CS 61A Lecture 11

Friday, September 26

Announcements

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  - Homework Party on Monday 9/29, time and place TBD

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  - Don't fall behind! Come to class (discussion, lab, & office hours)!
  - Regrades are due by Sunday 9/29 @ 11:59pm
- Guerrilla Section 2 is on Saturday. RSVP on Piazza if you want to come!
- Homework 3 due Wednesday 10/1 @ 11:59pm
  - Homework Party on Monday 9/29, time and place TBD
- Optional Hog Contest due Wednesday 10/1 @ 11:59pm

Sequences

The Sequence Abstraction

The Sequence Abstraction

`red, orange, yellow, green, blue, indigo, violet.`

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There isn't just one sequence class or data abstraction (in Python or in general).

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The sequence abstraction is a collection of behaviors:

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0.

The Sequence Abstraction

```
red, orange, yellow, green, blue, indigo, violet.

 0 ,   1    ,   2    ,   3   ,  4  ,   5    ,   6    .
```

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> **Length.** A sequence has a finite length.
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There is built-in syntax associated with this behavior, or we can use functions.

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Length. A sequence has a finite length.

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There is built-in syntax associated with this behavior, or we can use functions.

A list is a kind of built-in sequence

Lists

```
['Demo']
```

Lists are Sequences

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```
>>> digits = [1, 8, 2, 8]
```

Lists are Sequences

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
```

Lists are Sequences

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
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>>> digits = [1, 8, 2, 8]
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```

Length. A sequence has a finite length.

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```
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
```

Lists are Sequences

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
```

Length. A sequence has a finite length.

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```
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]

>>> pairs = [[10, 20], [30, 40]]
```

Lists are Sequences

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
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8
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Length. A sequence has a finite length.

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```
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]

>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
```

Lists are Sequences

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
```

Length. A sequence has a finite length.

Element selection. A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0.

```
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]

>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```

For Statements

(Demo)

Sequence Iteration

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```
def count(s, value):
    total = 0
    for element in s:
        if element == value:
            total = total + 1
    return total
```

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    total = 0
    for element in s:
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```

Name bound in the first frame
of the current environment
(not a new frame)

For Statement Execution Procedure

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```
for <name> in <expression>:
    <suite>
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2. For each element in that sequence, in order:

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for <name> in <expression>:
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   A. Bind <name> to that element in the current frame

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for <name> in <expression>:
    <suite>
```

1. Evaluate the header <expression>, which must yield an iterable value (a sequence)

2. For each element in that sequence, in order:

   A. Bind <name> to that element in the current frame

   B. Execute the <suite>

Sequence Unpacking in For Statements

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```
>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]

>>> same_count = 0
```

Sequence Unpacking in For Statements

A sequence of fixed-length sequences

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>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]

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Sequence Unpacking in For Statements

A sequence of fixed-length sequences

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>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]

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```

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1

>>> same_count
2
```

Sequence Unpacking in For Statements

A sequence of fixed-length sequences

```
>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]

>>> same_count = 0
```

A name for each element in a fixed-length sequence

```
>>> for x, y in pairs:
...     if x == y:
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>>> same_count
2
```

Sequence Unpacking in For Statements

A sequence of fixed-length sequences

```
>>> pairs = [[1, 2], [2, 2], [3, 2], [4, 4]]

>>> same_count = 0
```

A name for each element in a fixed-length sequence

Each name is bound to a value, as in multiple assignment

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1

>>> same_count
2
```

Ranges

The Range Type

A range is a sequence of consecutive integers.*

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..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

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```
..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...
```

```
range(-2, 2)
```

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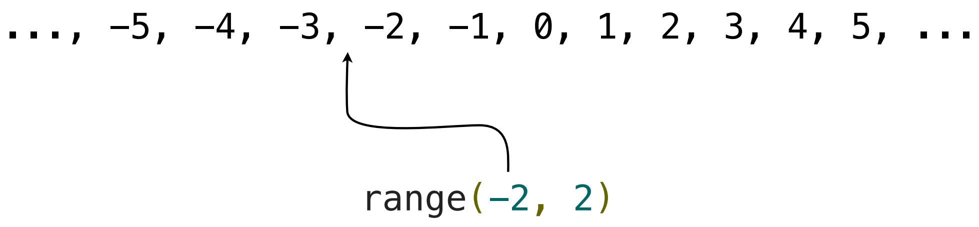

