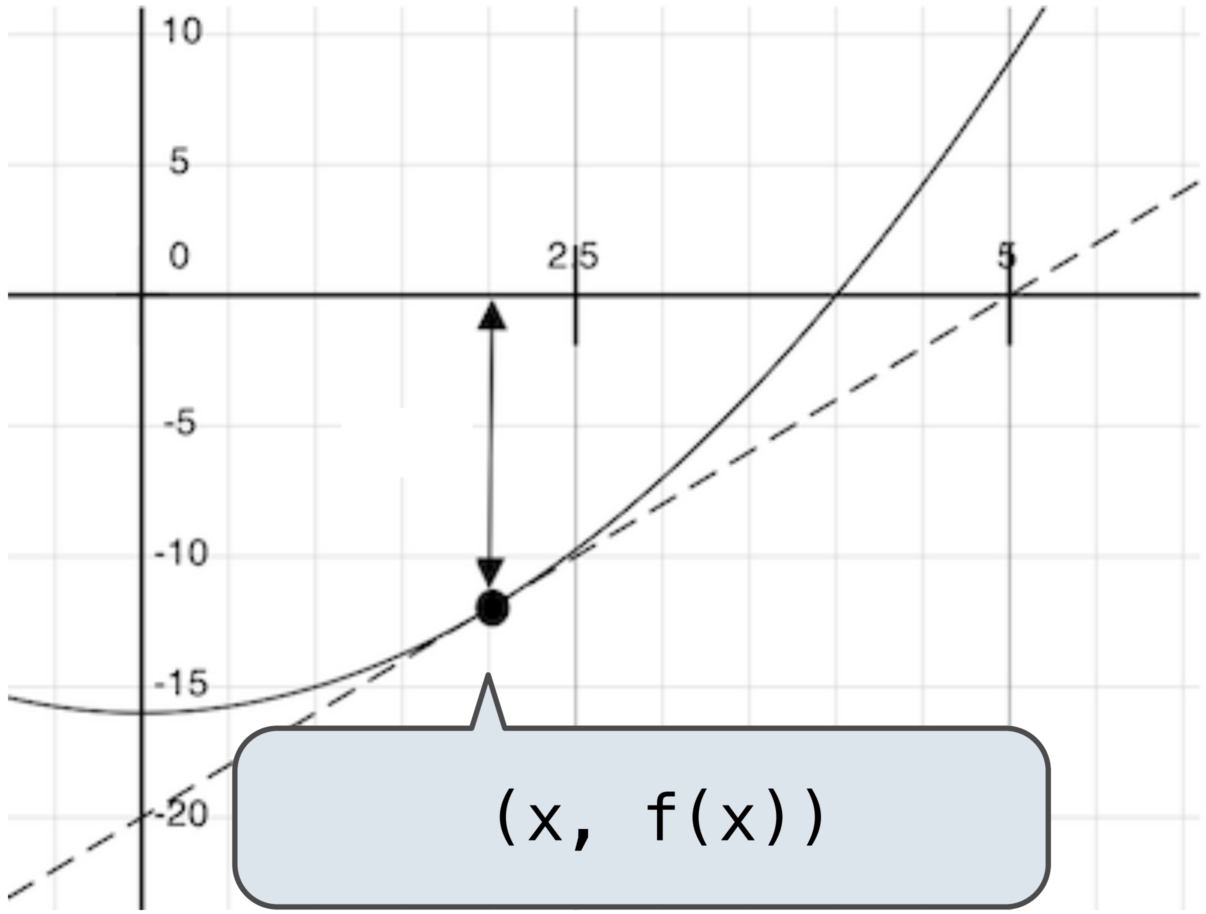

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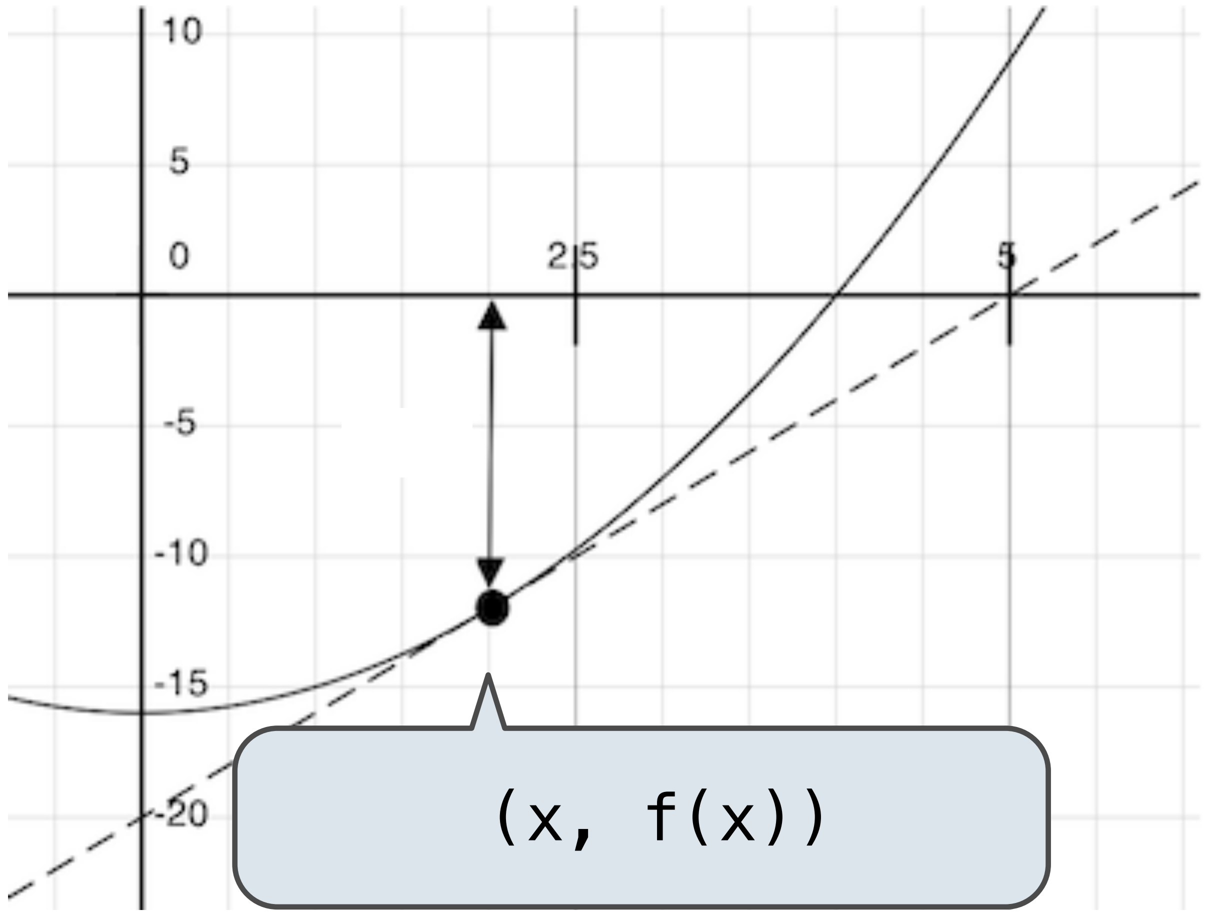

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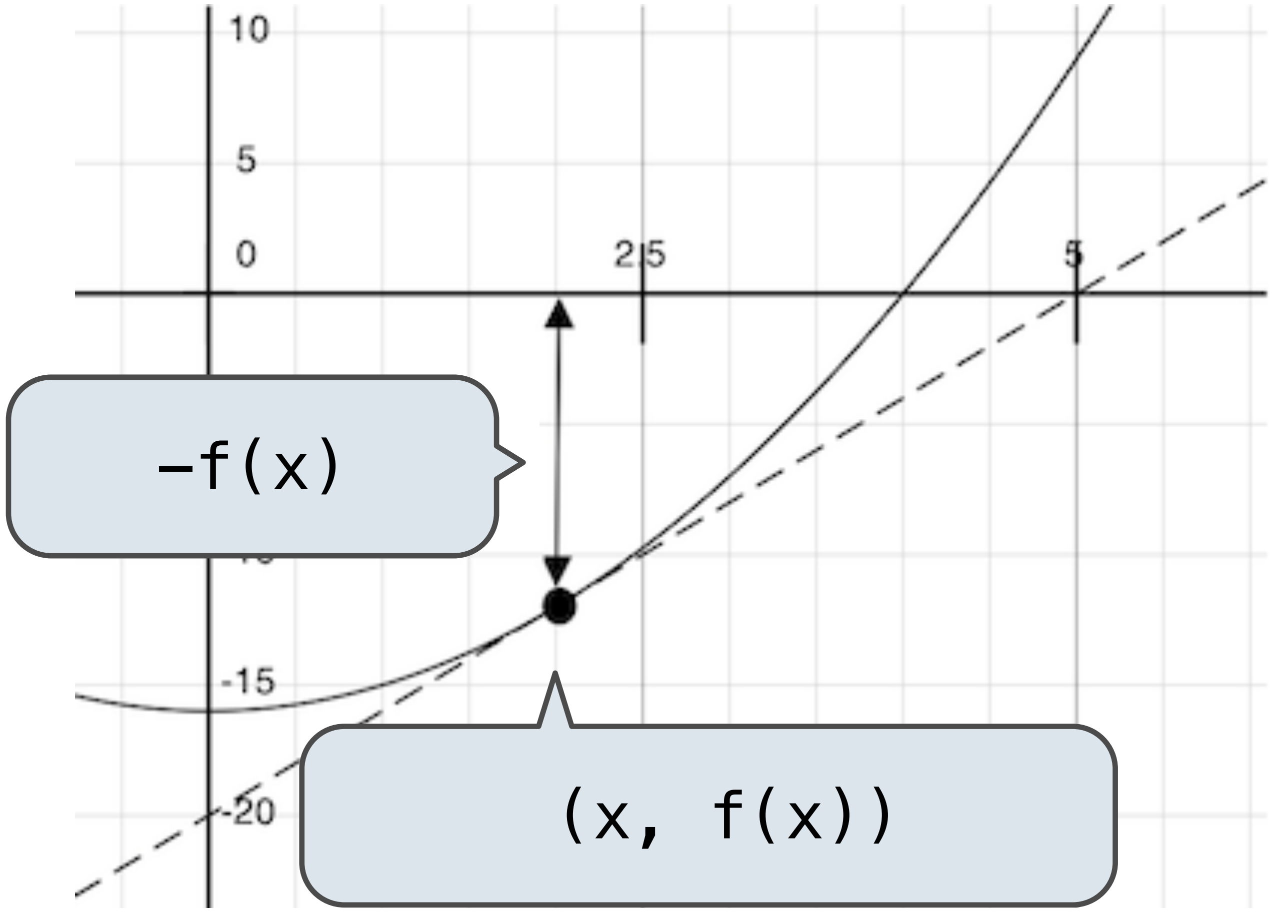

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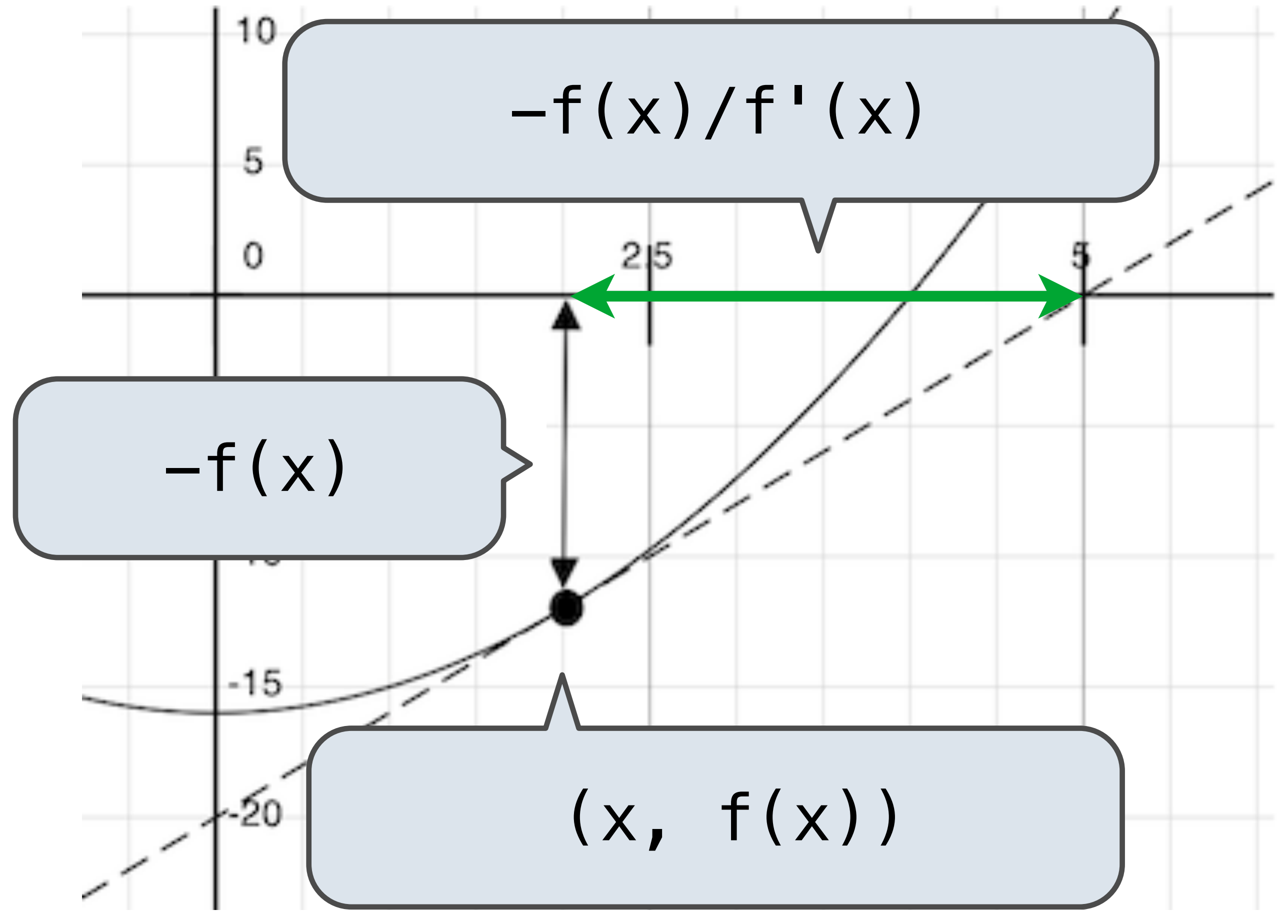

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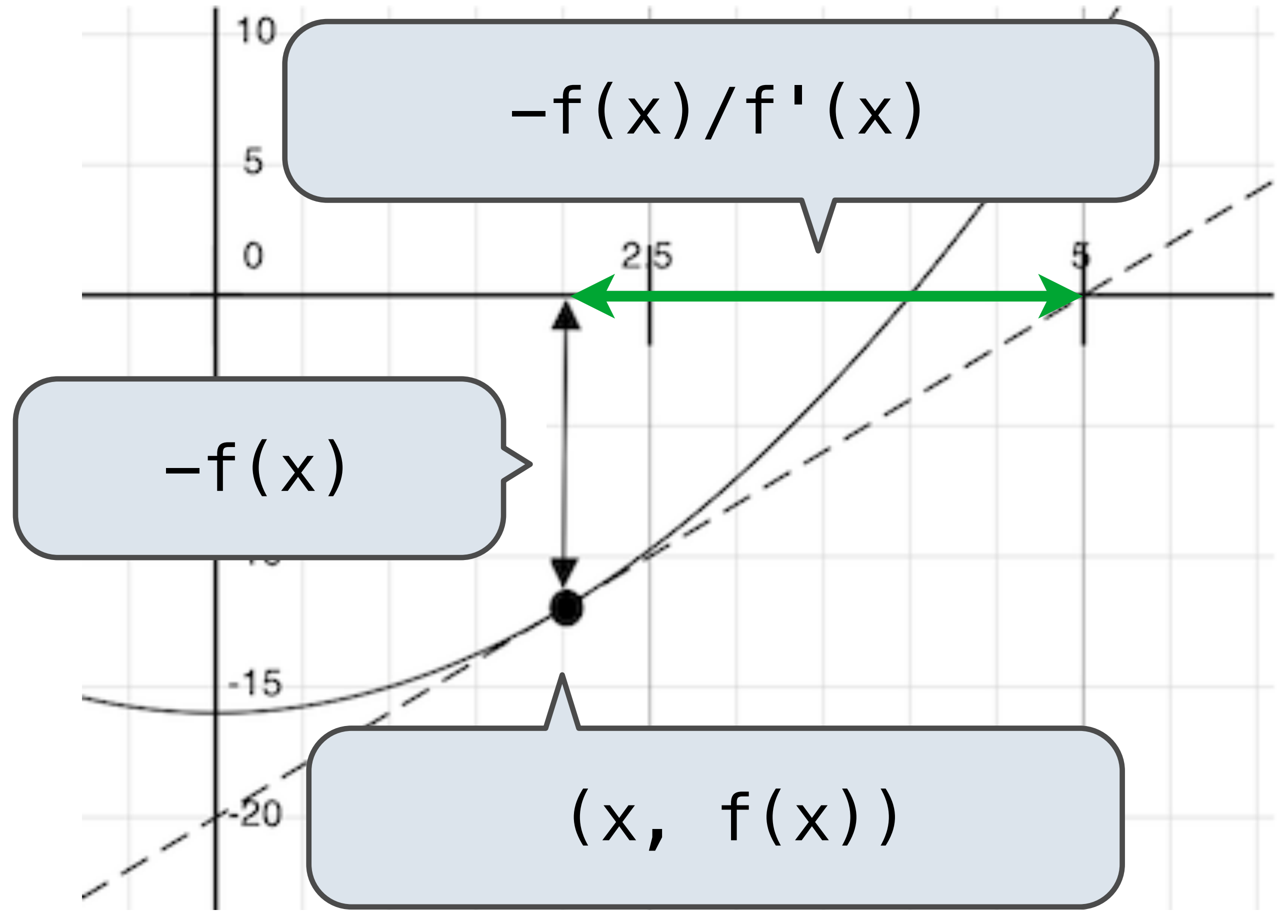

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Visualization of Newton's Method

(Demo)

http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

$f(x)=x^2 - 2$

Using Newton's Method

How to find the square root of 2?

