61A Lecture 21

Monday, October 20

Announcements

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- Midterm 2 is on Monday 10/27 7pm–9pm

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  - Class Conflict? Fill out the conflict form at the top of http://cs61a.org

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  - Review session on Saturday 10/25 3pm–6pm in 2050 VLSB

Time

The Consumption of Time

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For every k, n/k is also a factor!

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(Demo)

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(Demo)

Interactive Diagram

Fibonacci Space Consumption

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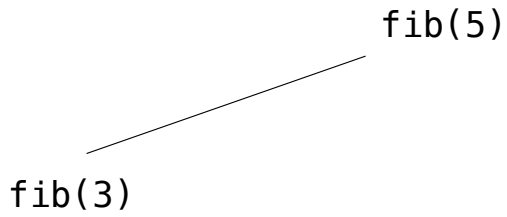
```
fib(5)
```

Fibonacci Space Consumption



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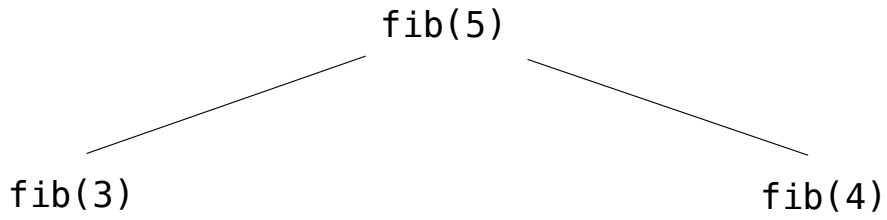

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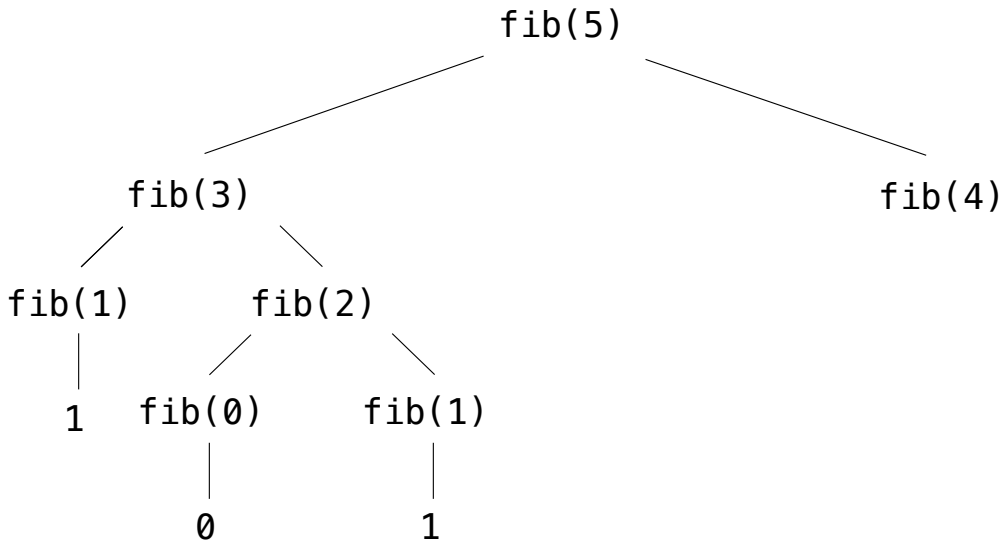