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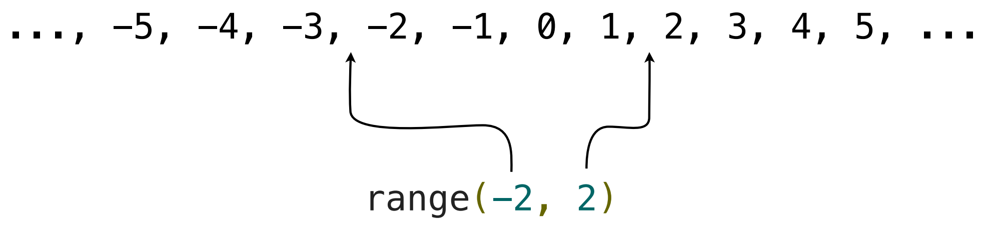

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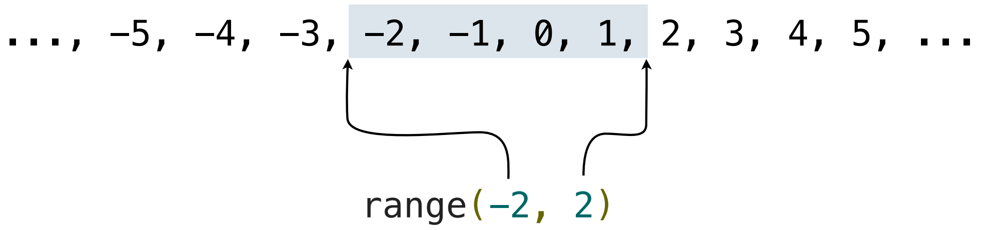

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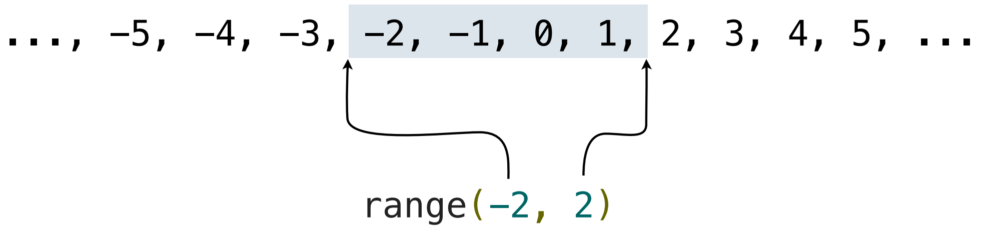

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```
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Length: ending value - starting value

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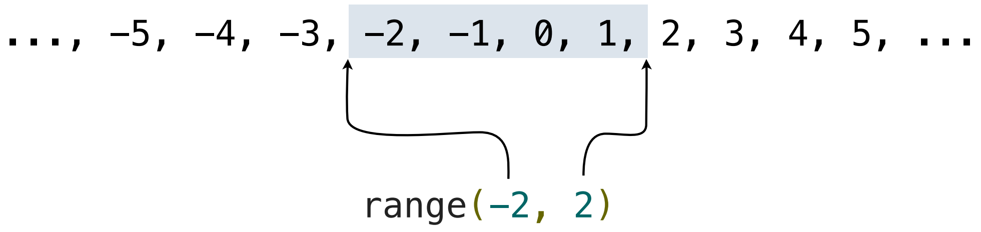

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range(-2, 2)

Length: ending value - starting value

Element selection: starting value + index

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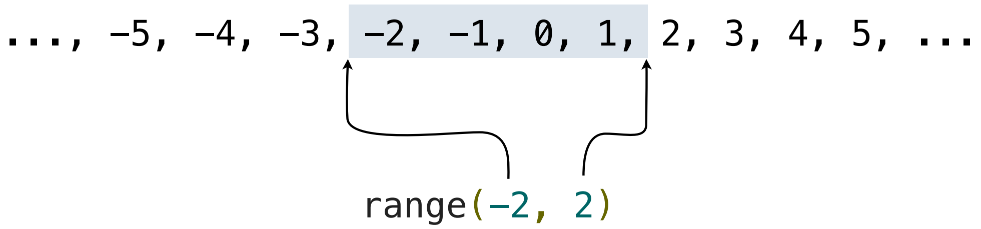

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range(-2, 2)