$f(x)=x^2 - 2$

Using Newton's Method

How to find the square root of 2?

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>>> f = lambda x: x*x - 2
>>> find_root(f, 1)
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How to find the log base 2 of 1024?

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>>> g = lambda x: pow(2, x) - 1024
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What number is one less than its square?

Using Newton's Method

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What number is one less than its square?

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>>> h = lambda x: x*x - (x+1)
>>> find_root(h, 1)
```

Special Case: Square Roots

$$x = \frac{x + \frac{a}{x}}{2}$$

Babylonian Method

Special Case: Square Roots

How to compute square_root(a)

Idea: Iteratively refine a guess x about the square root of a

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Babylonian Method

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Babylonian Method

Implementation questions:

What *guess* should start the computation?

How do we know when we are finished?

Special Case: Square Roots

$$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$

Special Case: Square Roots

`How to compute cube_root(a)`

Idea: Iteratively refine a guess x about the cube root of a

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Special Case: Square Roots

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Iterative Improvement

(Demo)

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(Demo)

```
def iter_improve(update, done, guess=1, max_updates=1000):
    """Iteratively improve guess with update until done returns a true value.

    guess -- An initial guess
    update -- A function from guesses to guesses; updates the guess
    done -- A function from guesses to boolean values; tests if guess is good

    >>> iter_improve(golden_update, golden_test)
    1.618033988749895
    """
    k = 0
    while not done(guess) and k < max_updates:
        guess = update(guess)
        k = k + 1
    return guess
```

Iterative Improvement

(Demo)

```
def golden_update(guess):
    return 1/guess + 1
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Iterative Improvement

(Demo)

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    return guess * guess == guess + 1
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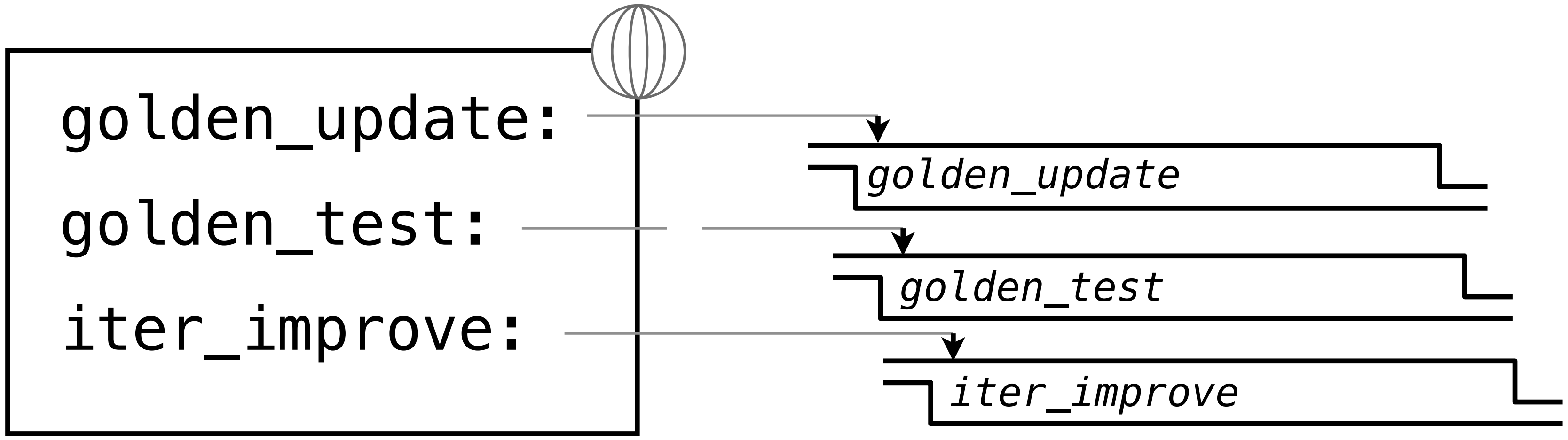

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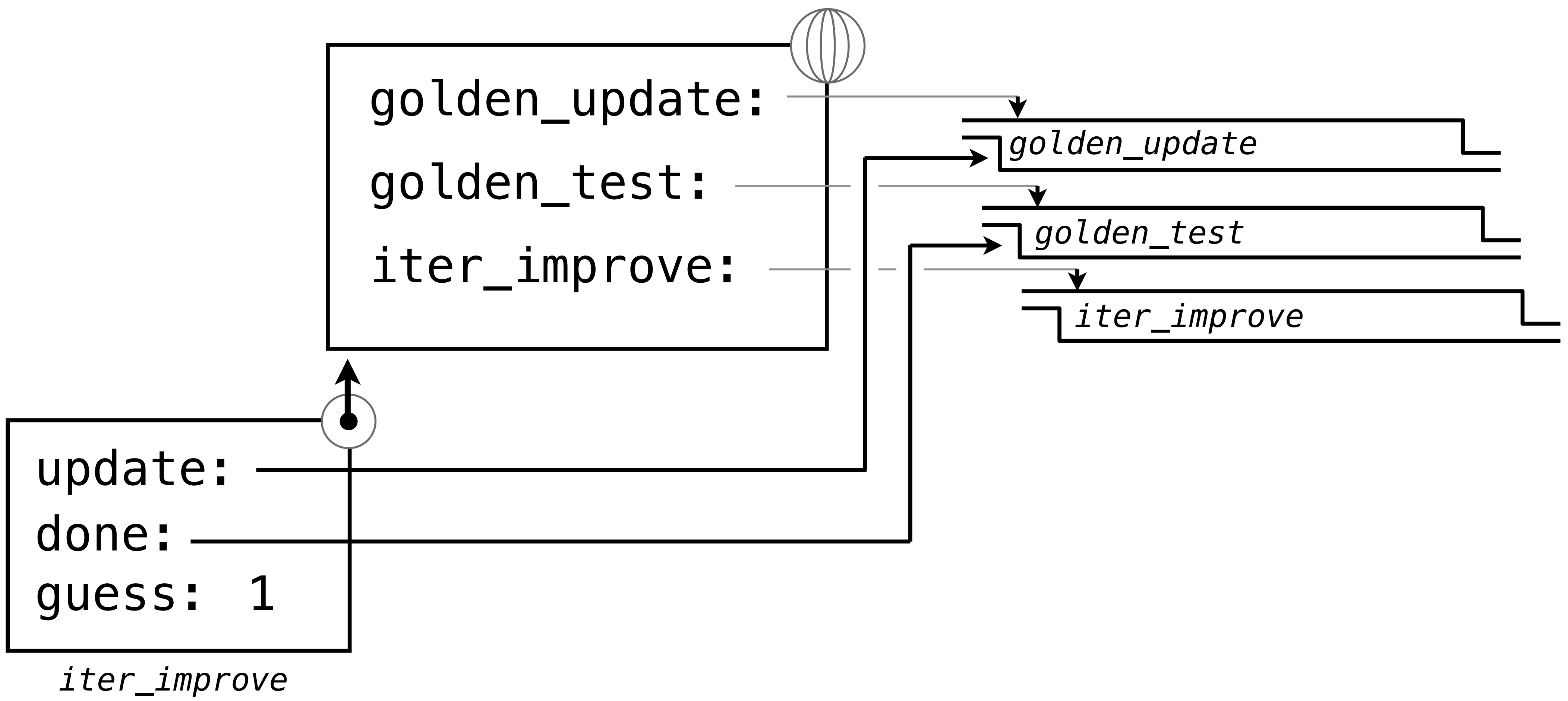

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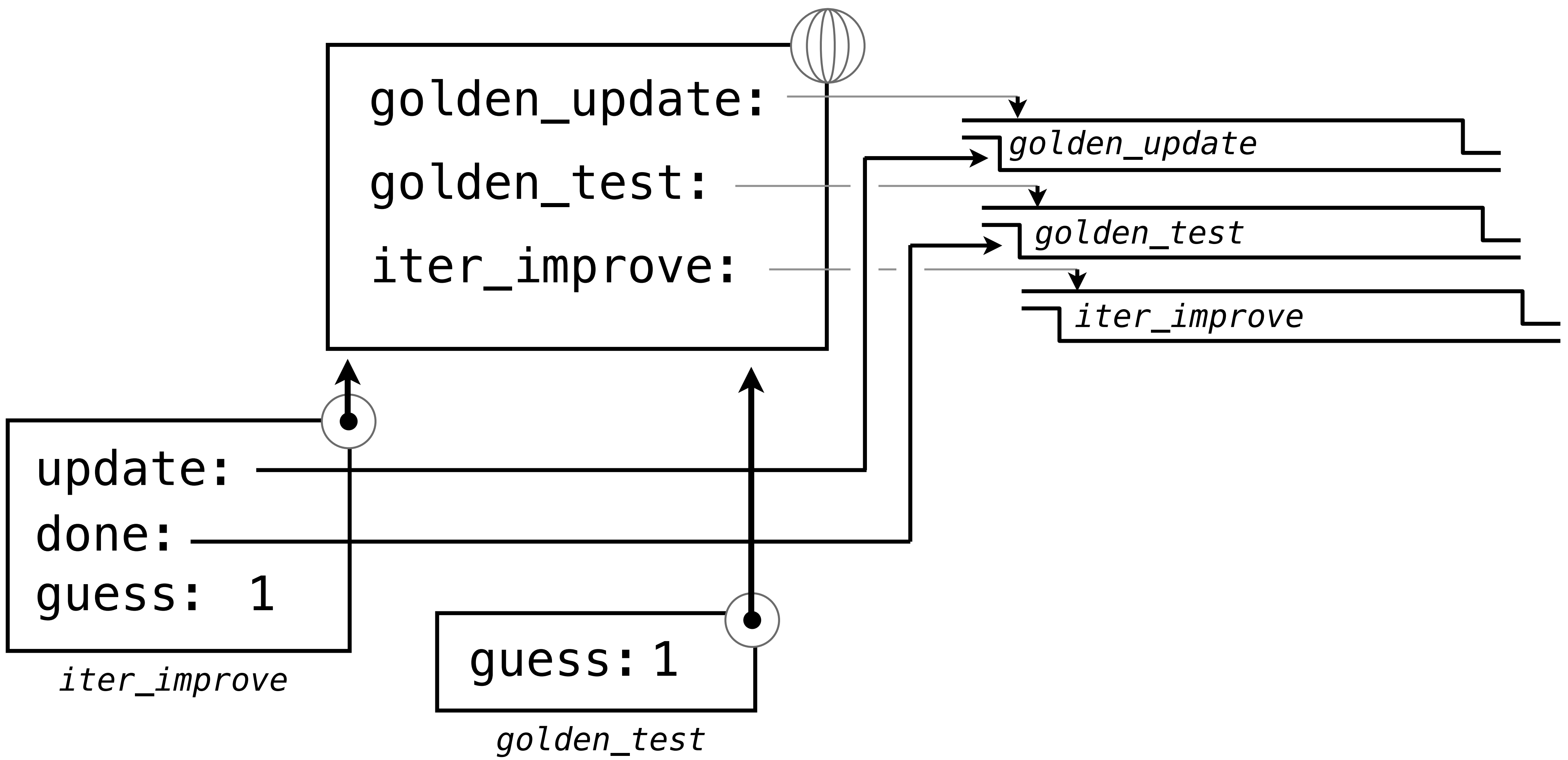

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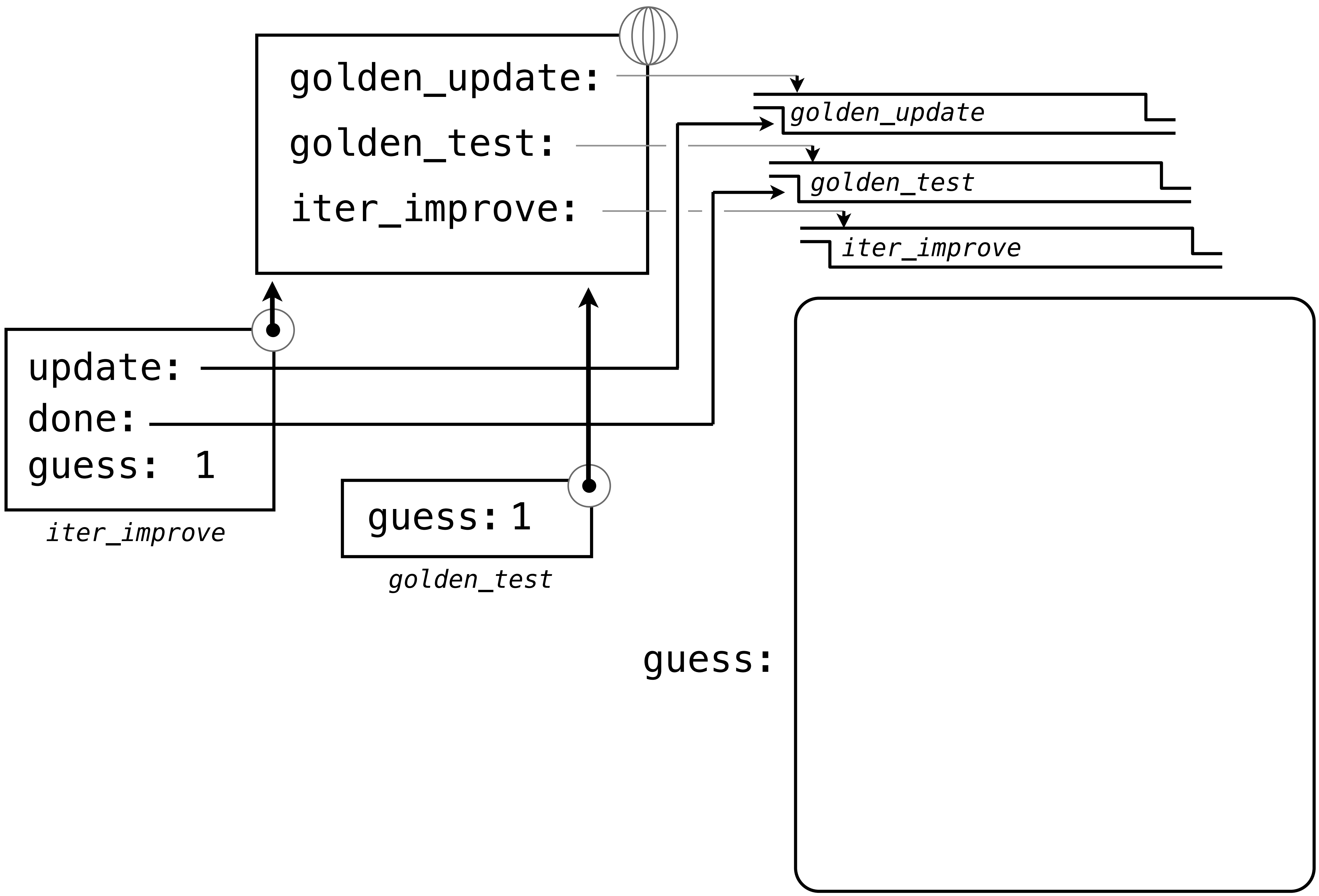

