61A Lecture 32

Friday, November 22

Announcements

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- Homework 10 due Tuesday 11/26 @ 11:59pm

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- No lecture on Wednesday 11/27 or Friday 11/29

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- Recursive art contest entries due Monday 12/2 @ 11:59pm

Appending Lists

(Demo)

Lists in Logic

Lists in Logic

Expressions begin with *query* or *fact* followed by relations.

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Expressions and their relations are Scheme lists.

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Simple fact: Conclusion

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Hypothesis

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(query (append-to-form ?left (c d) (e b c d)))
Success!
left: (e b)
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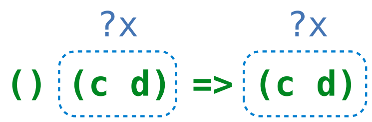

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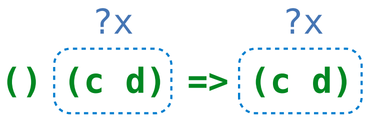

(b) (c d) => (b c d)

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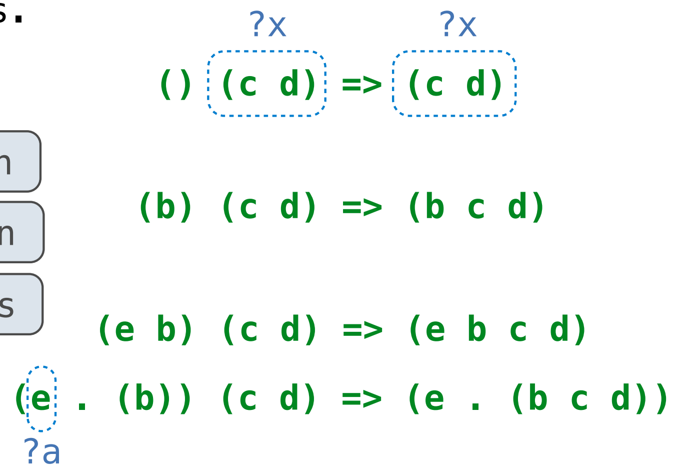

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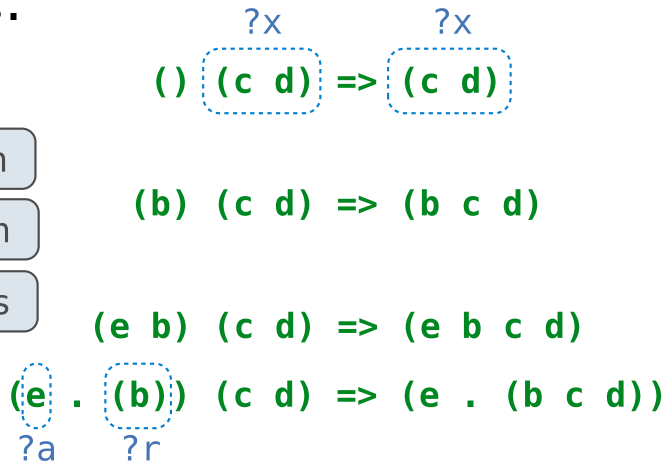

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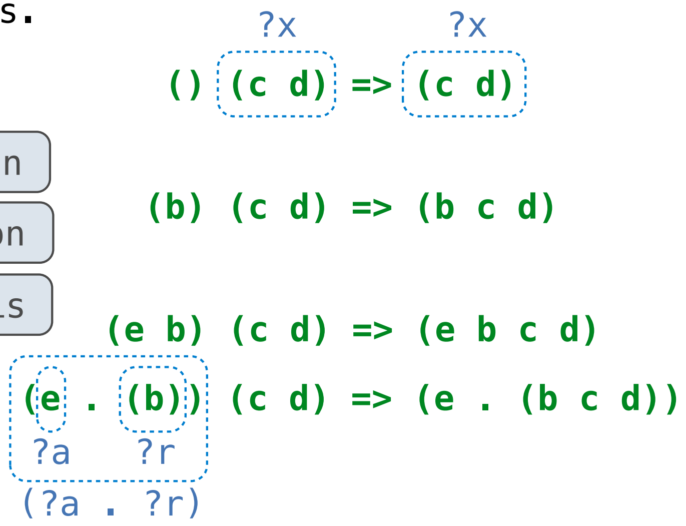