Length: ending value - starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]

>>> list(range(4))
[0, 1, 2, 3]
```

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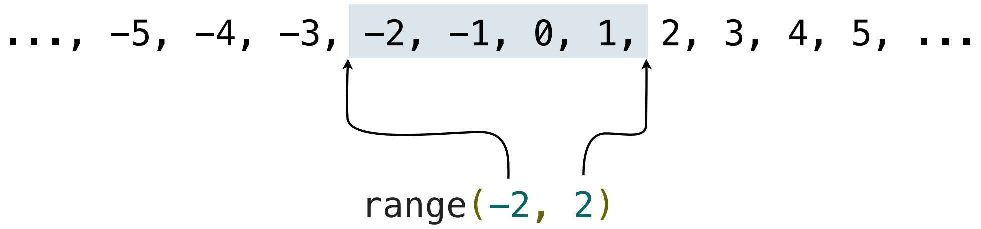

..., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ...

range(−2, 2)

Length: ending value − starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]

>>> list(range(4))
[0, 1, 2, 3]
```

List constructor

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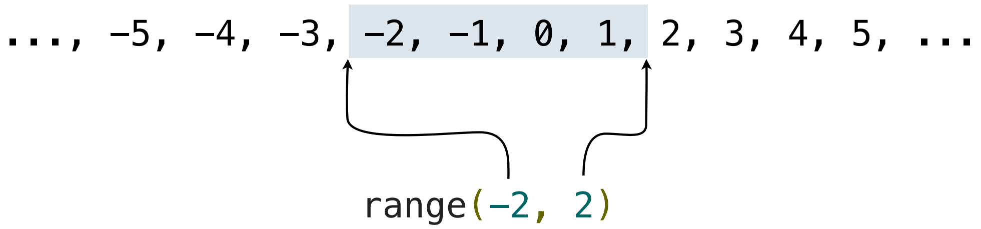

..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

range(-2, 2)

Length: ending value - starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

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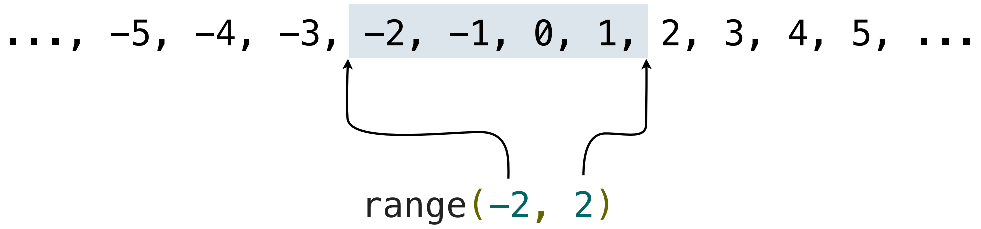

Length: ending value - starting value

Element selection: starting value + index

(Demo)

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

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List Comprehensions

List Comprehensions

```
>>> letters = ['a', 'b', 'c', 'd', 'e', 'f', 'm', 'n', 'o', 'p']
>>> [letters[i] for i in [3, 4, 6, 8]]
```

List Comprehensions

```
>>> letters = ['a', 'b', 'c', 'd', 'e', 'f', 'm', 'n', 'o', 'p']
>>> [letters[i] for i in [3, 4, 6, 8]]
```

```
['d', 'e', 'm', 'o']
```

List Comprehensions

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```
[<map exp> for <name> in <iter exp> if <filter exp>]
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Short version: [<map exp> for <name> in <iter exp>]
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A combined expression that evaluates to a list using this evaluation procedure:

List Comprehensions

```
[<map exp> for <name> in <iter exp> if <filter exp>]

Short version: [<map exp> for <name> in <iter exp>]
```

A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent

List Comprehensions

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[<map exp> for <name> in <iter exp> if <filter exp>]

Short version: [<map exp> for <name> in <iter exp>]
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A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent

2. Create an empty *result list* that is the value of the expression

List Comprehensions

```
[<map exp> for <name> in <iter exp> if <filter exp>]

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A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent

2. Create an empty *result list* that is the value of the expression

3. For each element in the iterable value of `<iter exp>`:

List Comprehensions

```
[<map exp> for <name> in <iter exp> if <filter exp>]
```

Short version: `[<map exp> for <name> in <iter exp>]`

A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent

2. Create an empty *result list* that is the value of the expression

3. For each element in the iterable value of `<iter exp>`:

   A. Bind `<name>` to that element in the new frame from step 1

List Comprehensions