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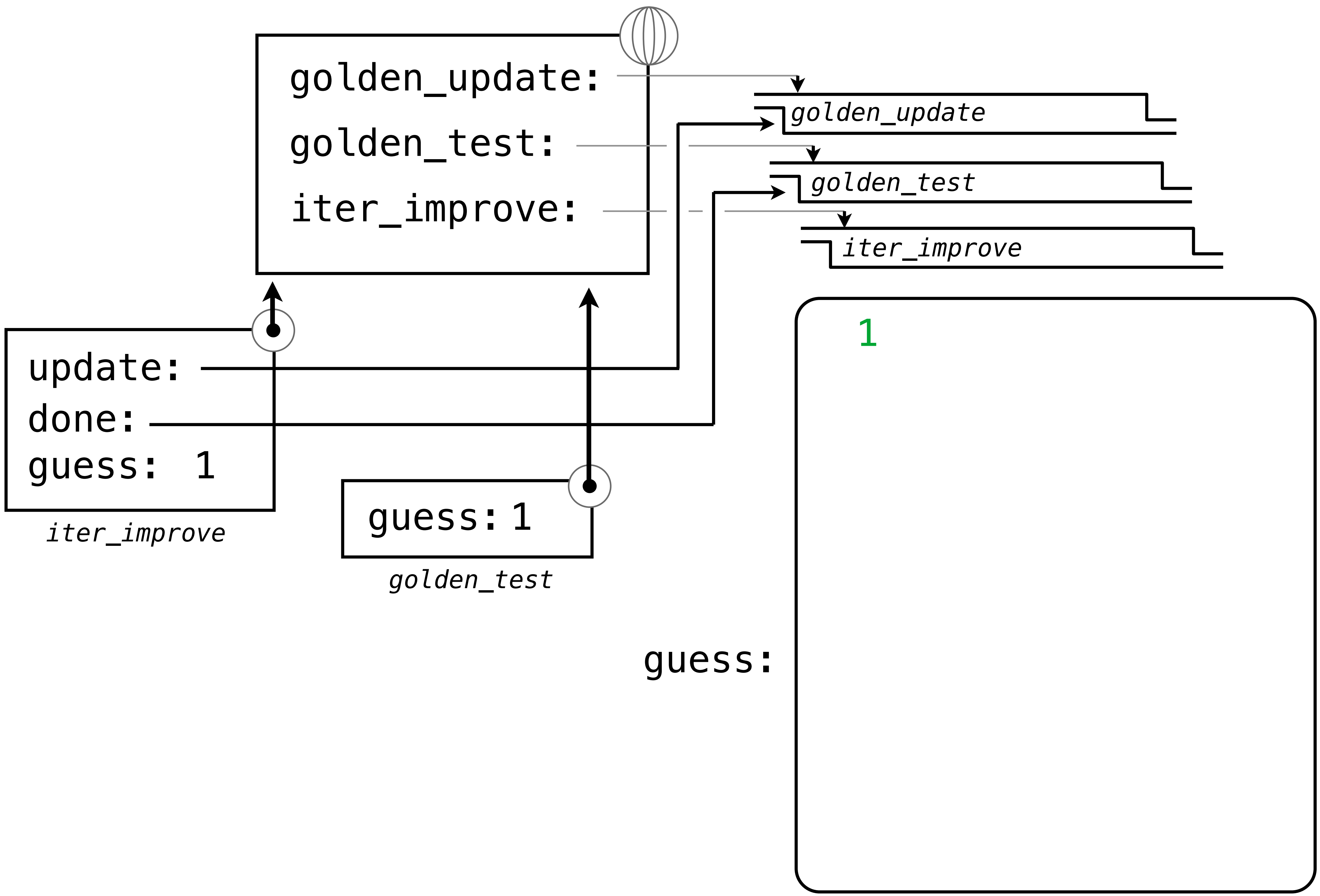

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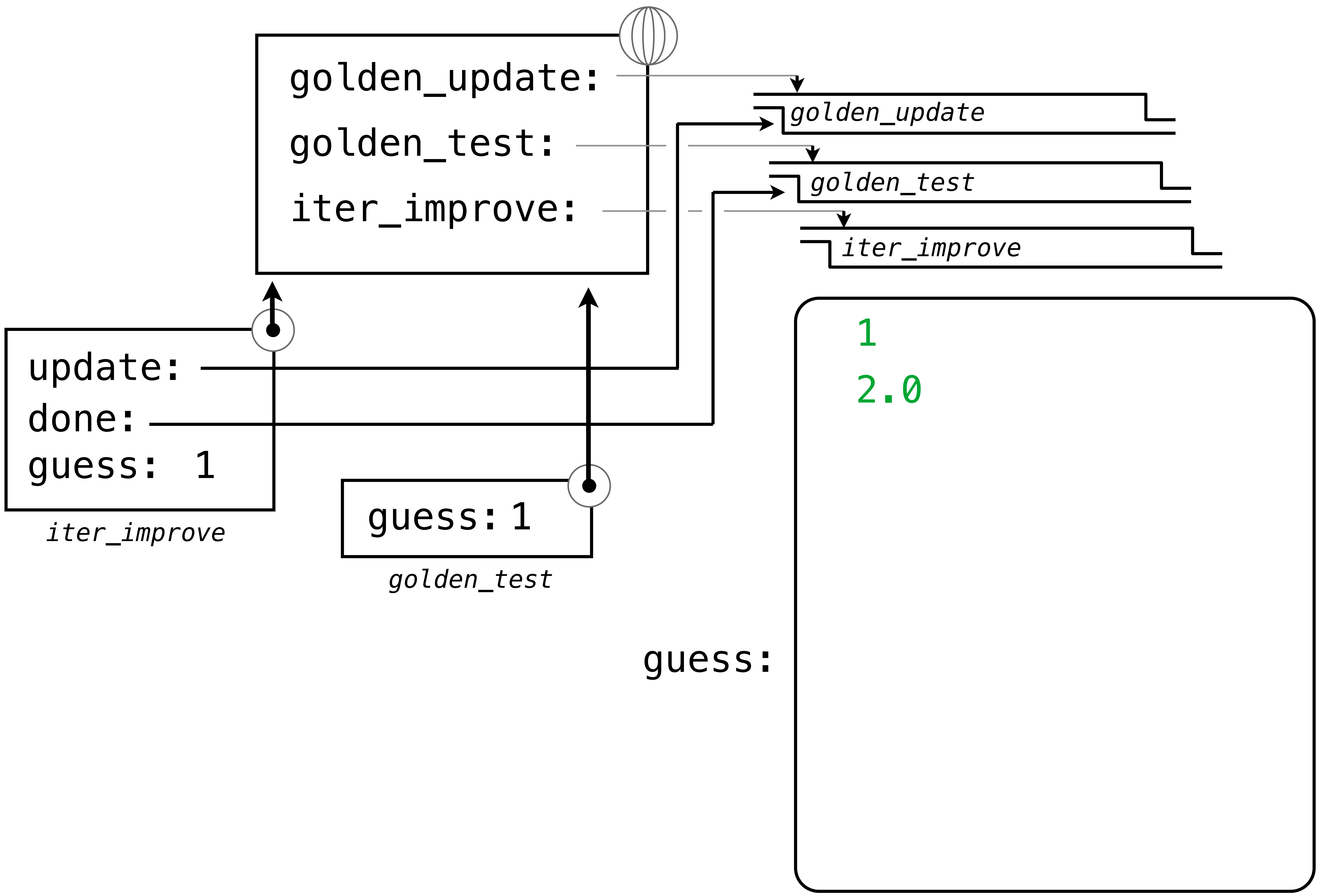

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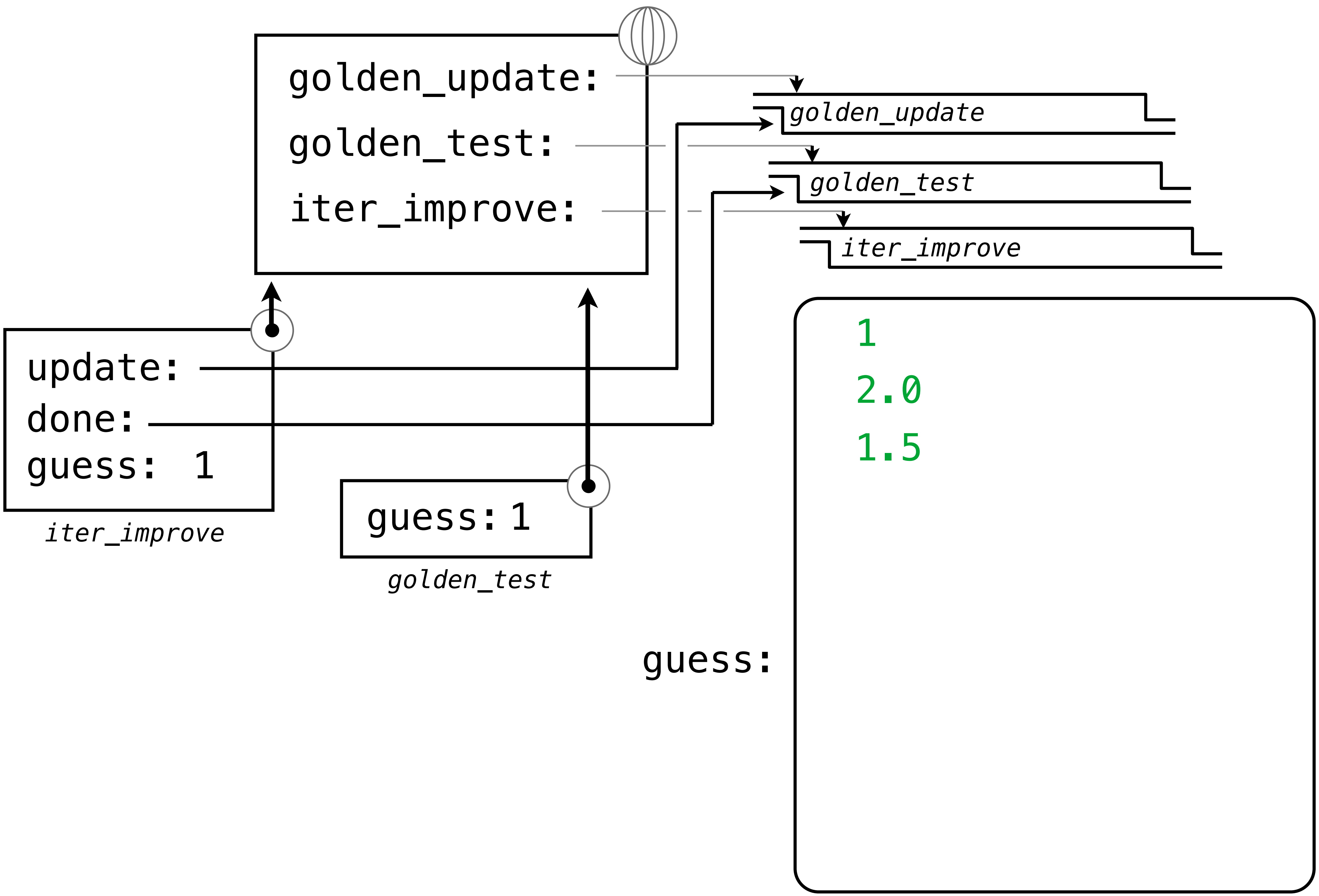

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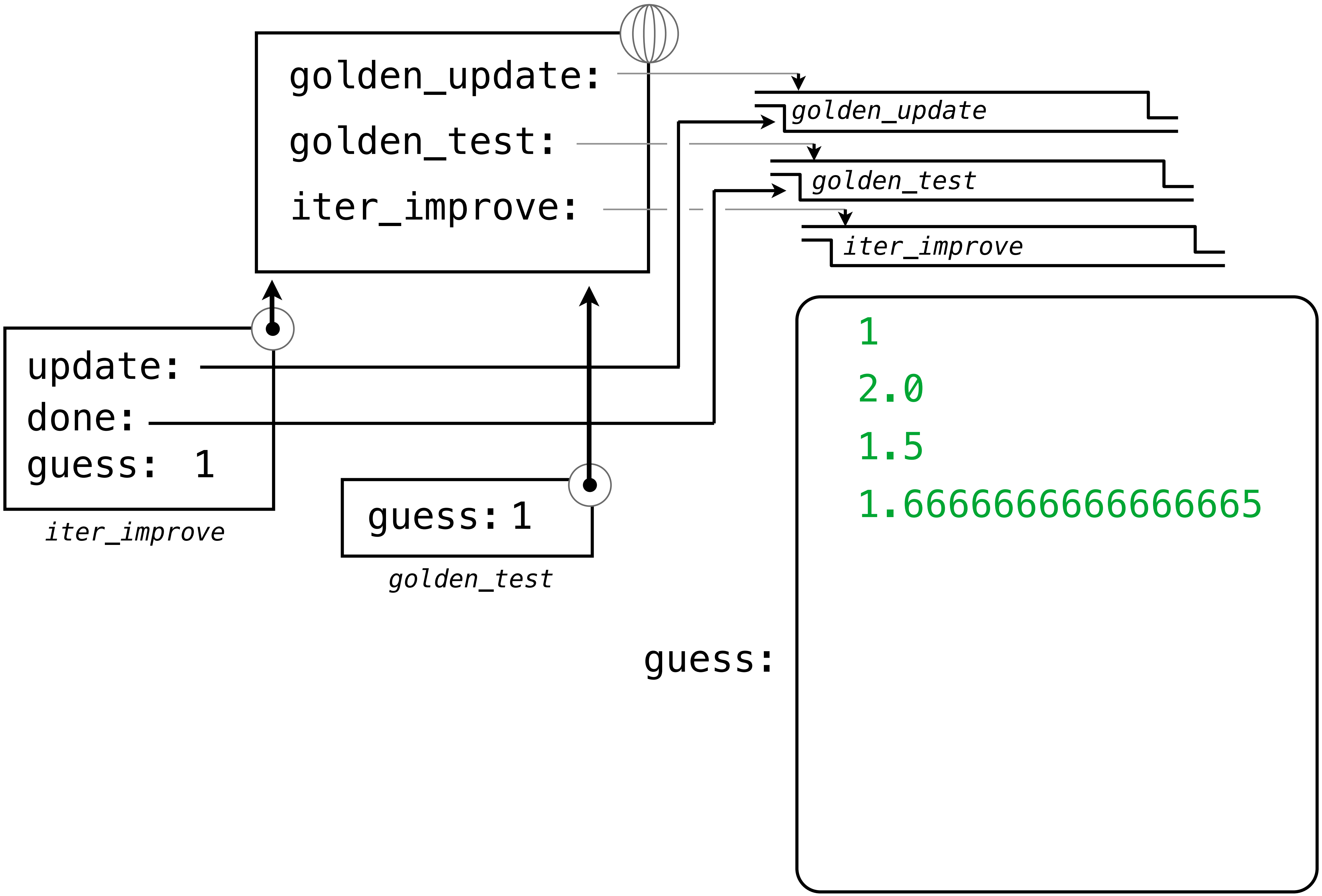