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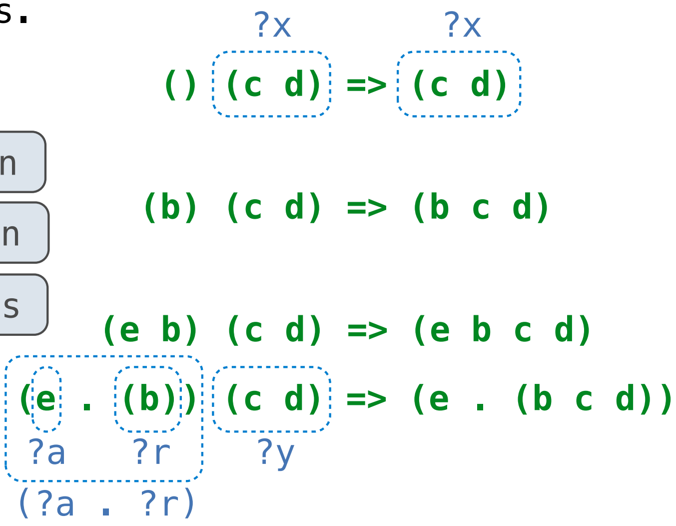

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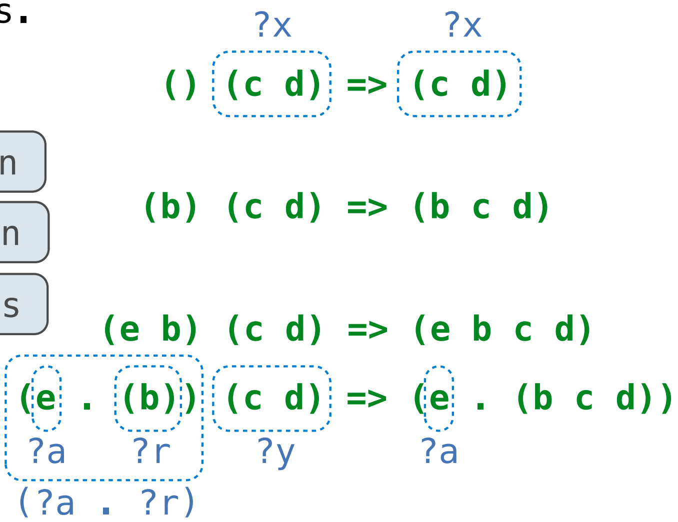

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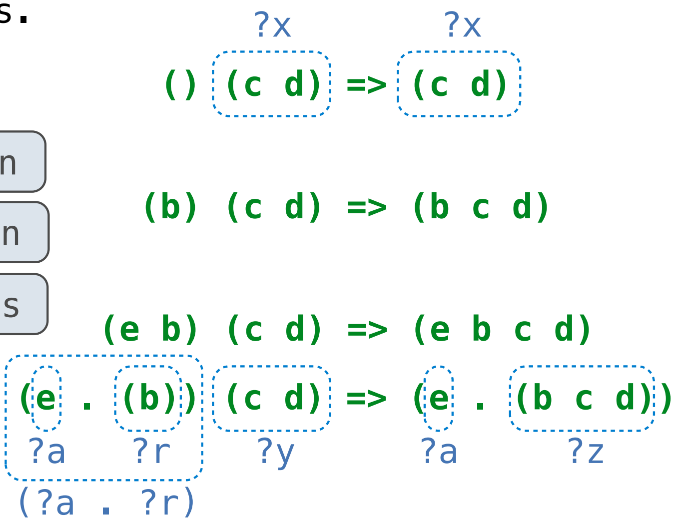

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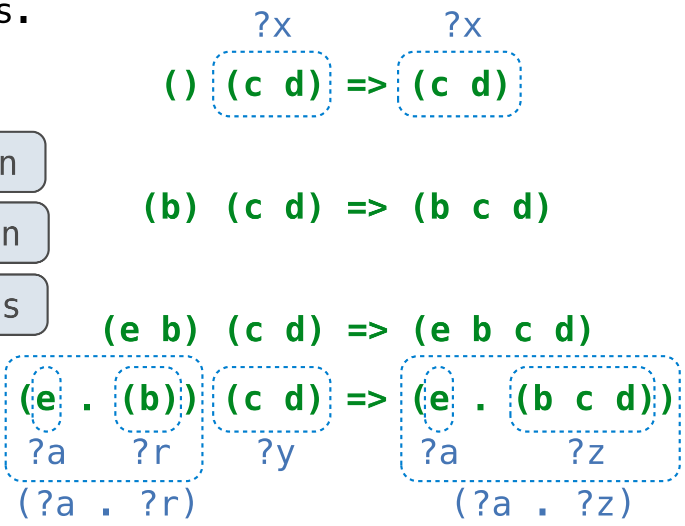