```
[<map exp> for <name> in <iter exp> if <filter exp>]
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```
Short version: [<map exp> for <name> in <iter exp>]
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A combined expression that evaluates to a list using this evaluation procedure:

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2. Create an empty *result list* that is the value of the expression

3. For each element in the iterable value of `<iter exp>`:

   A. Bind `<name>` to that element in the new frame from step 1

   B. If `<filter exp>` evaluates to a true value, then add the value of `<map exp>` to the result list

Higher-Order Sequence Functions

Functions that Perform List Comprehensions

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```
def apply_to_all(map_fn, s):
    """Apply map_fn to each element of s.



    """
    return [map_fn(x) for x in s]
```

Functions that Perform List Comprehensions

```
def apply_to_all(map_fn, s):
    """Apply map_fn to each element of s.

                     lambda x: x*3

    """
    return [map_fn(x) for x in s]
```

Functions that Perform List Comprehensions

```
def apply_to_all(map_fn, s):
    """Apply map_fn to each element of s.

    >>> apply_to_all(lambda x: x*3, range(5))
    [0, 3, 6, 9, 12]
    """
    return [map_fn(x) for x in s]
```

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def apply_to_all(map_fn, s):
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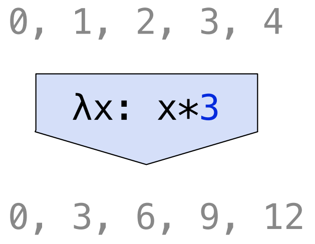

0, 1, 2, 3, 4

λx: x*3

0, 3, 6, 9, 12

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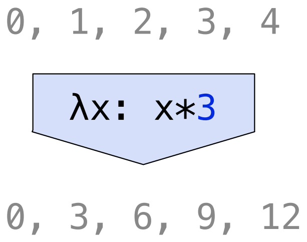

```
def keep_if(filter_fn, s):
    """List all elements x of s for which filter_fn(x) is true.



    """
    return [x for x in s if filter_fn(x)]
```

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```
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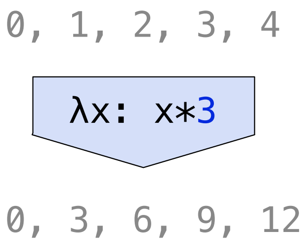

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λx: x*3

0, 3, 6, 9, 12

```
def keep_if(filter_fn, s):
    """List all elements x of s for which filter_fn(x) is true.

                lambda x: x>5

    """
    return [x for x in s if filter_fn(x)]
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def apply_to_all(map_fn, s):
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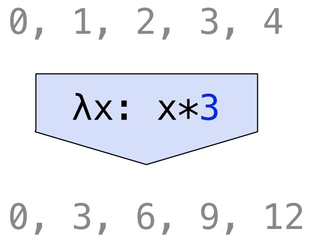

0, 1, 2, 3, 4

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0, 3, 6, 9, 12

```
def keep_if(filter_fn, s):
    """List all elements x of s for which filter_fn(x) is true.

    >>> keep_if(lambda x: x>5, range(10))
    [6, 7, 8, 9]
    """
    return [x for x in s if filter_fn(x)]
```

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def apply_to_all(map_fn, s):
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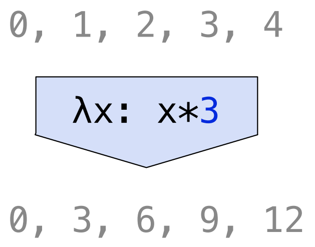

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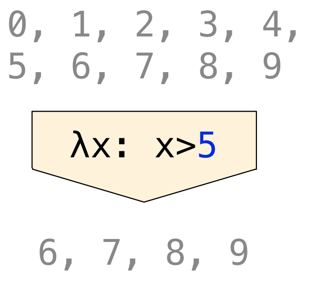
```



0, 1, 2, 3, 4,
5, 6, 7, 8, 9

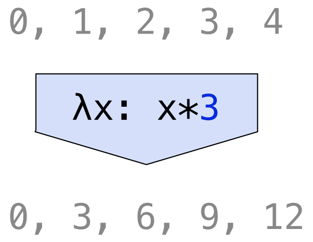
λx: x>5

6, 7, 8, 9

Functions that Perform List Comprehensions

```
def apply_to_all(map_fn, s):
    """Apply map_fn to each element of s.