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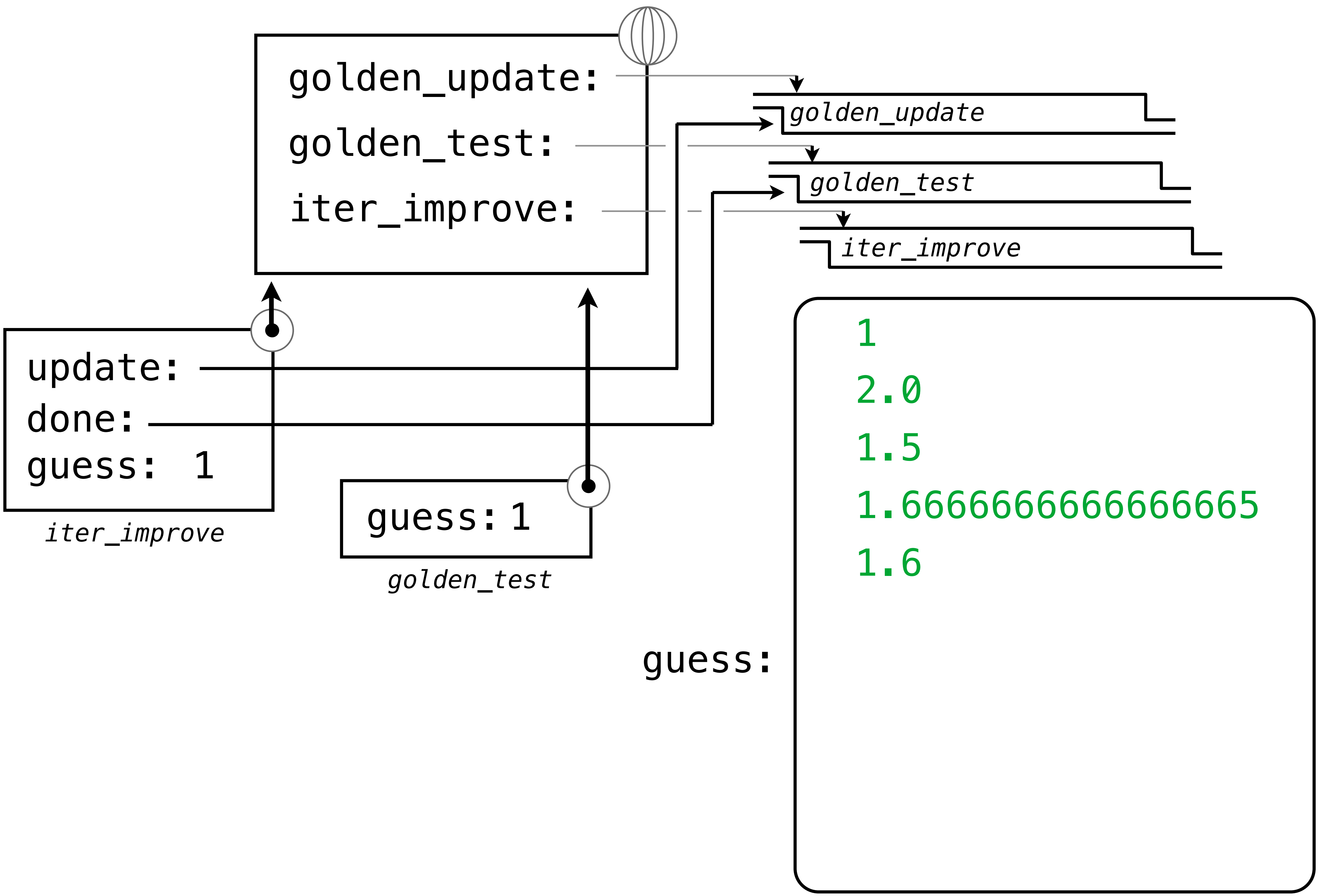

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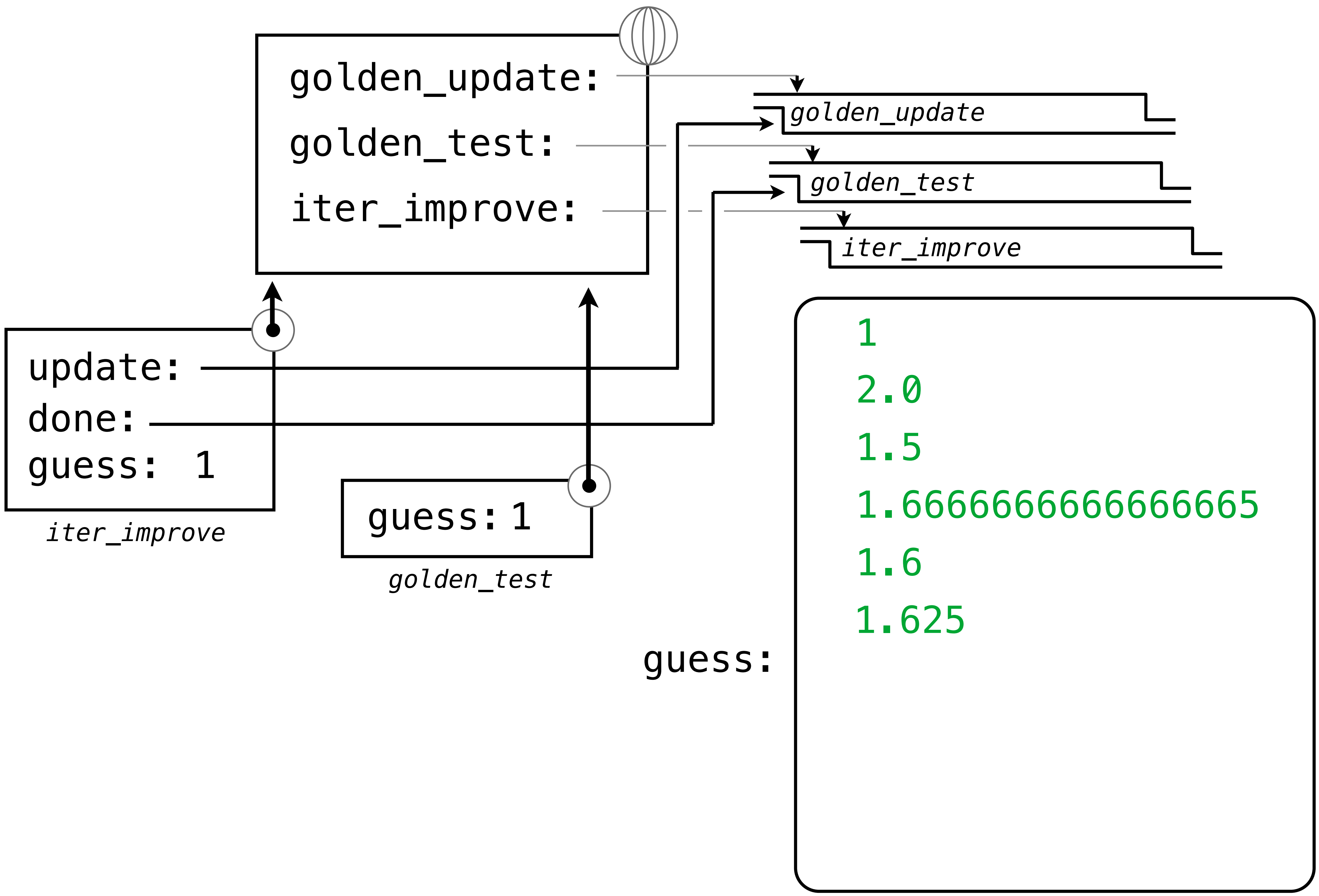

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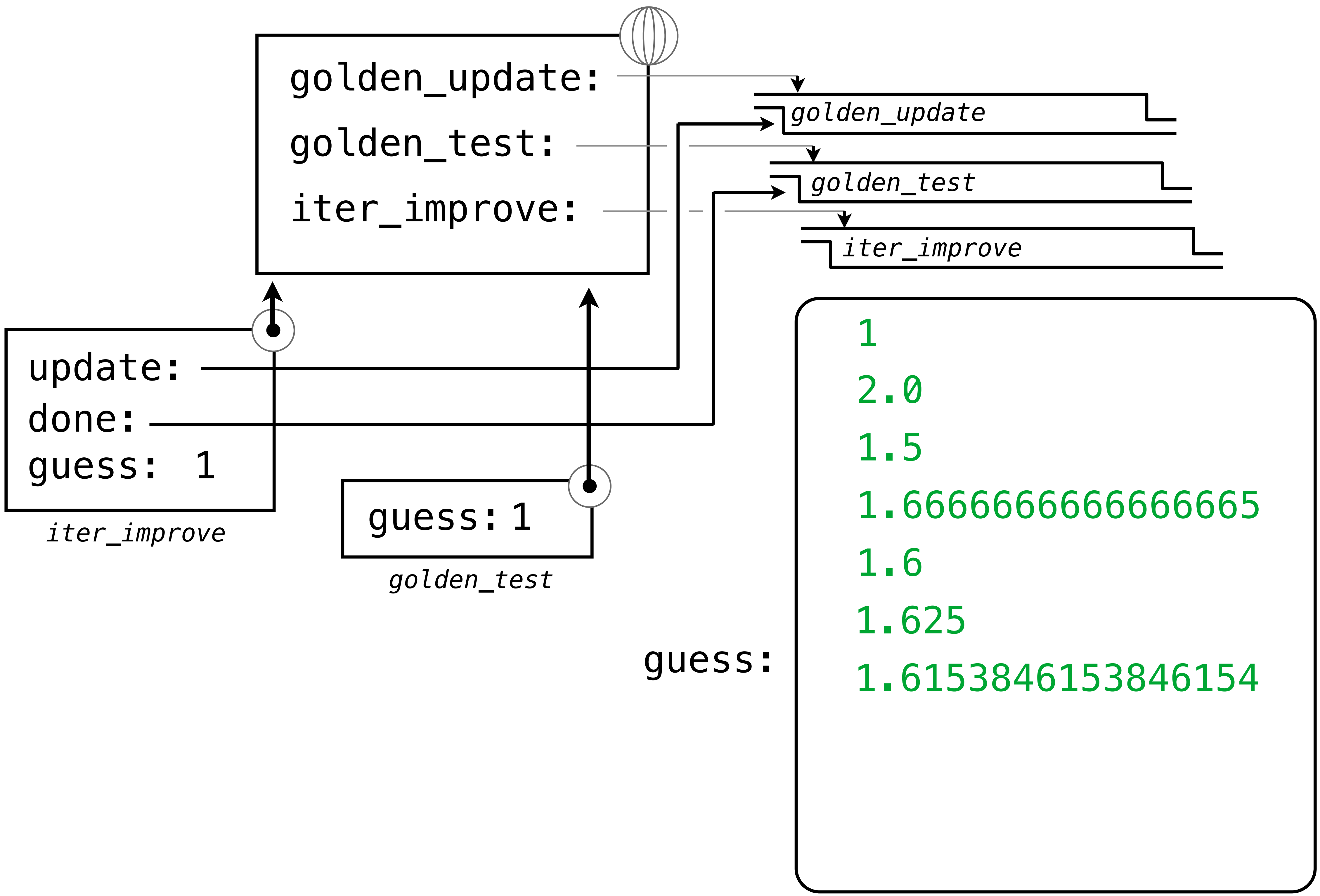

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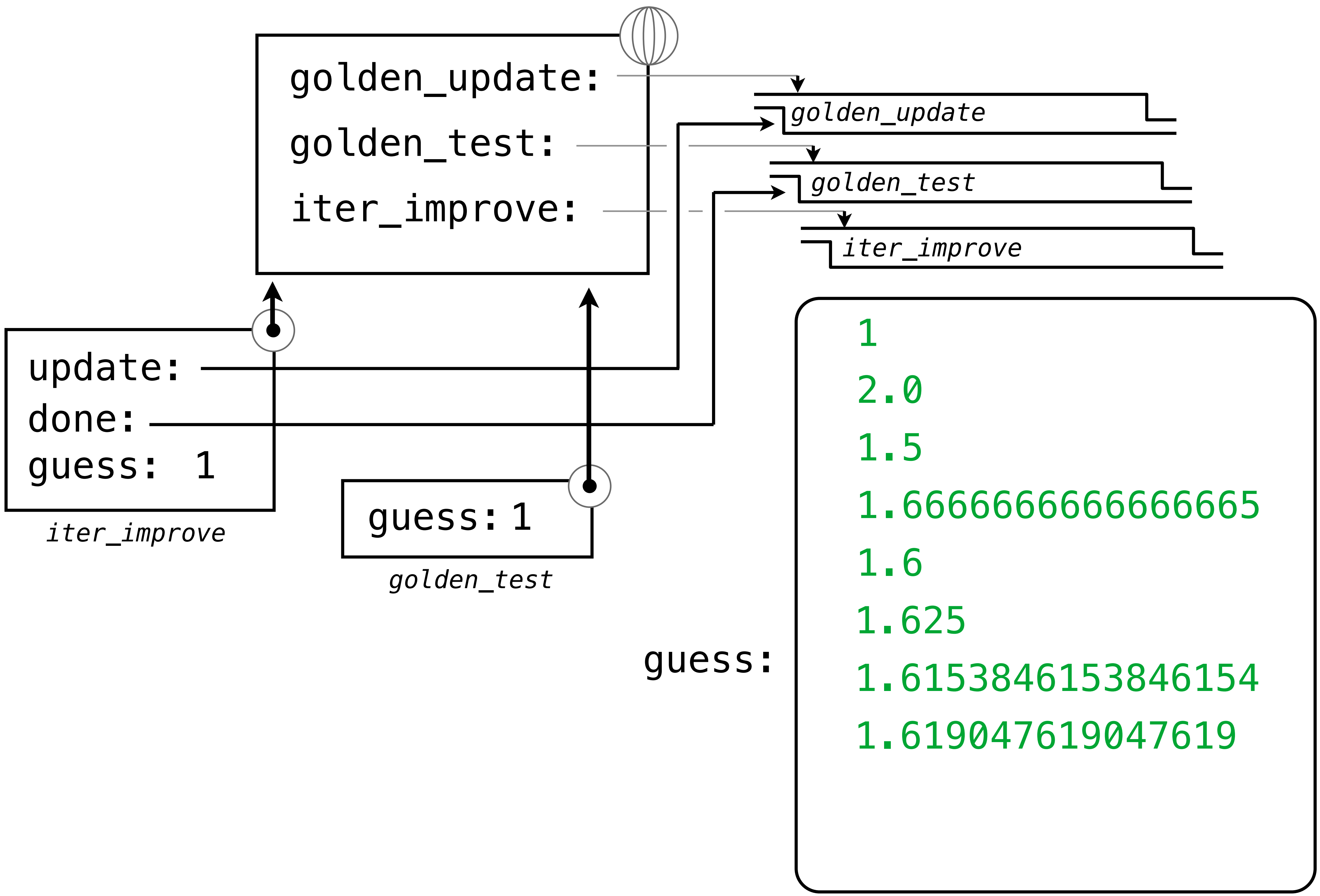