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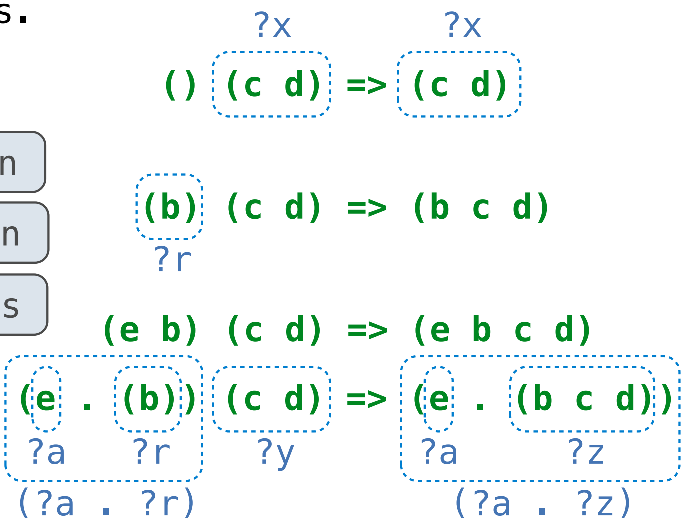

?x ?x

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?r

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(e . (b)) (c d) => (e . (b c d))

?a ?r ?y ?a ?z

(?a . ?r) (?a . ?z)

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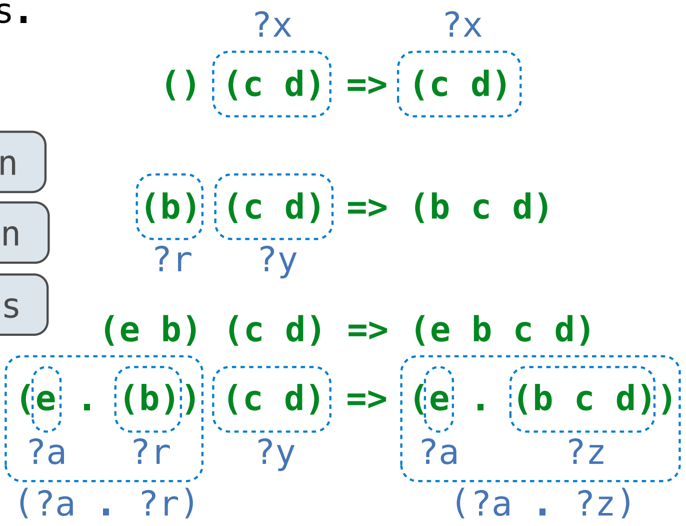

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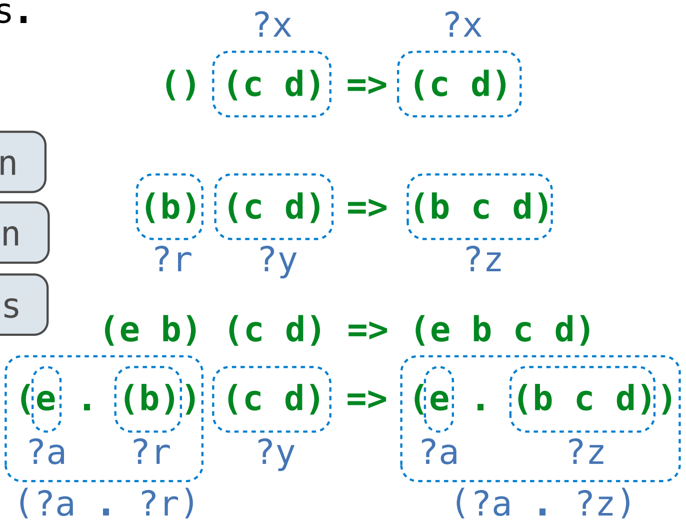