    >>> apply_to_all(lambda x: x*3, range(5))
    [0, 3, 6, 9, 12]
    """
    return [map_fn(x) for x in s]
```



0, 1, 2, 3, 4

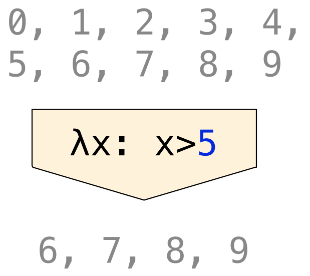
λx: x*3

0, 3, 6, 9, 12

Same number
of different
elements

```
def keep_if(filter_fn, s):
    """List all elements x of s for which filter_fn(x) is true.

    >>> keep_if(lambda x: x>5, range(10))
    [6, 7, 8, 9]
    """
    return [x for x in s if filter_fn(x)]
```



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5, 6, 7, 8, 9

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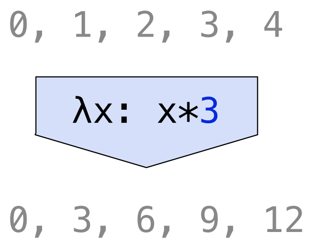

Same number
of different
elements

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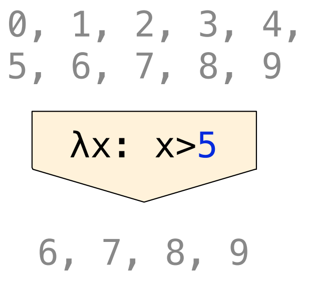

Smaller number
of same elements

Reducing a Sequence to a Value

Reducing a Sequence to a Value

```
def reduce(reduce_fn, s, initial):
    """Combine elements of s pairwise using reduce_fn, starting with initial.
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def reduce(reduce_fn, s, initial):
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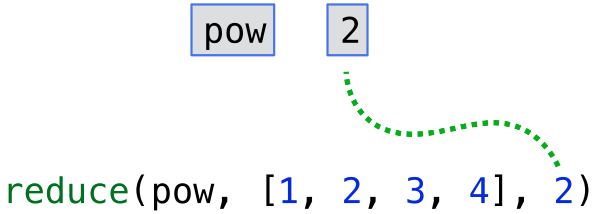

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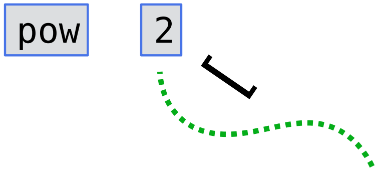

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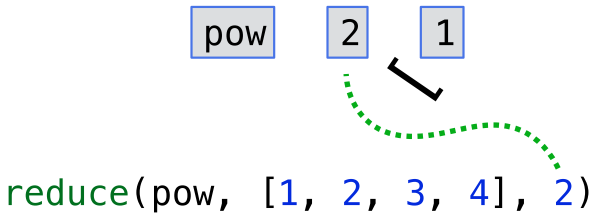

pow 2 1

reduce(pow, [1, 2, 3, 4], 2)

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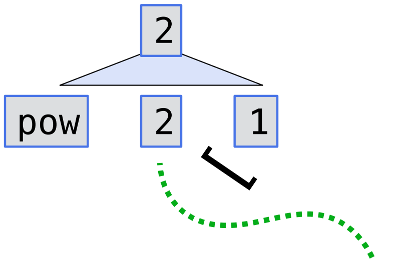

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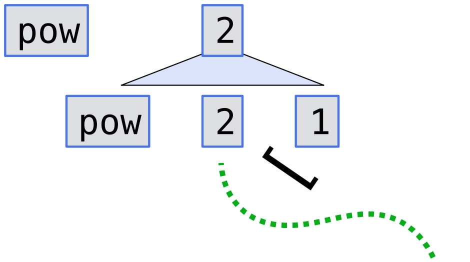

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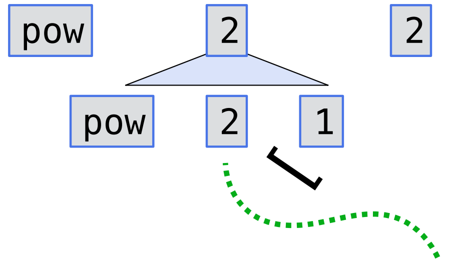