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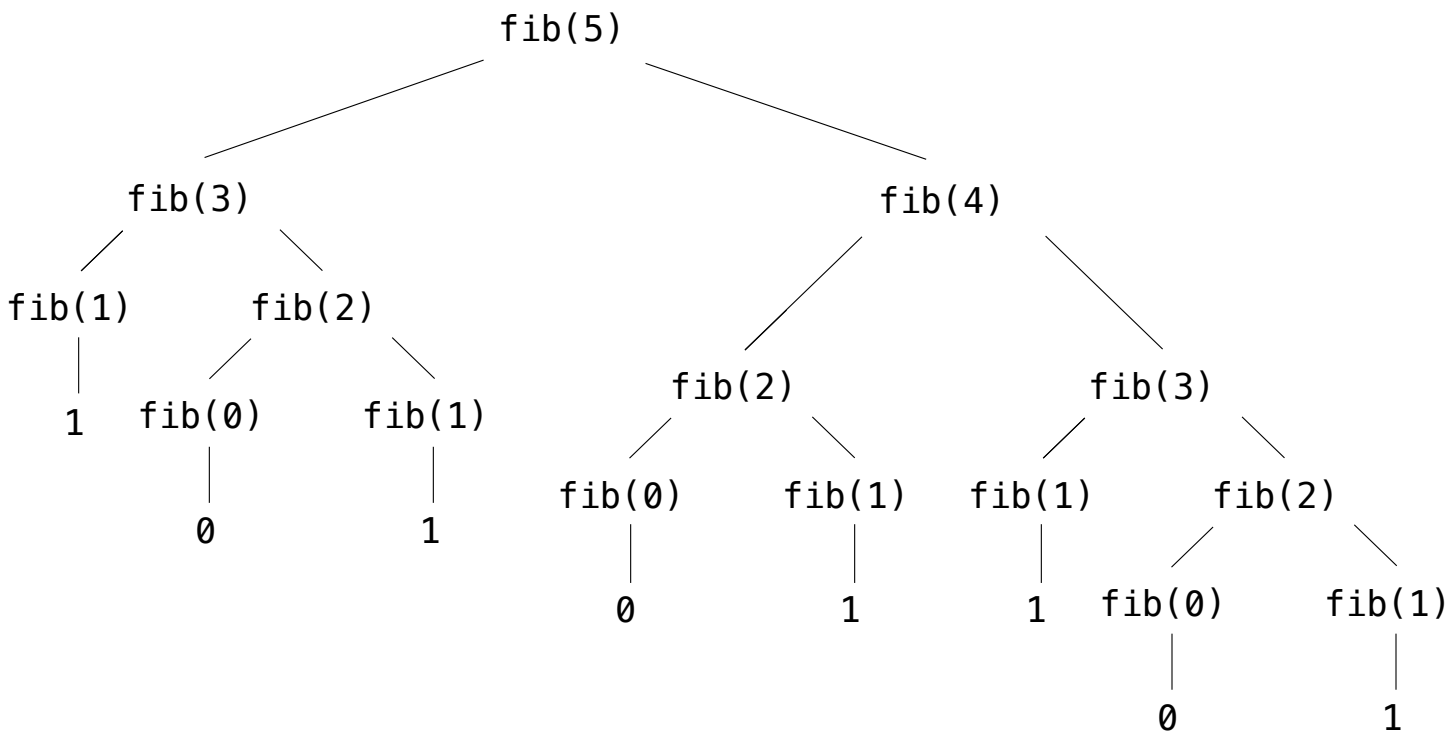

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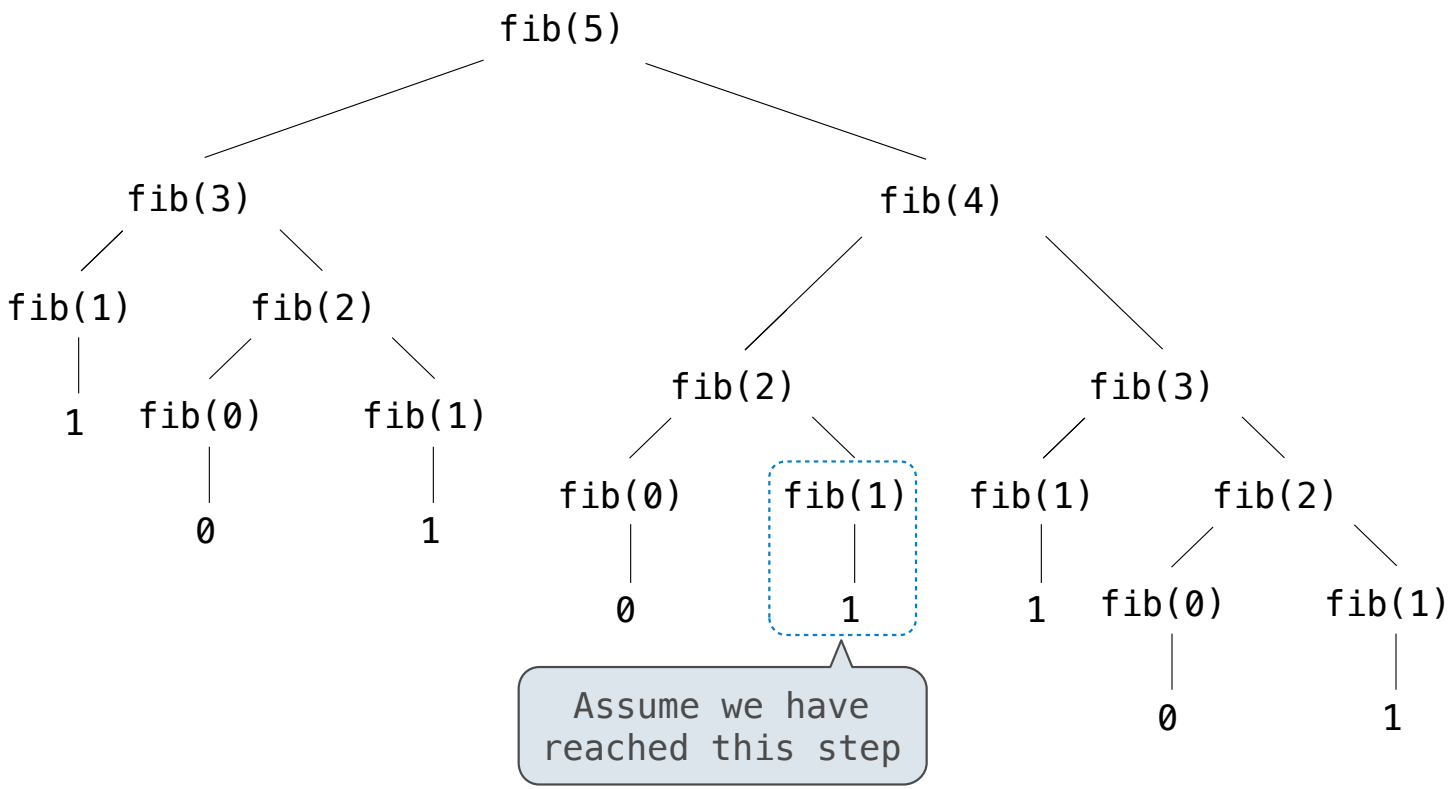