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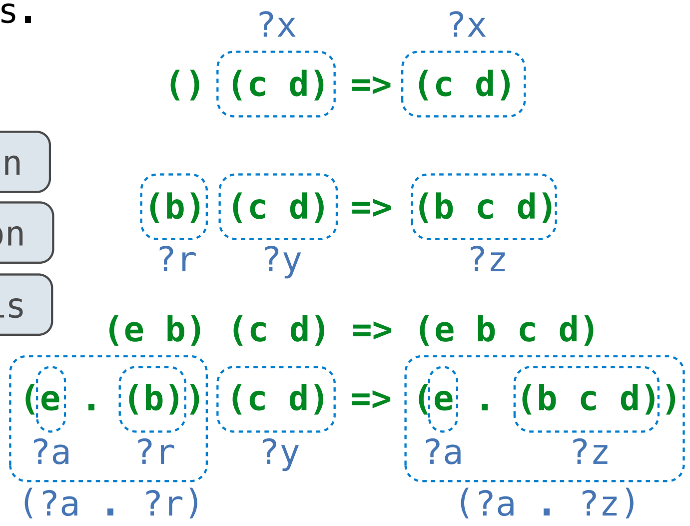

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(Demo)

Permuting Lists

Anagrams in Logic

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A permutation (i.e., anagram) of a list is:

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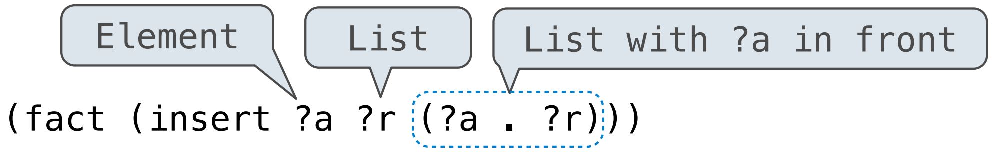

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Anagrams in Logic

A permutation (i.e., anagram) of a list is:

- The empty list for an empty list.
- The first element of the list inserted into an anagram of the rest of the list.

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
      (insert ?a       ?r        ?s))
```

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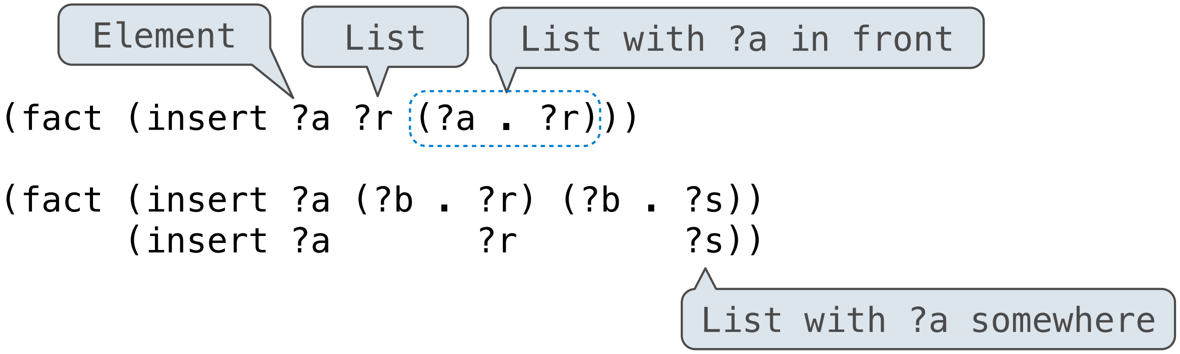

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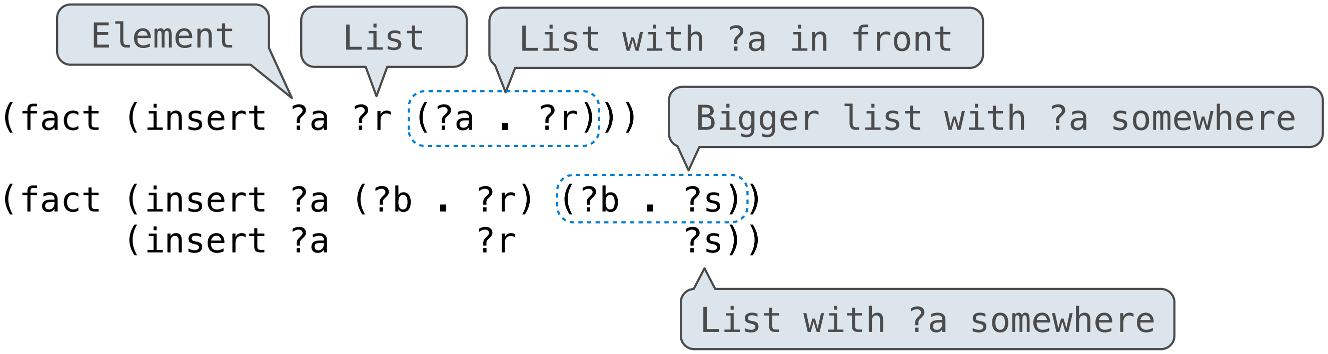

Element

List

List with ?a in front