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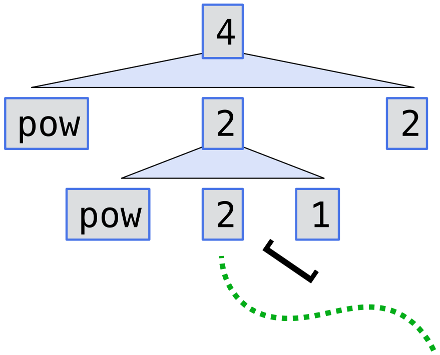

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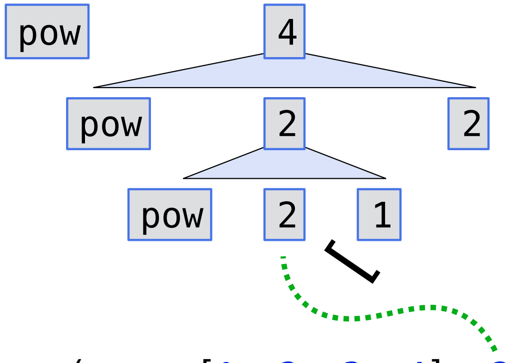

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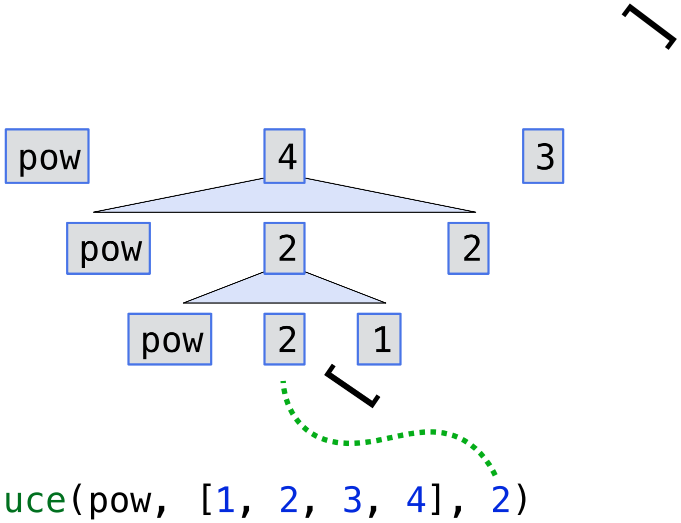

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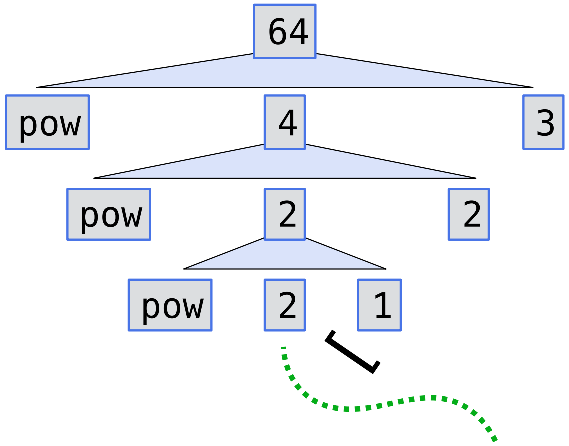

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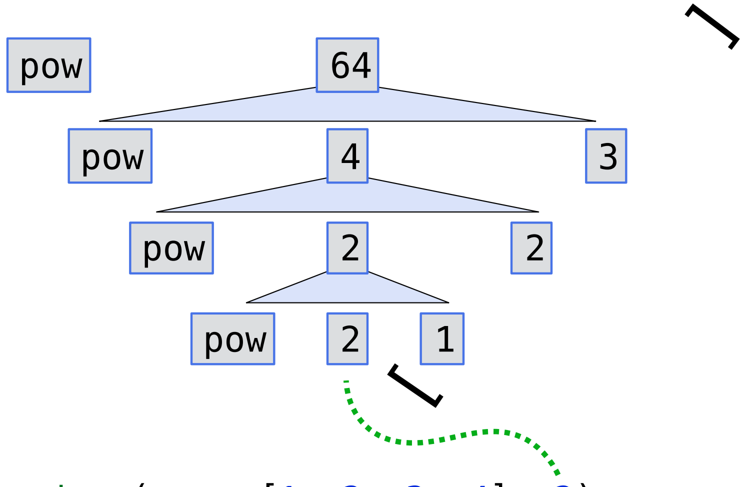