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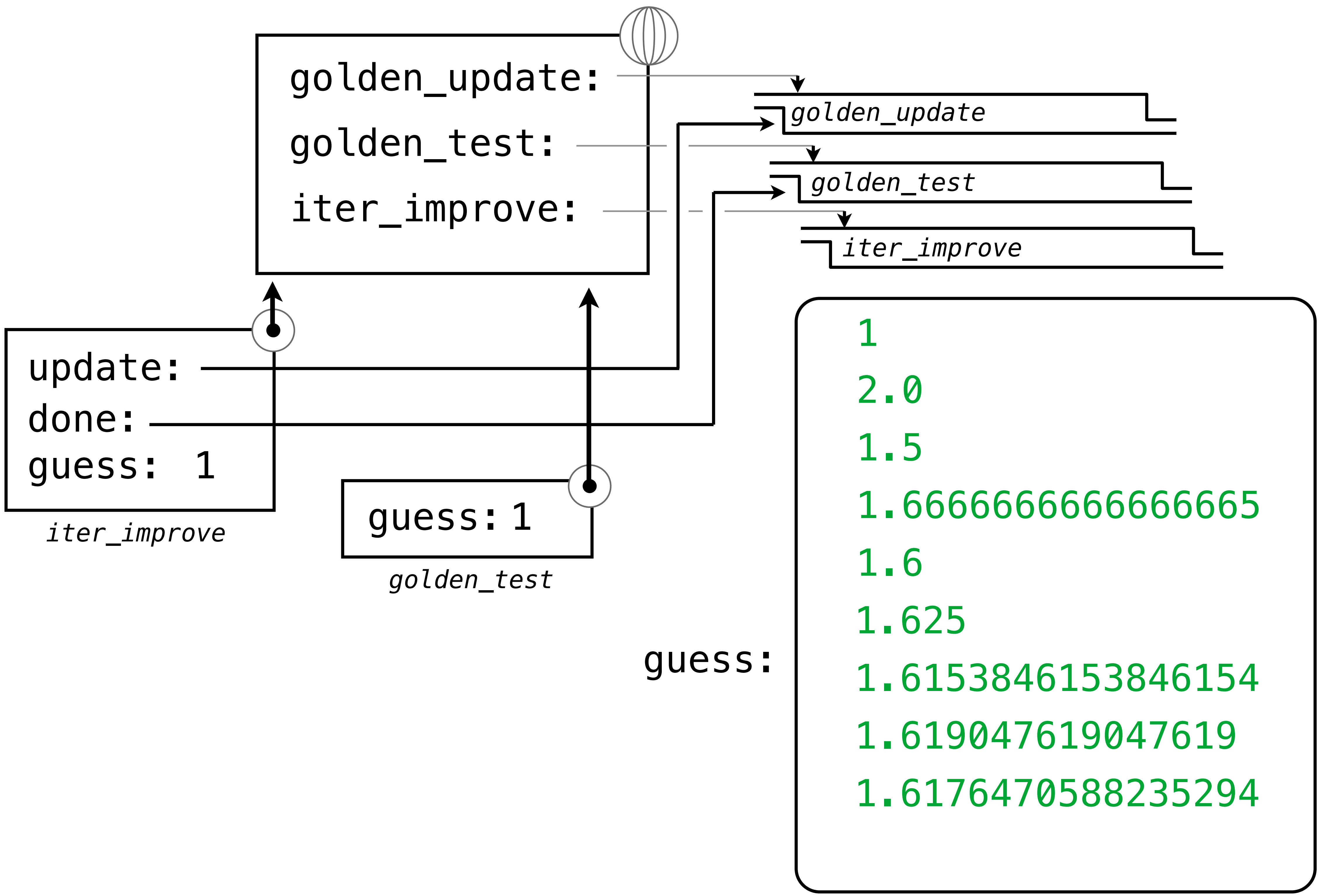

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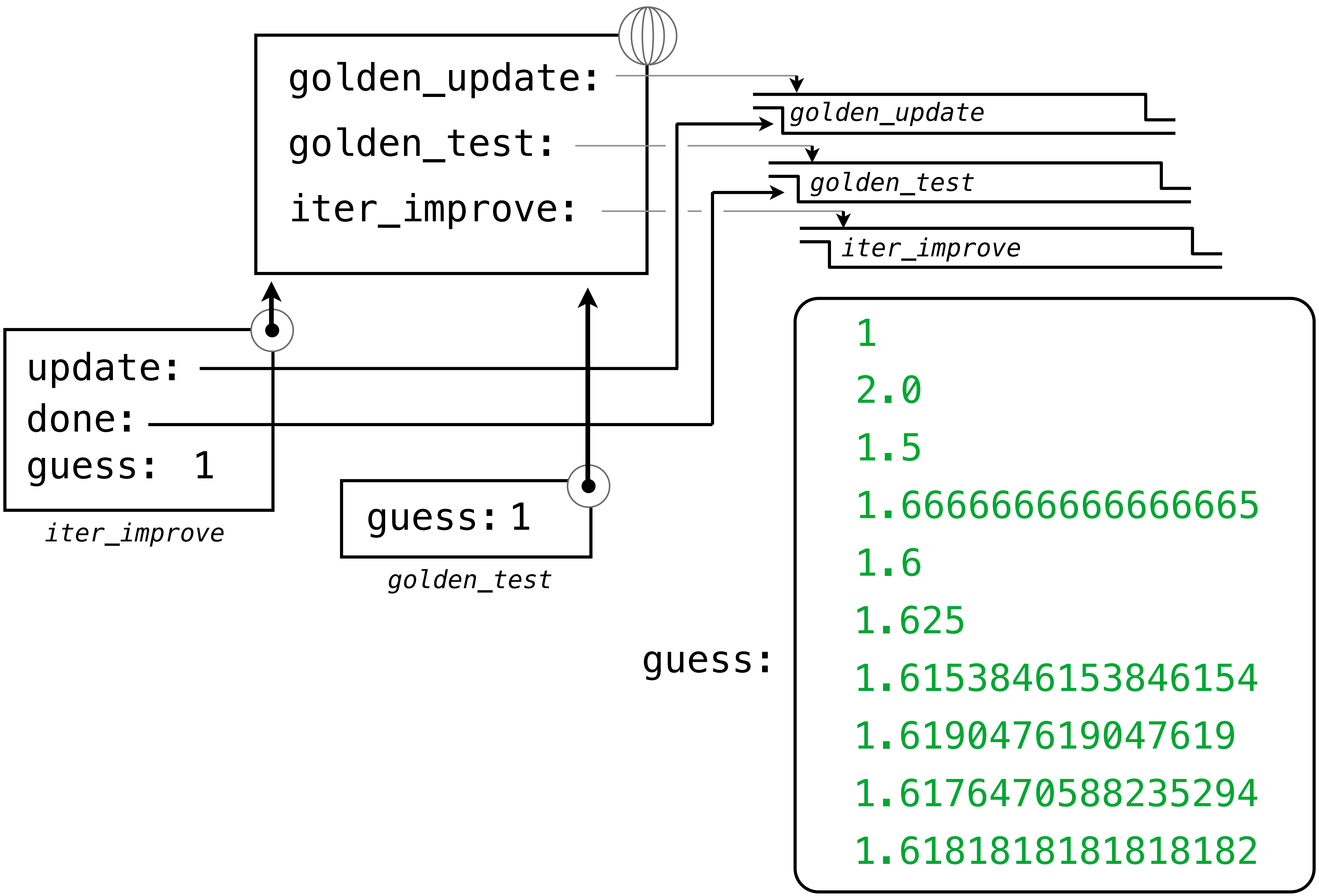
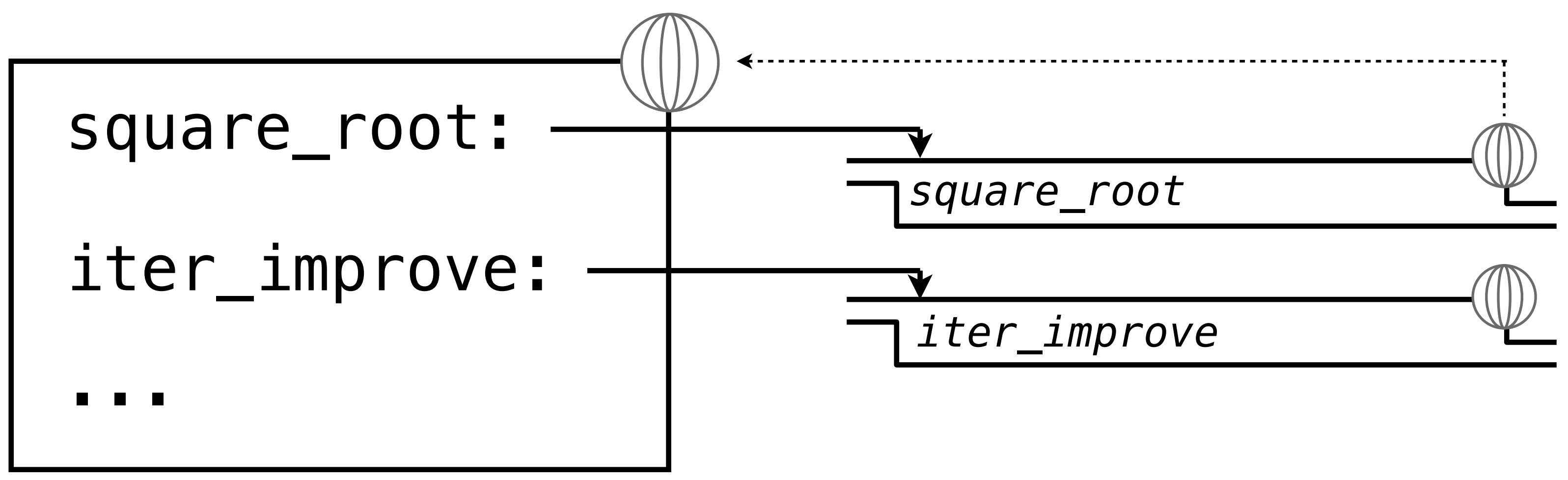


Square Roots by Iterative Improvement

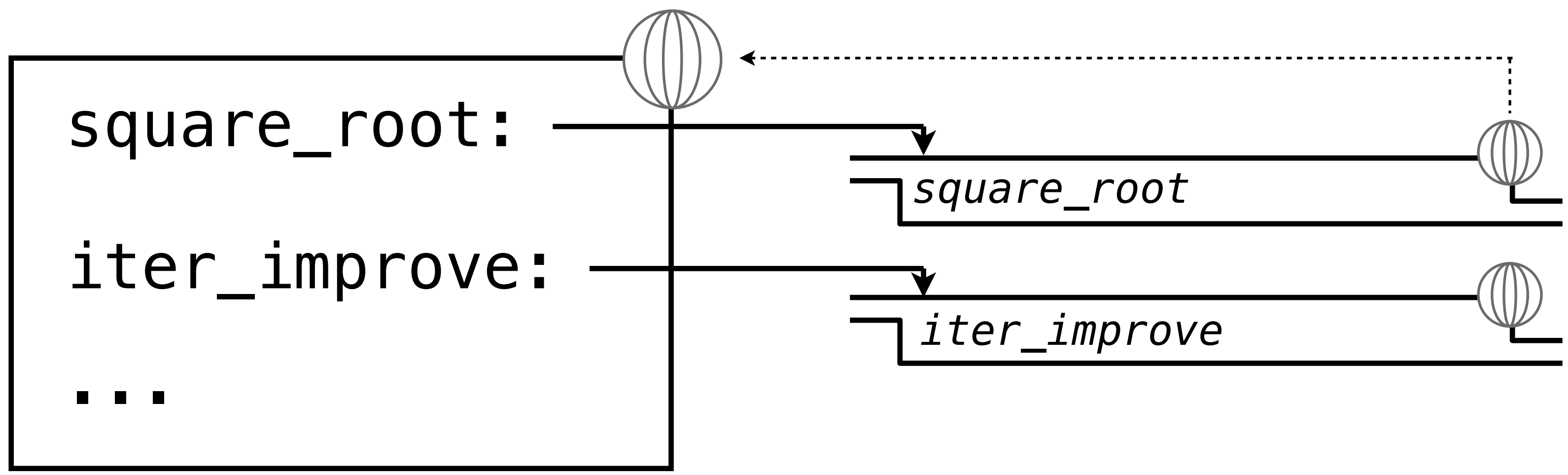
(Demo)

Square Roots by Iterative Improvement



(Demo)

Square Roots by Iterative Improvement



(Demo)

square_root(256)

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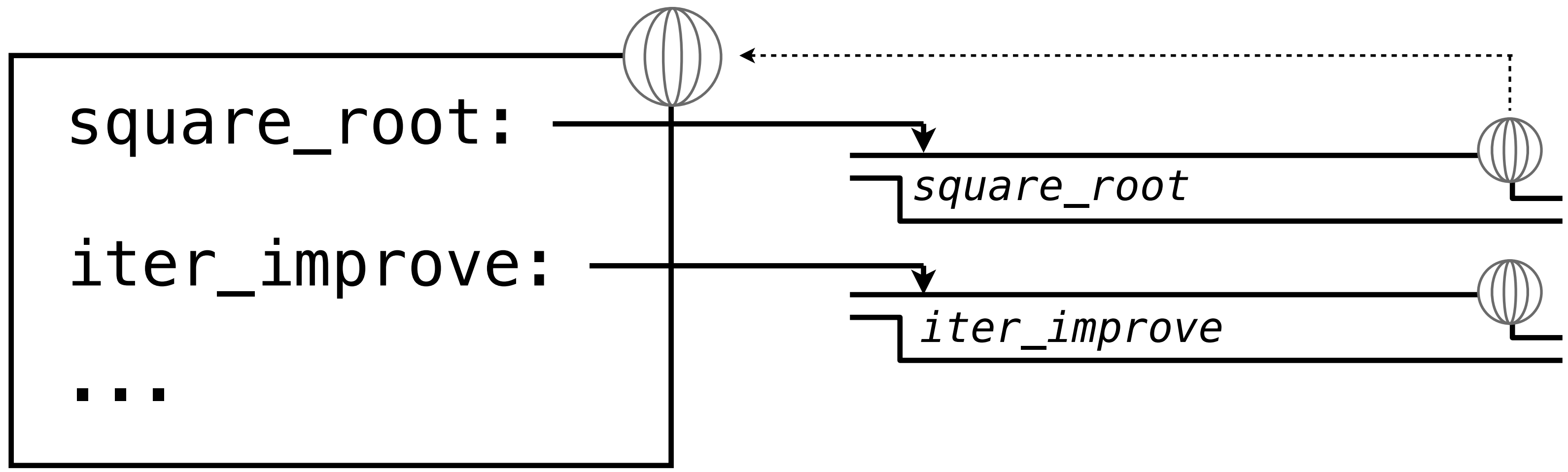

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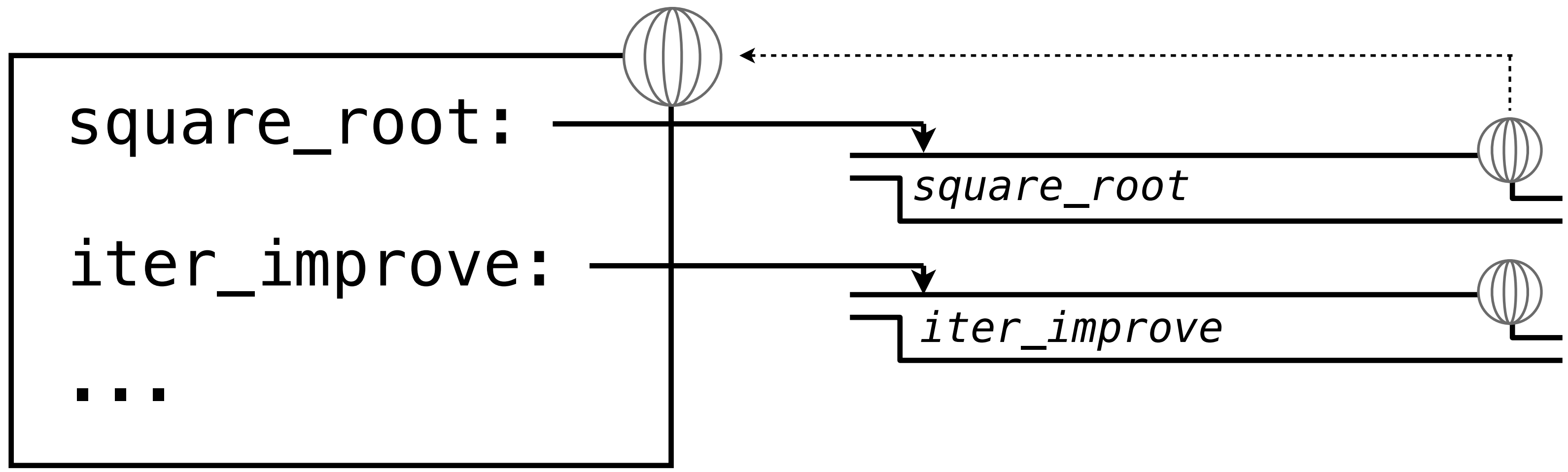