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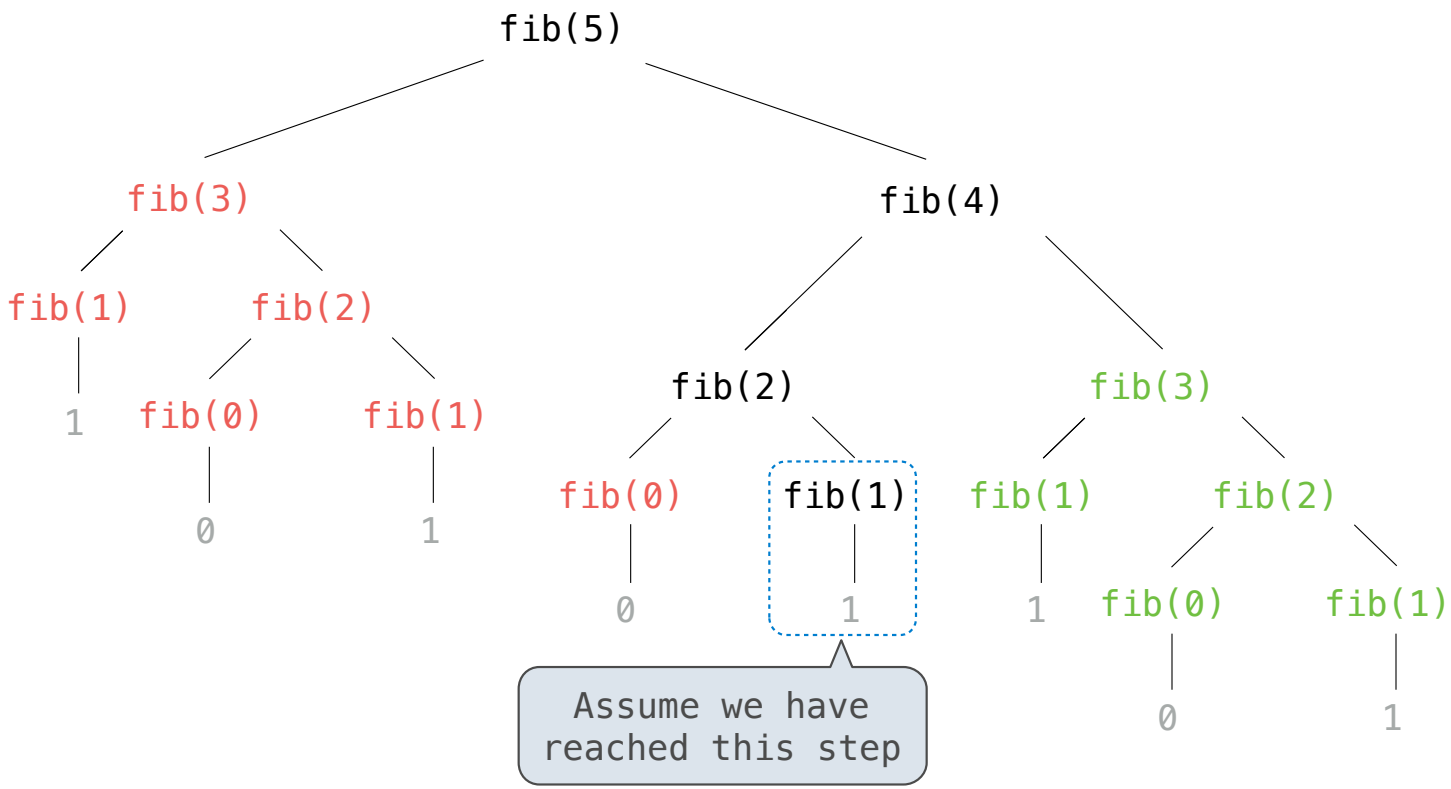