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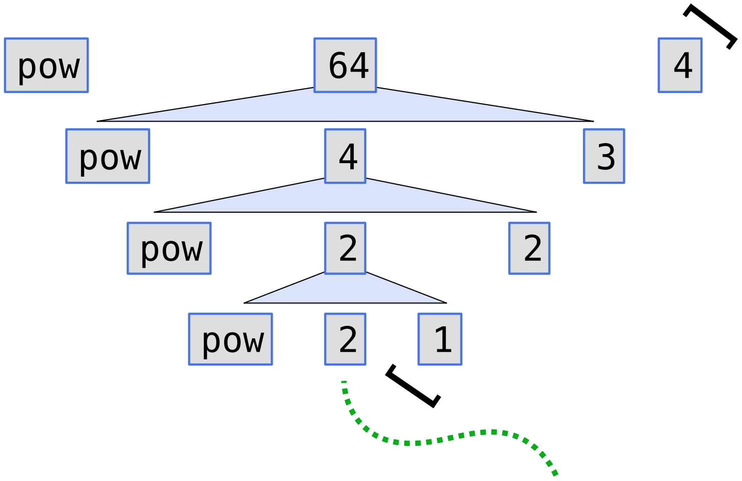

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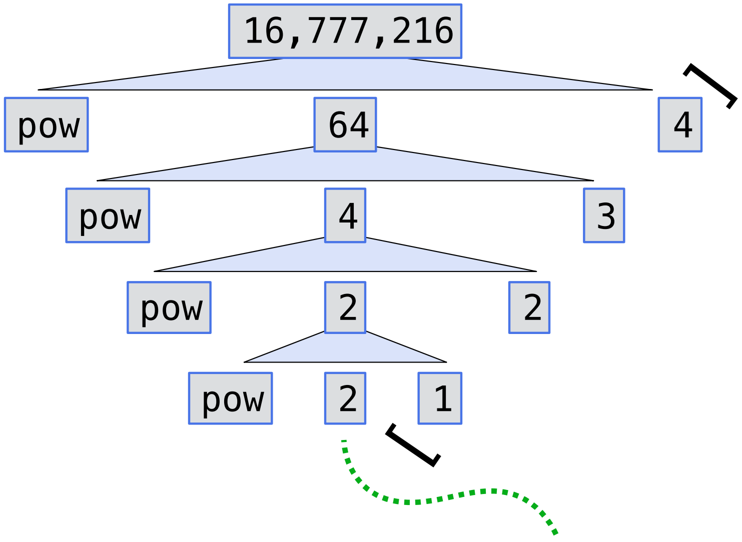

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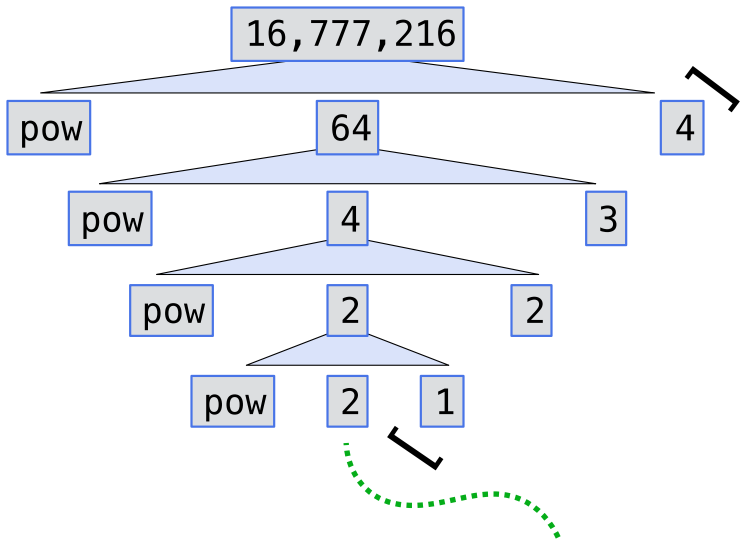

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(Demo)

Typical Names for Higher-Order Sequence Functions

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map and filter are built into Python,
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Most Python programmers just use list comprehensions