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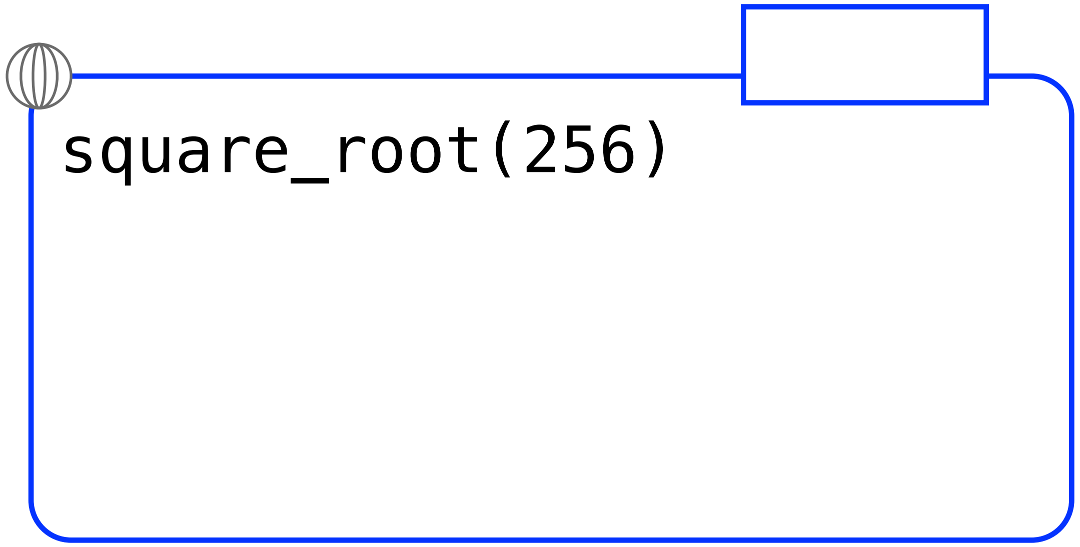

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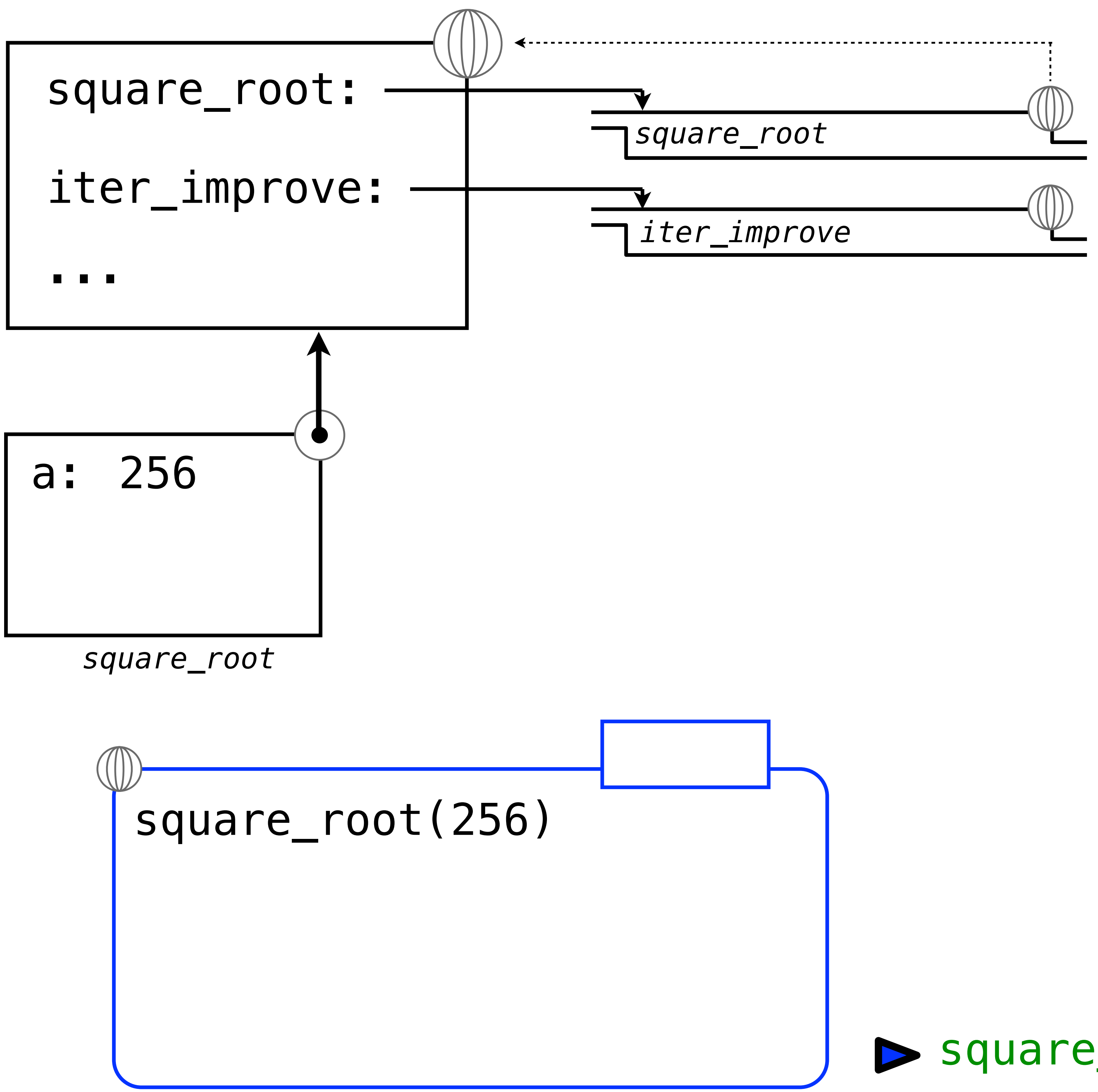

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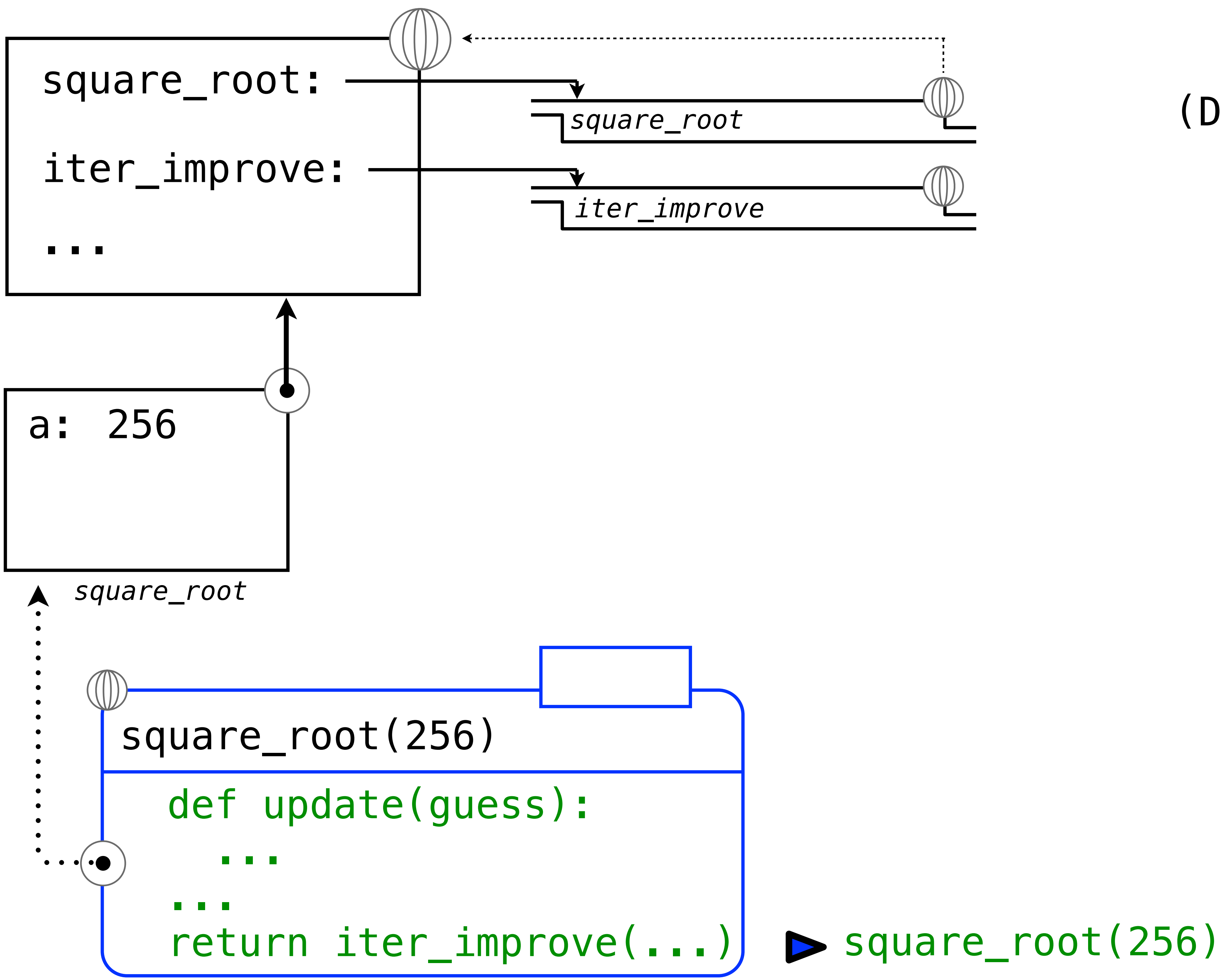

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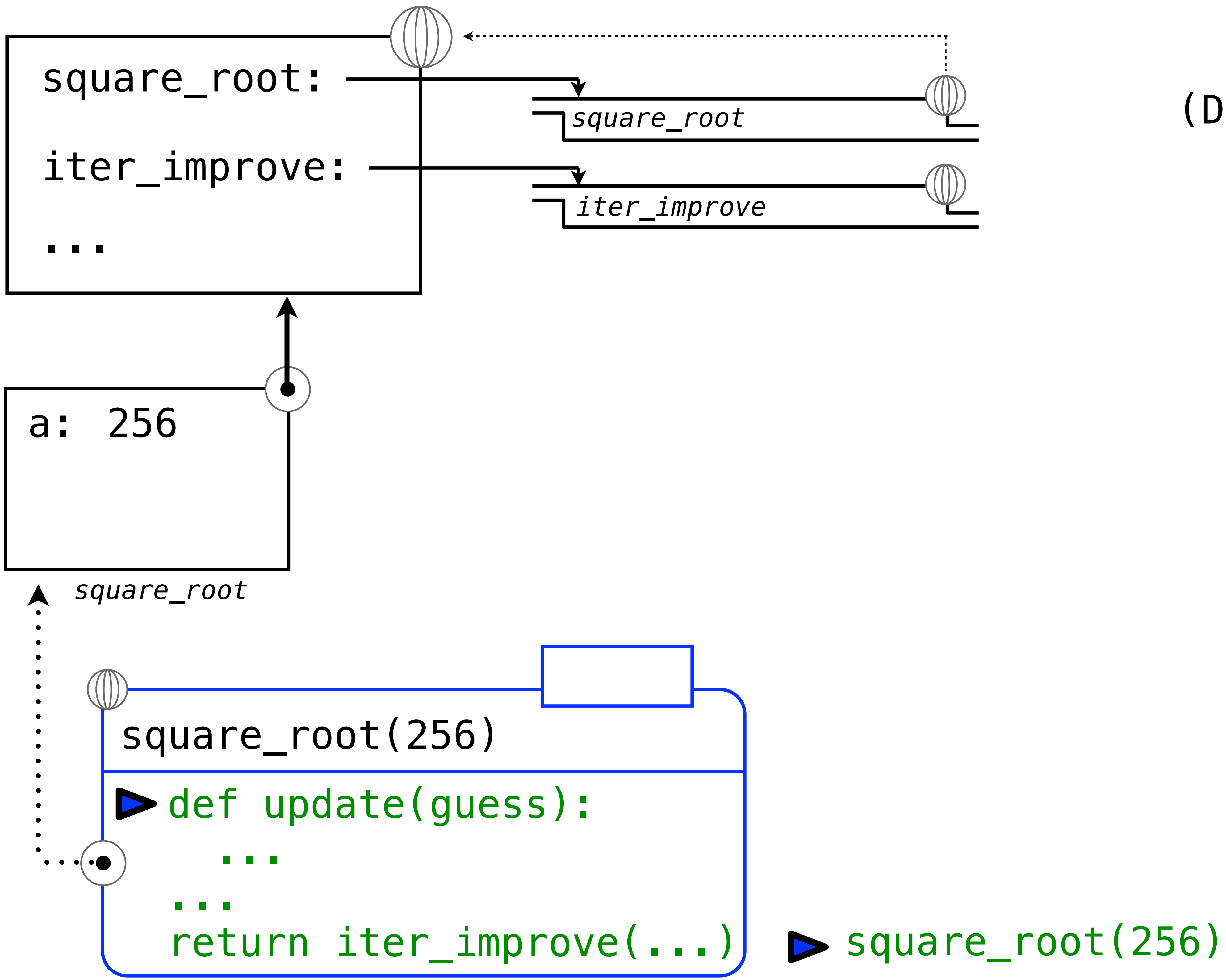

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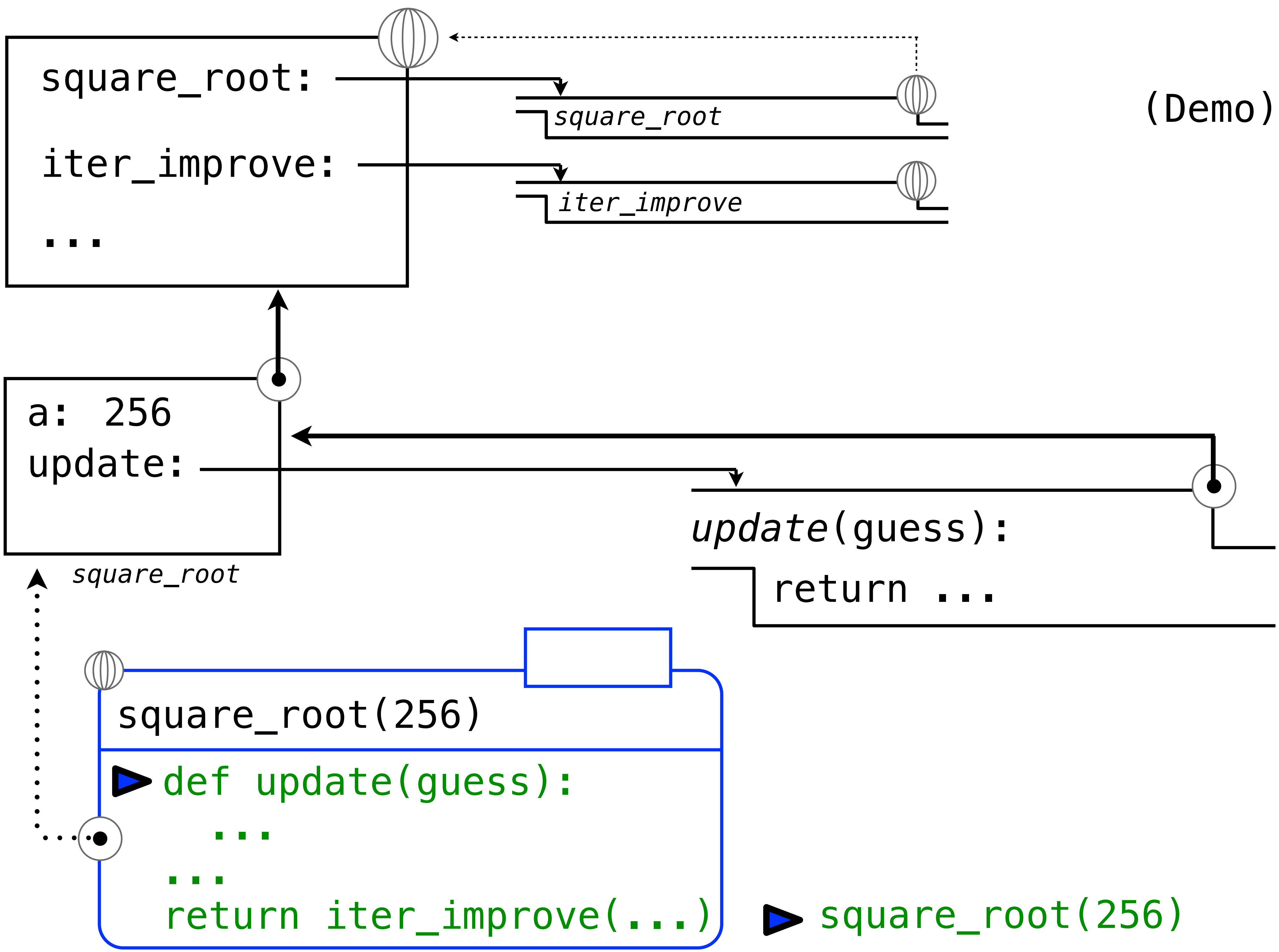