```
(fact (insert ?a ?r (?a . ?r)))
```

Bigger list with ?a somewhere

```
(fact (insert ?a (?b . ?r) (?b . ?s))
      (insert ?a       ?r        ?s))
```

List with ?a somewhere

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Bigger list with ?a somewhere

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```

List with ?a somewhere

```
(fact (anagram () ()))
```

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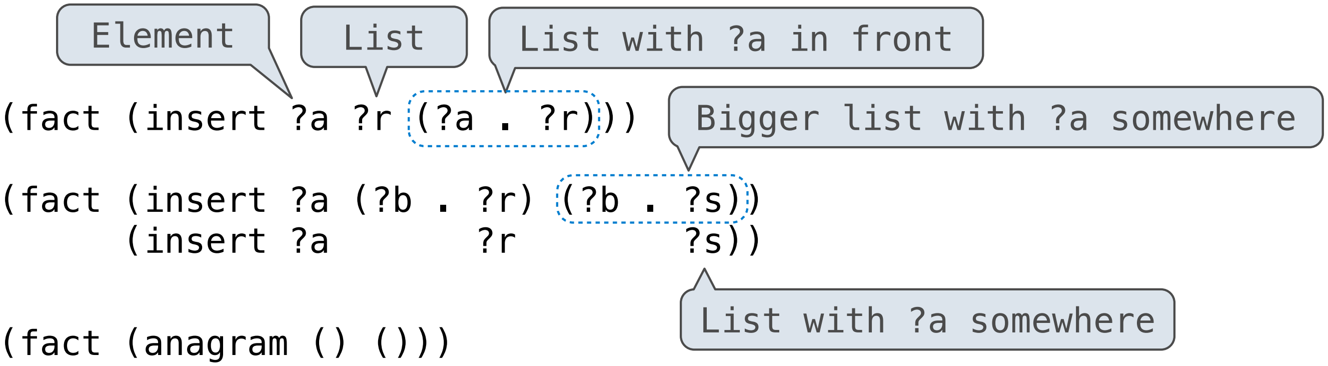

```
(fact (insert ?a ?r (?a . ?r)))

(fact (insert ?a (?b . ?r) (?b . ?s))
      (insert ?a       ?r        ?s))


(fact (anagram () ()))

(fact (anagram (?a . ?r) ?b)
```

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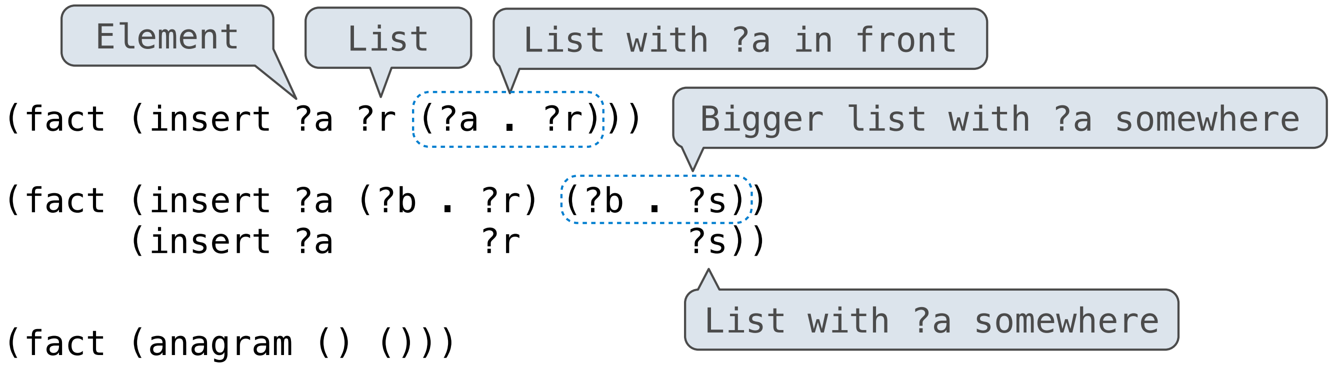