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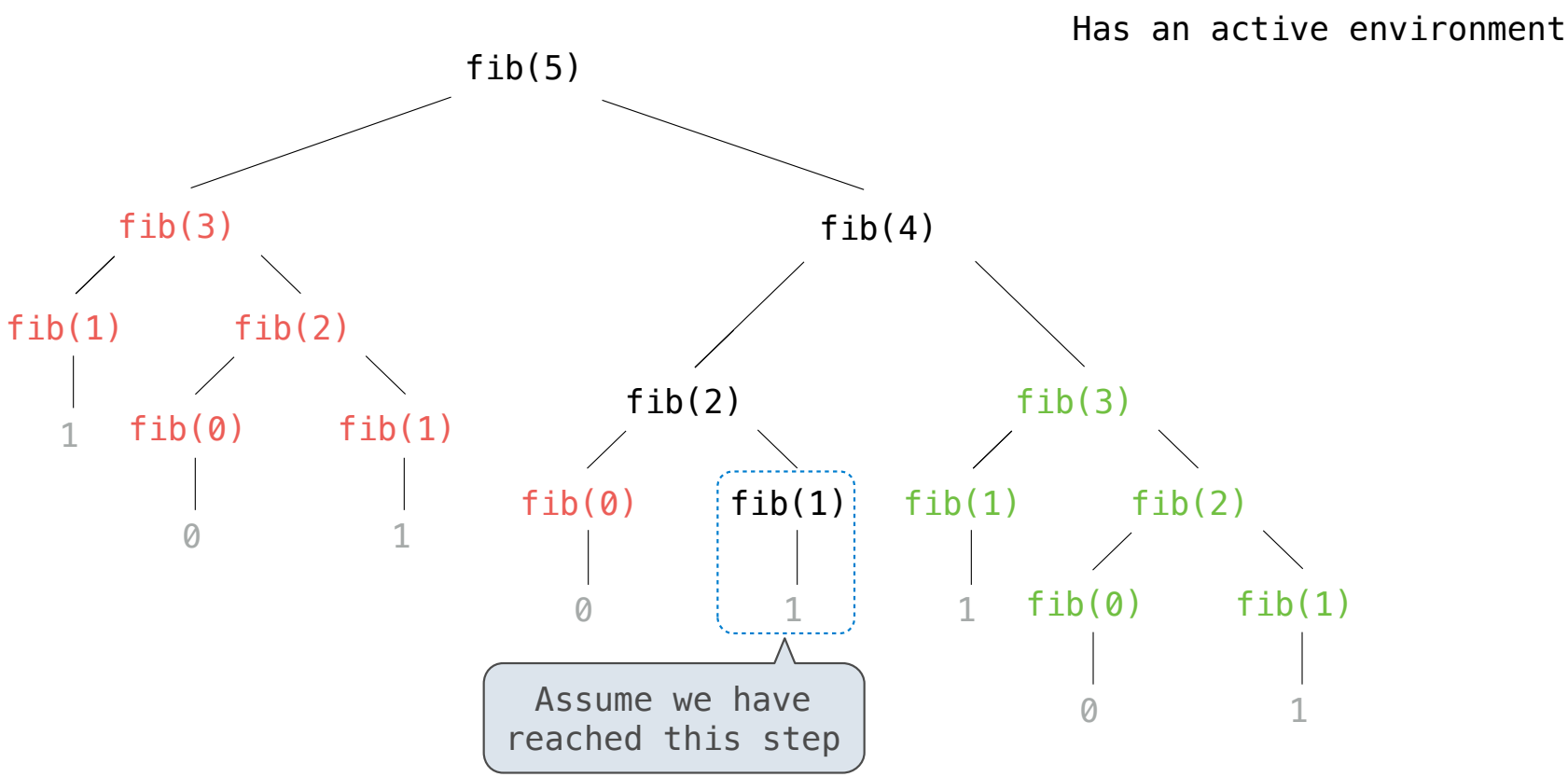
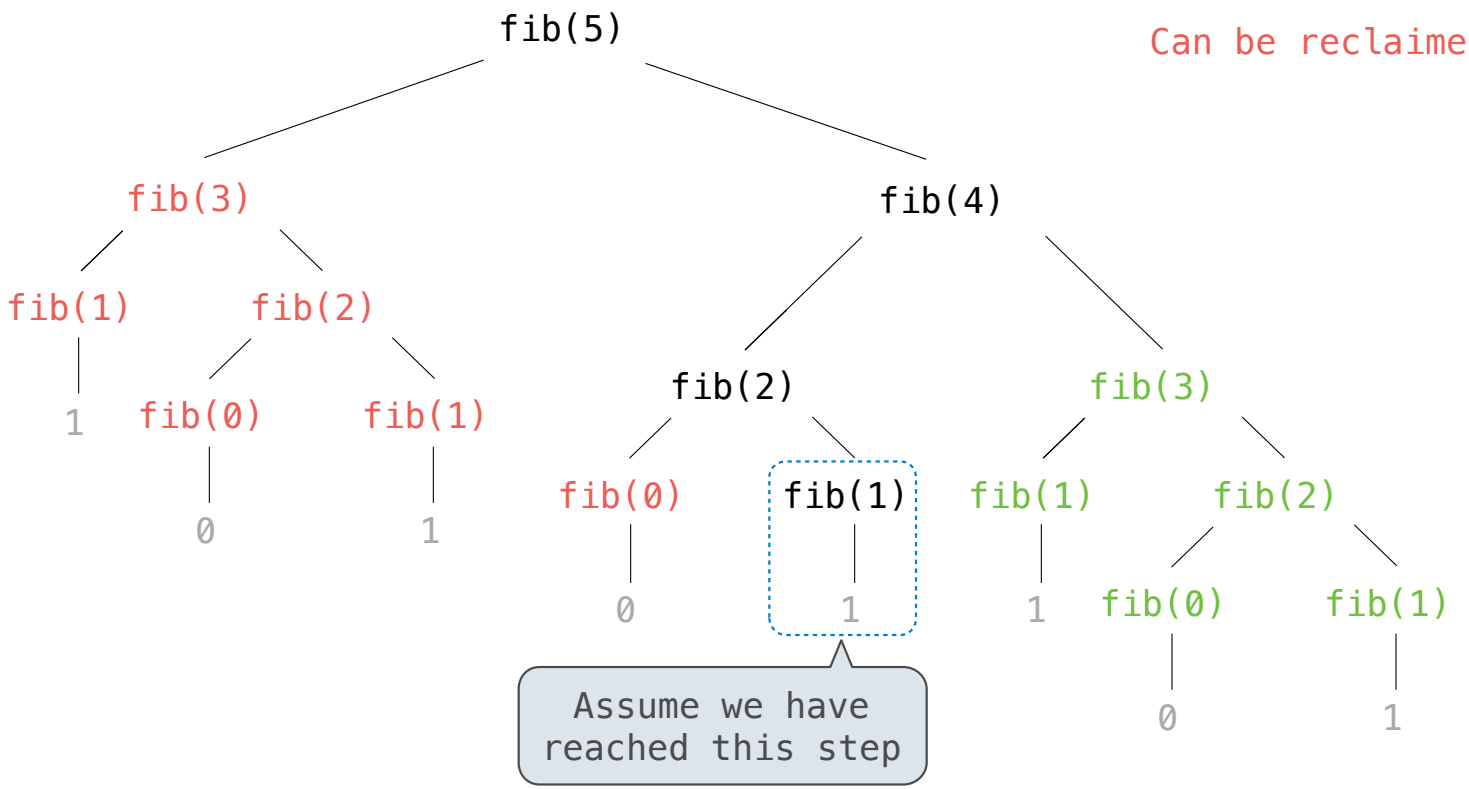