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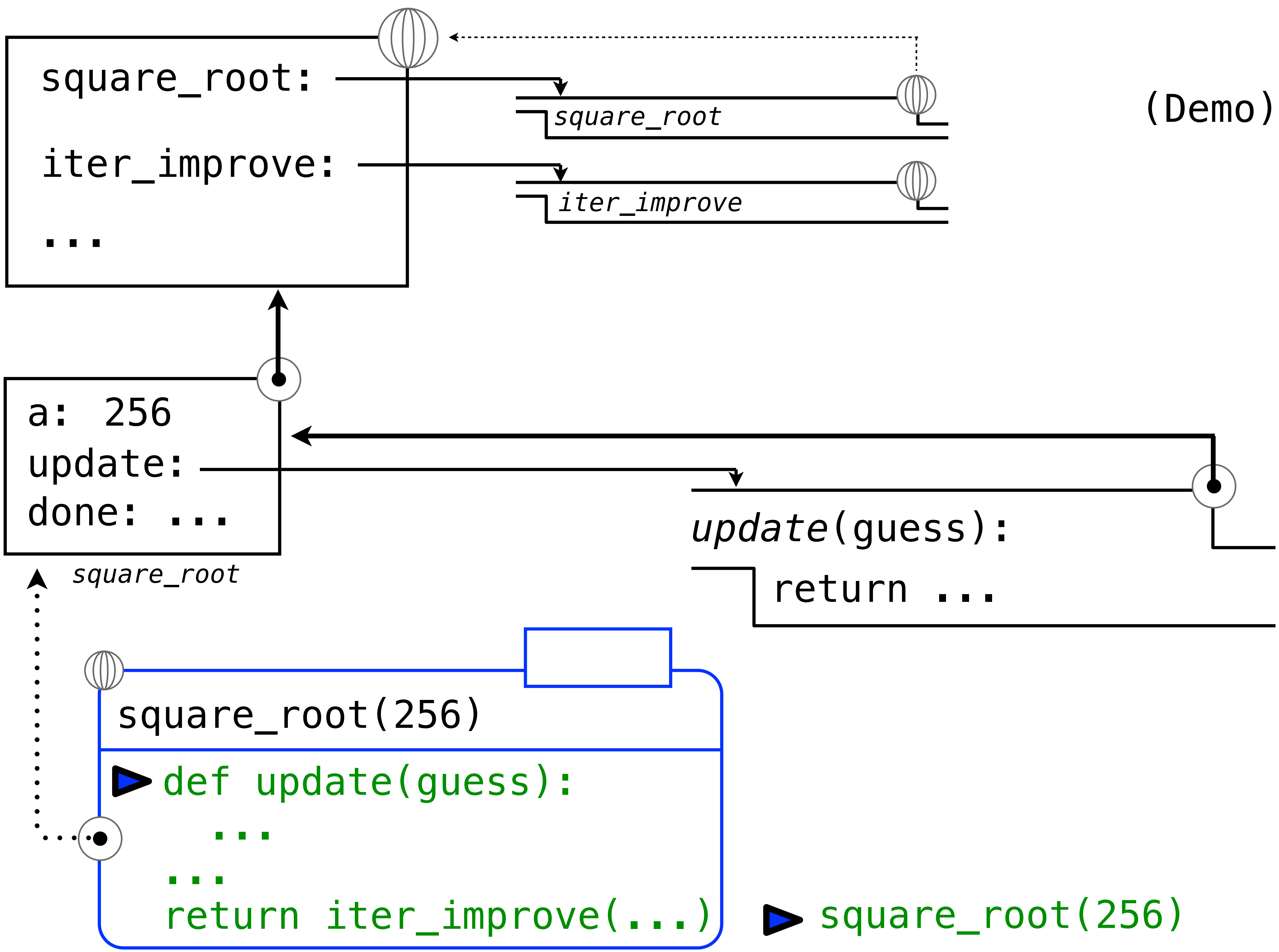

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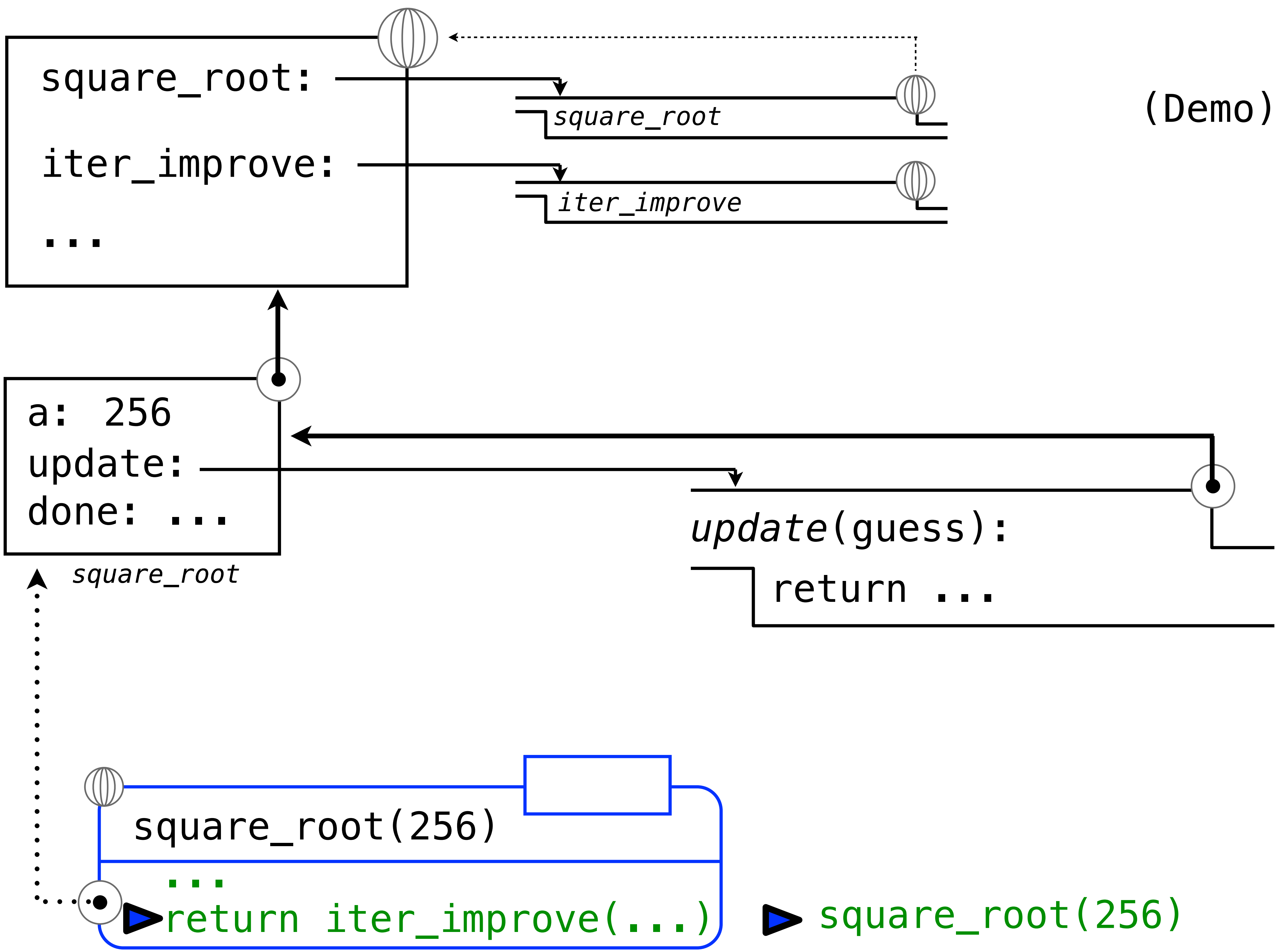

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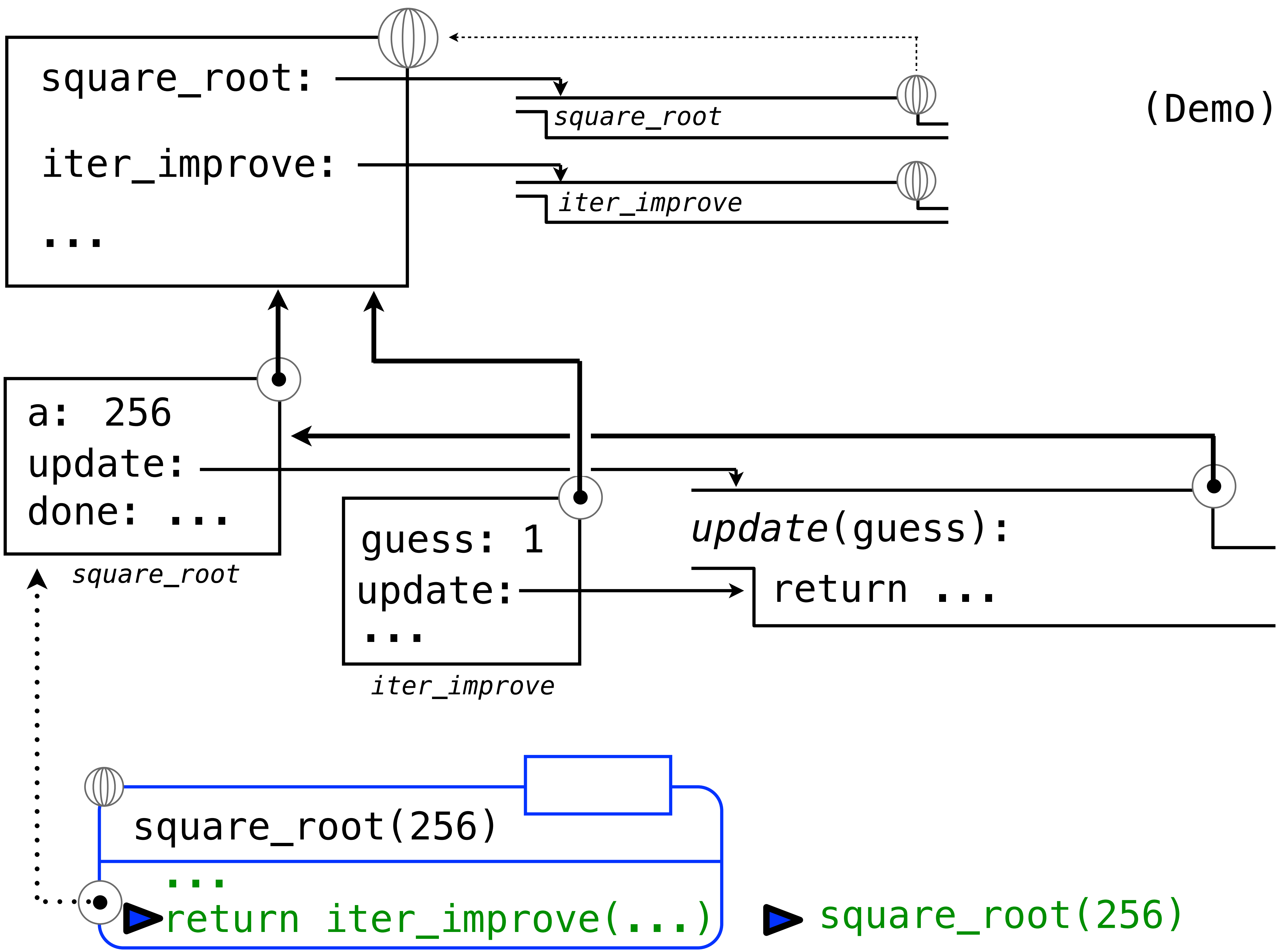

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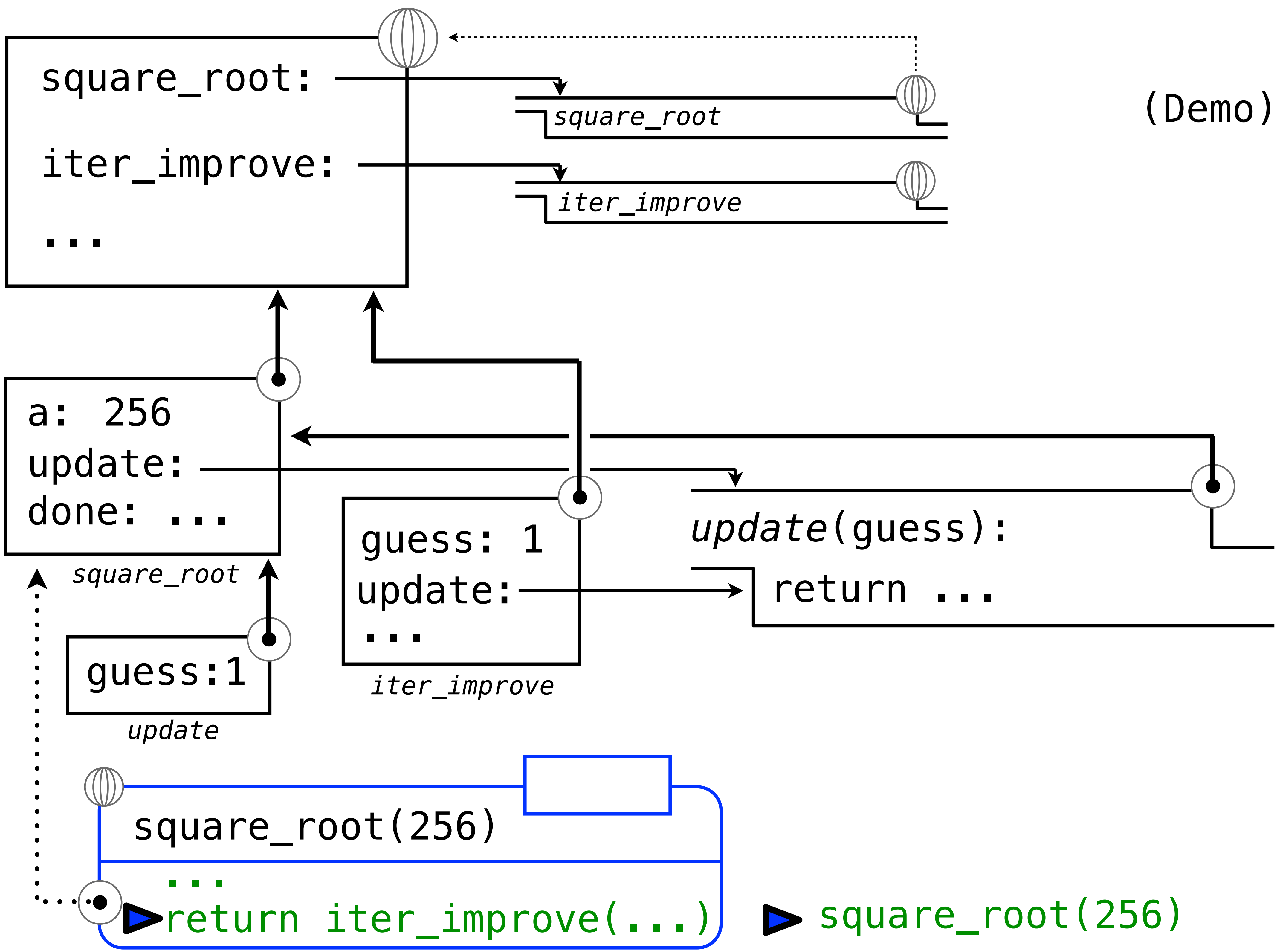