```
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(fact (insert ?a (?b . ?r) (?b . ?s))
      (insert ?a       ?r         ?s))


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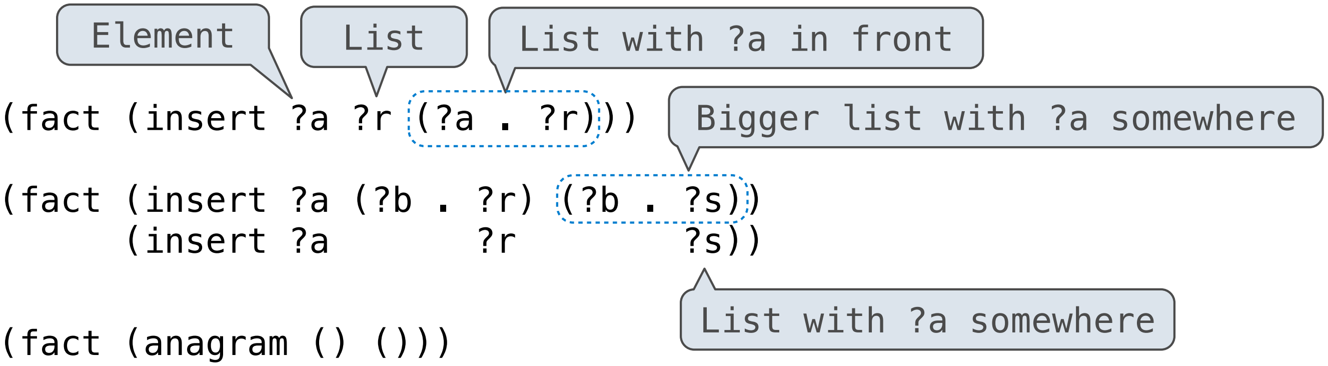
a r t



Element

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List with ?a in front

Bigger list with ?a somewhere

List with ?a somewhere

```
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```

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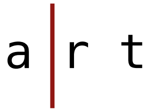

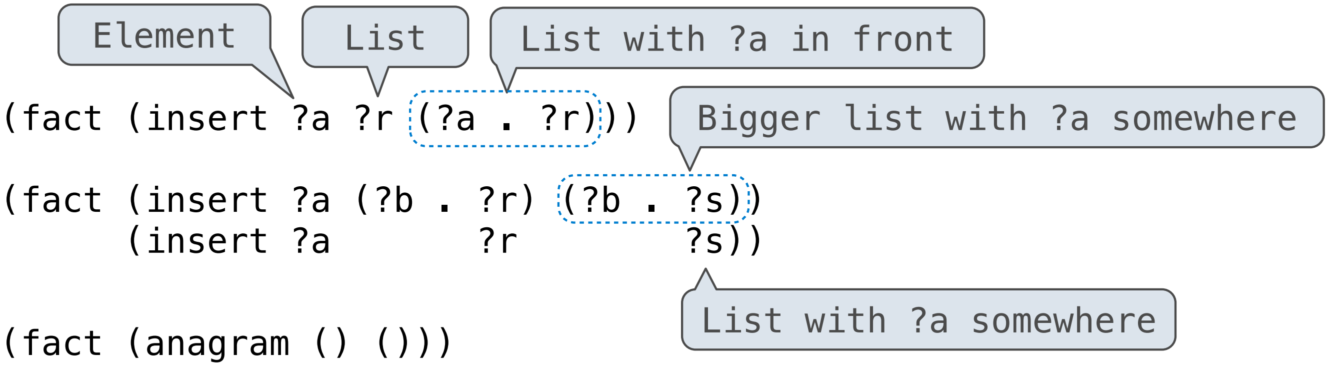

Element

List

List with ?a in front

Bigger list with ?a somewhere

List with ?a somewhere