Fibonacci Space Consumption

Has an active environment

Can be reclaimed



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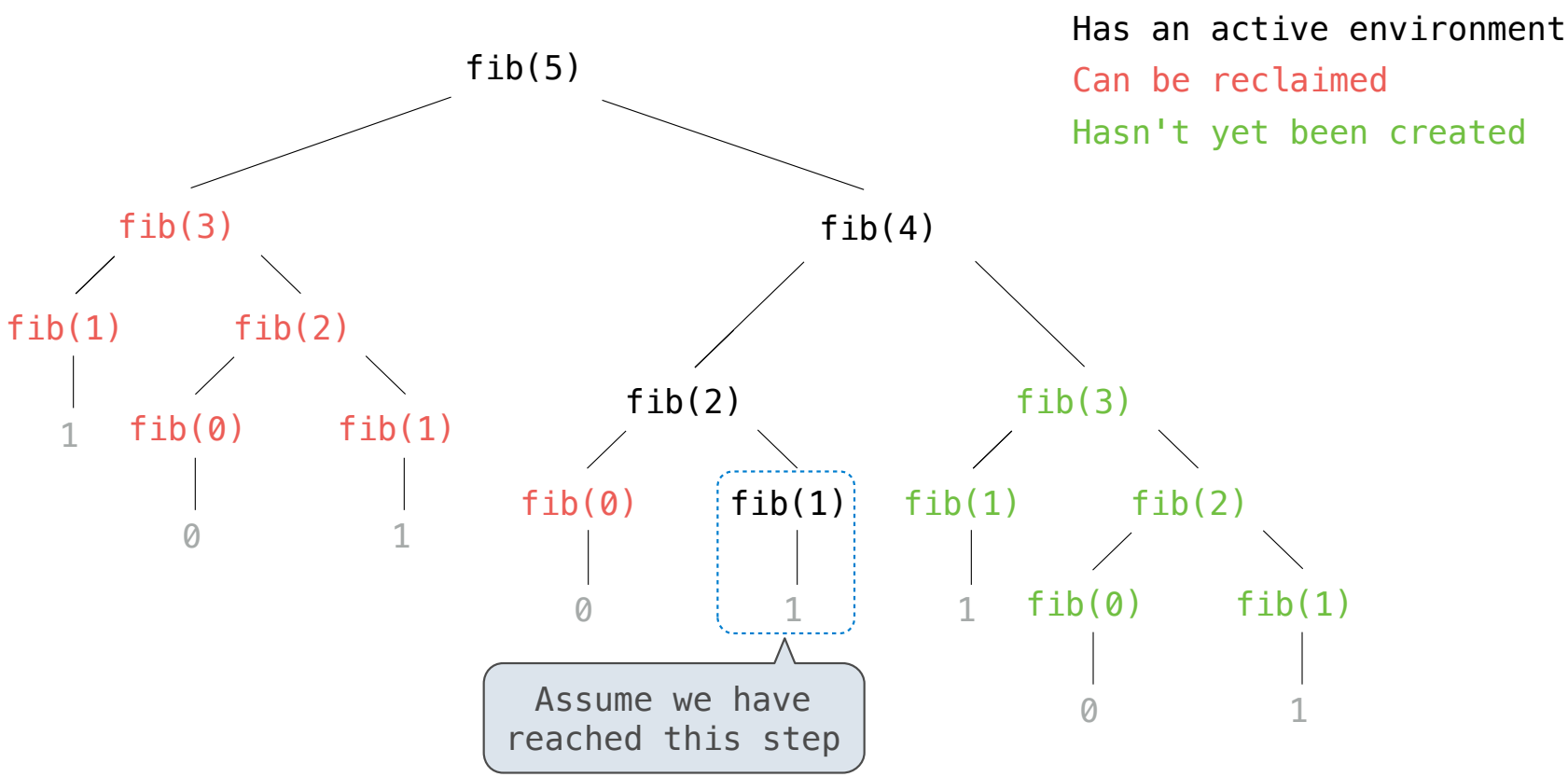

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- We choose $\mathbf{k_2}$ = 5**p** and **m** = 25

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Order of Growth of Counting Factors

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Assumption: integers occupy a fixed amount of space

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def square(x):
    return x*x

def exp_fast(b, n):
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        return b * exp_fast(b, n-1)
```

$\Theta(\log n)$	$\Theta(\log n)$

Comparing Orders of Growth

Properties of Orders of Growth

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    count = 0
    for item in a:
        if item in b:
            count += 1
    return count
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If a and b are both length **n**, then overlap takes $\Theta(n^2)$ steps

Comparing orders of growth (n is the problem size)

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