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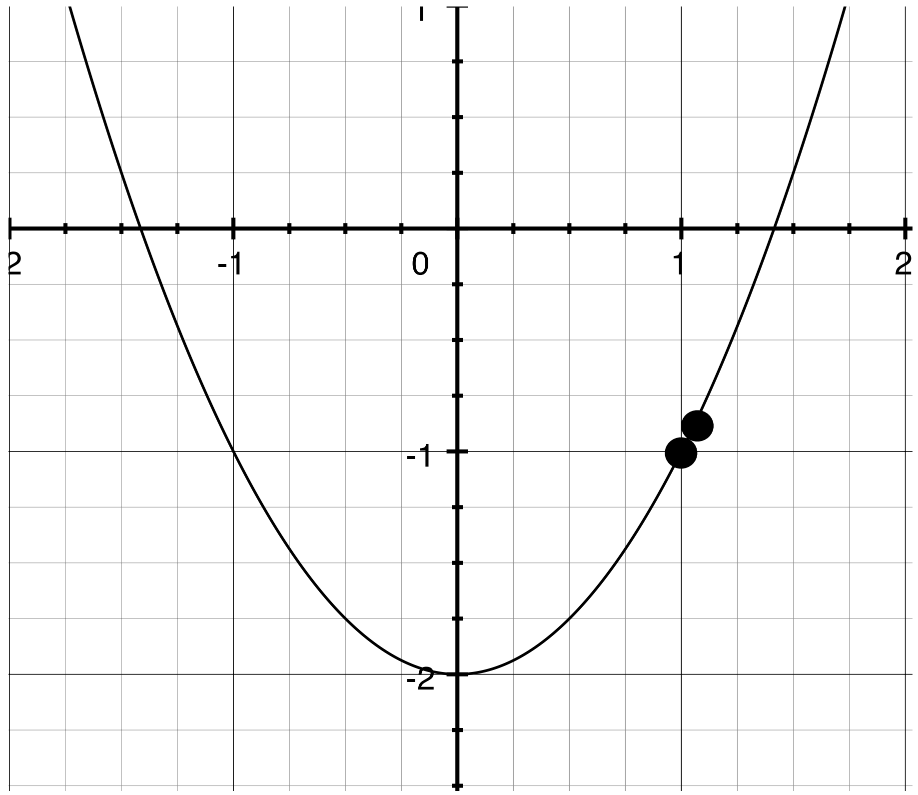

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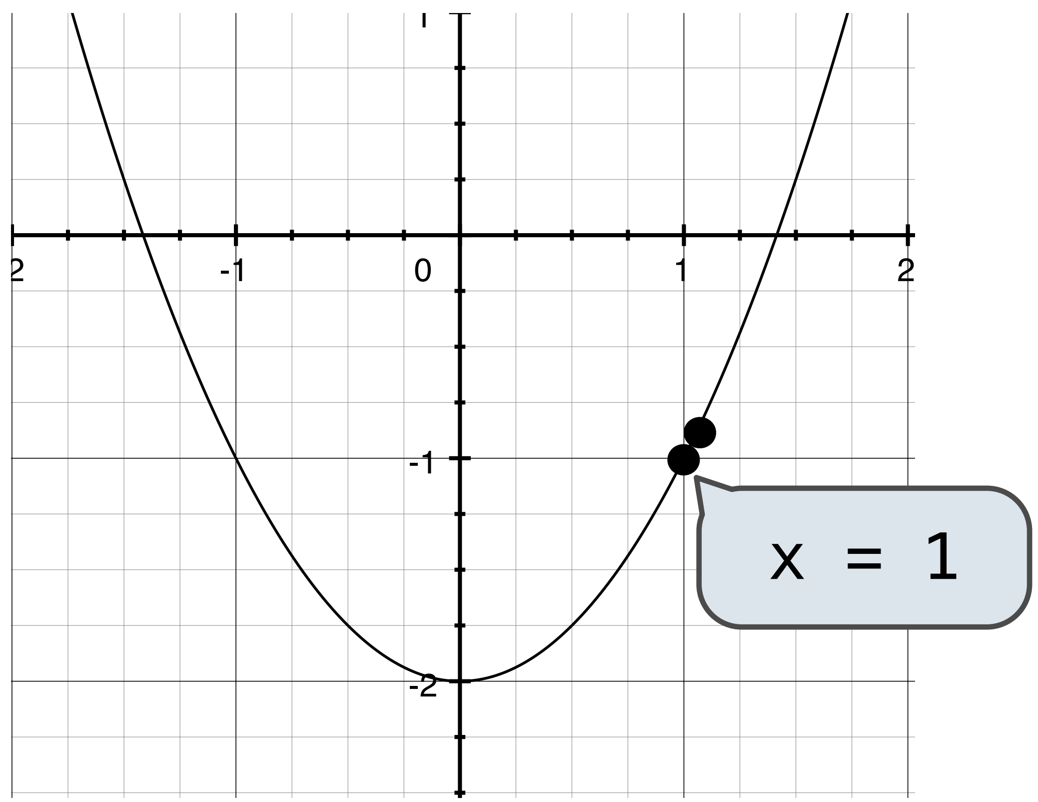

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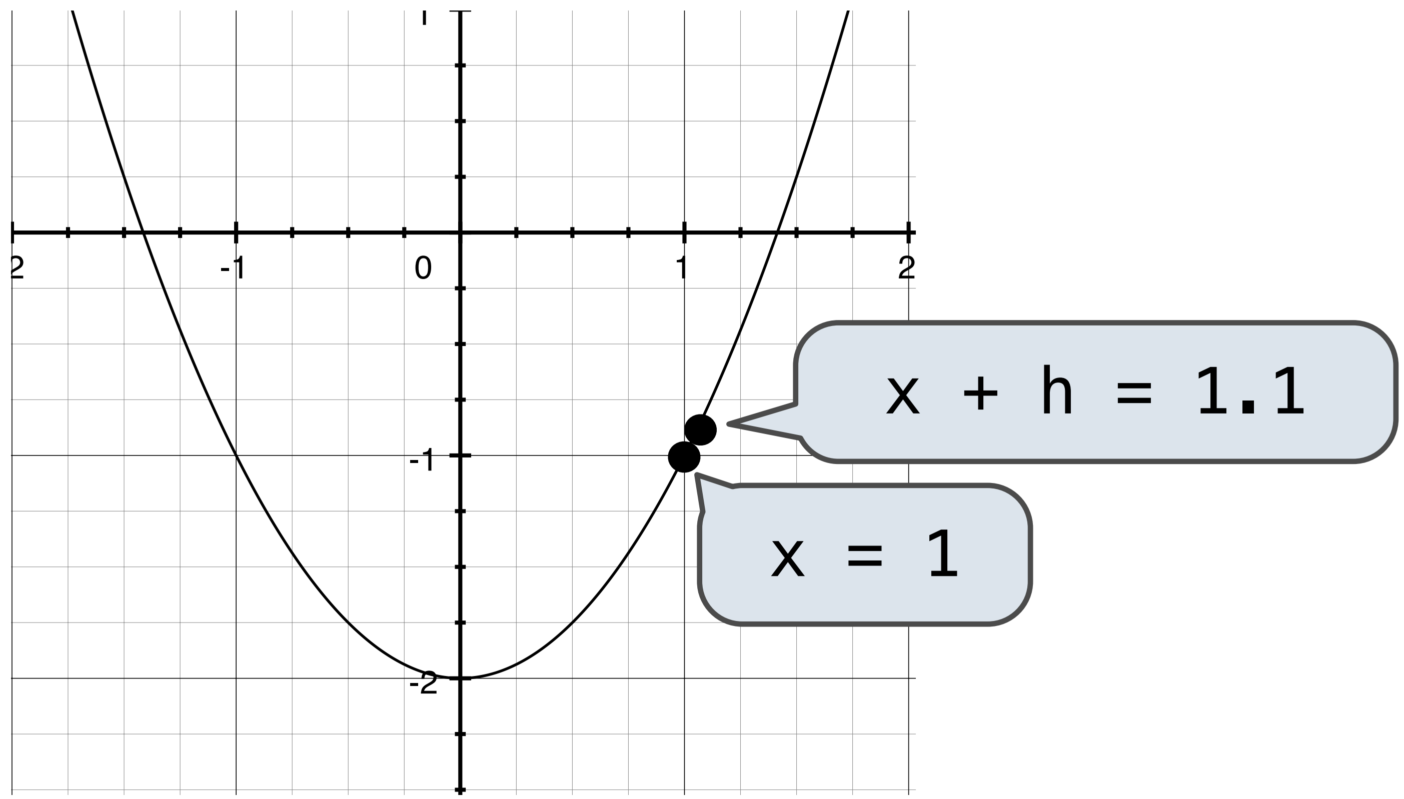

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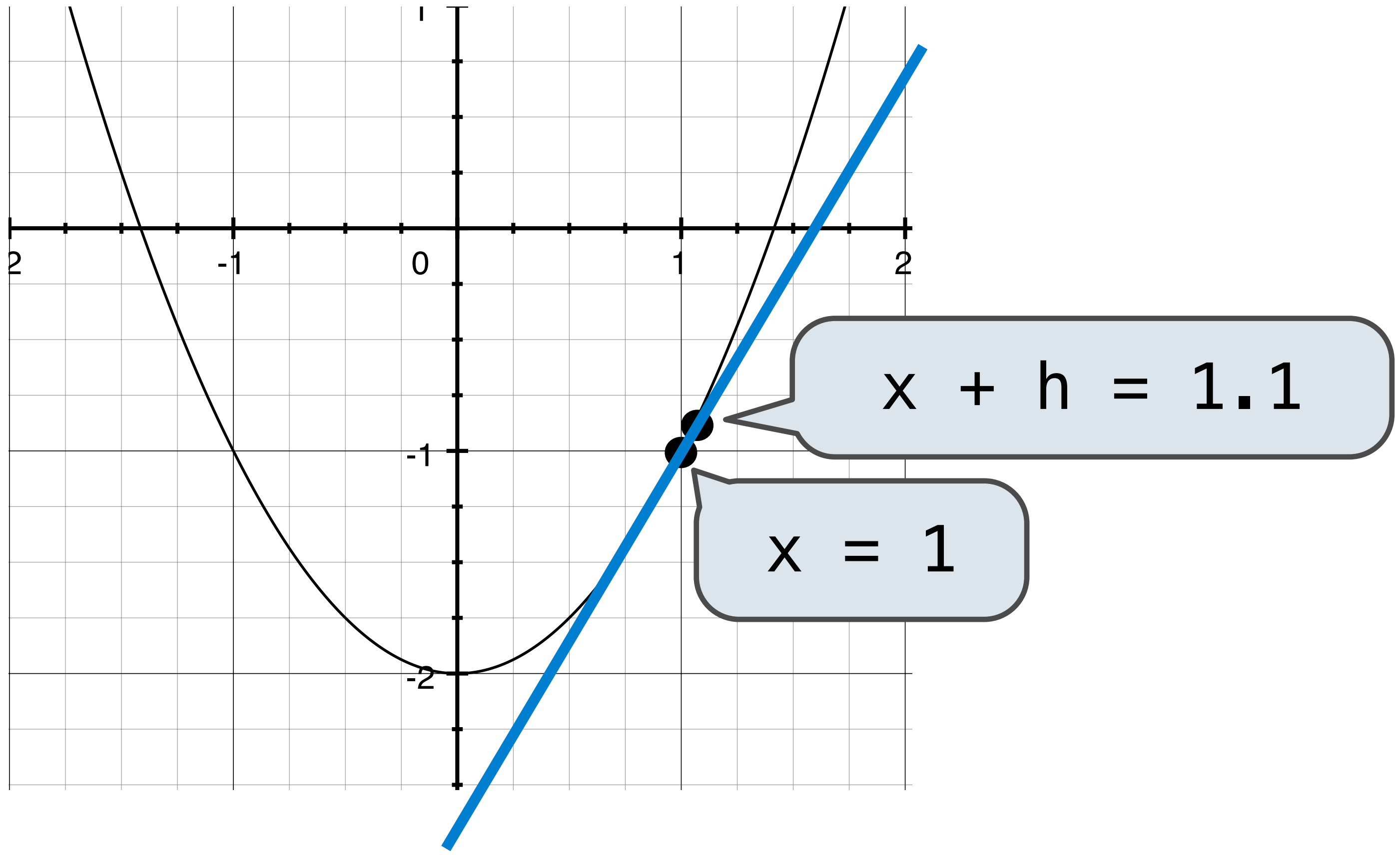

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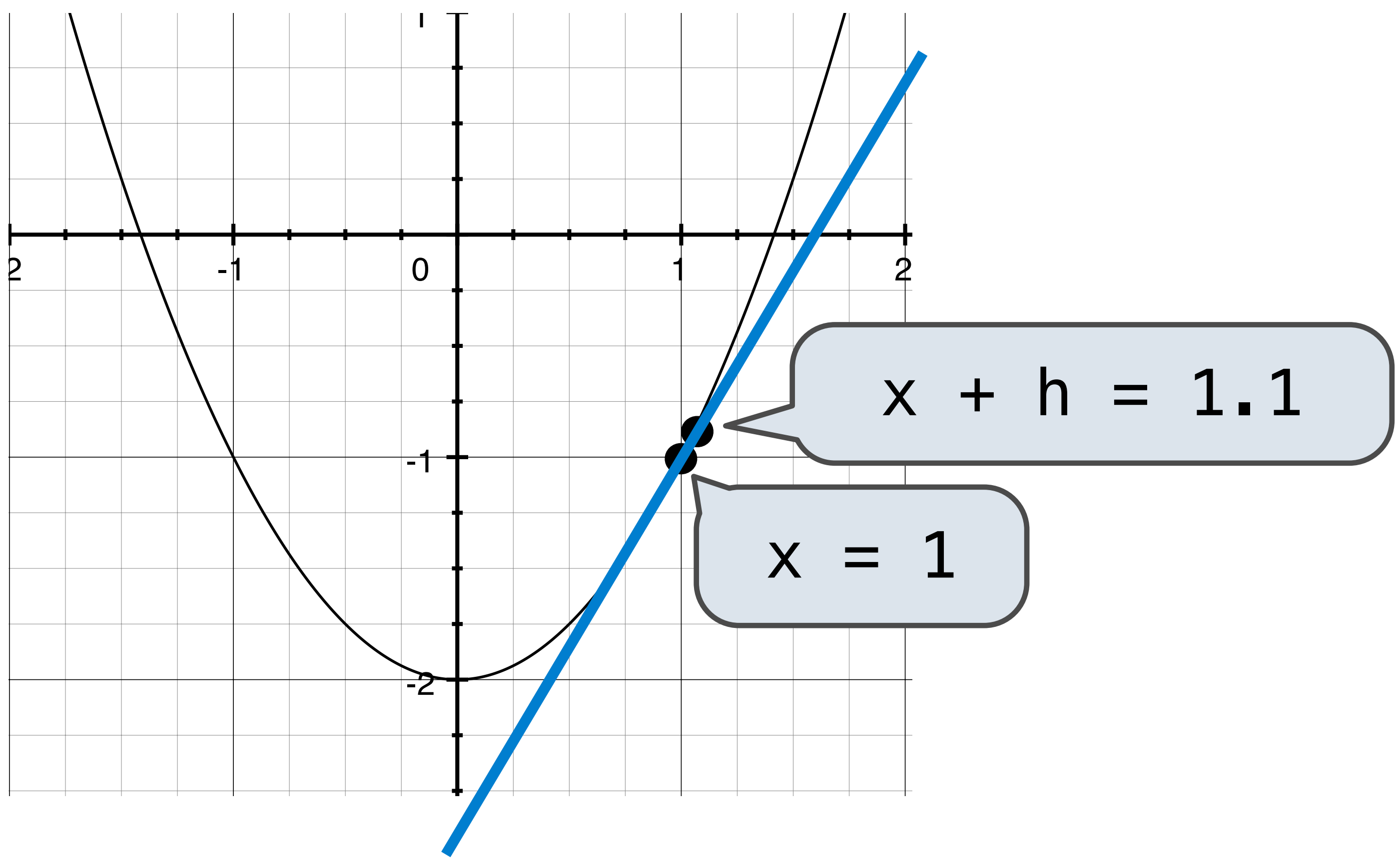

(Demo)

http://en.wikipedia.org/wiki/File:Graph_of_sliding_derivative_line.gif

Approximating Derivatives

`(Demo)`

Implementing Newton's Method

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Definition of a function zero