```
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(fact (insert ?a (?b . ?r) (?b . ?s))
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```

a | r t

r t

ar t

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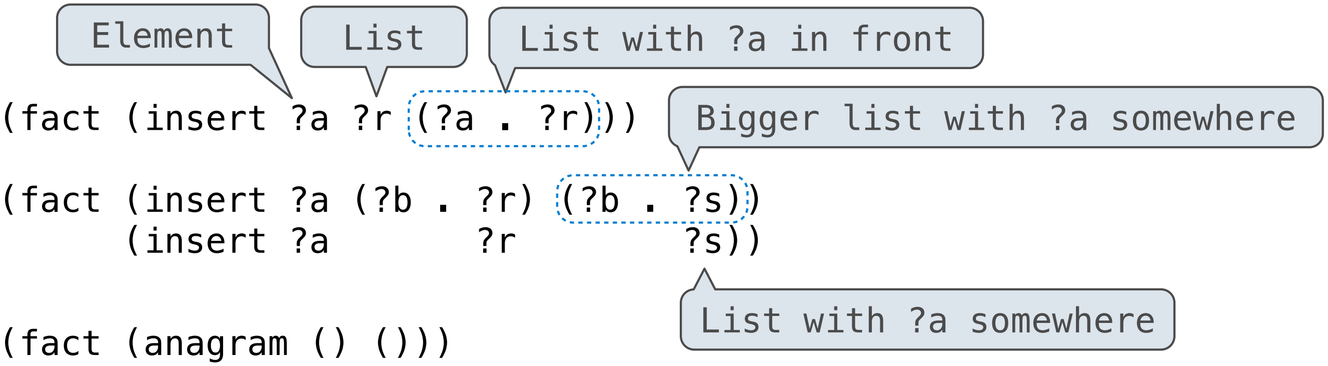

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```

a | r t

r t

ar t

r**a**t

r t**a**

Anagrams in Logic

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List with ?a somewhere

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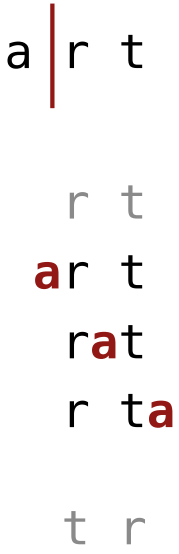

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```

a | r t

r t

ar t

r**a**t

r t**a**

t r

at r

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```

a | r t

r t

ar t

r**a**t

r t**a**

t r

at r

t**a**r

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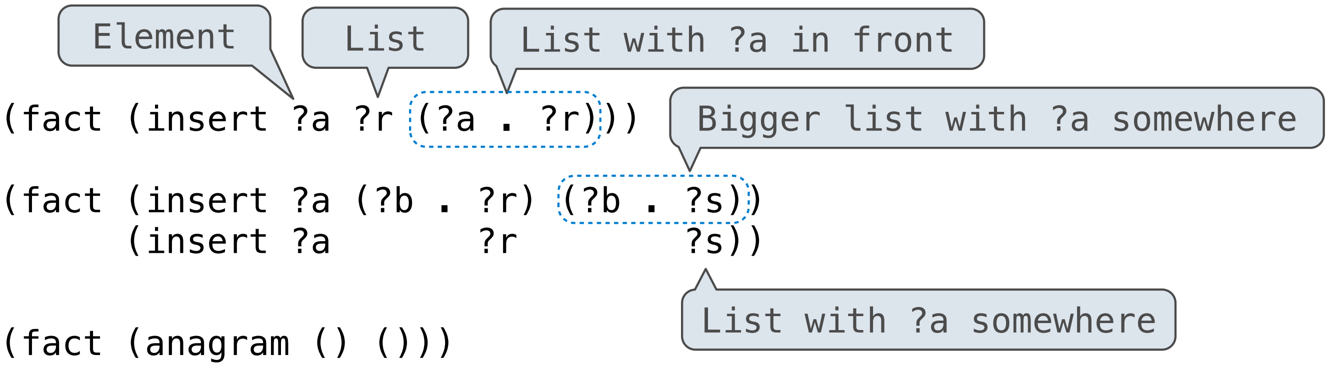

```
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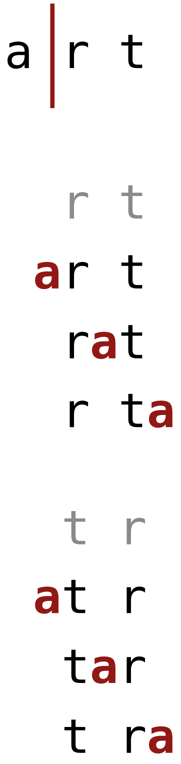

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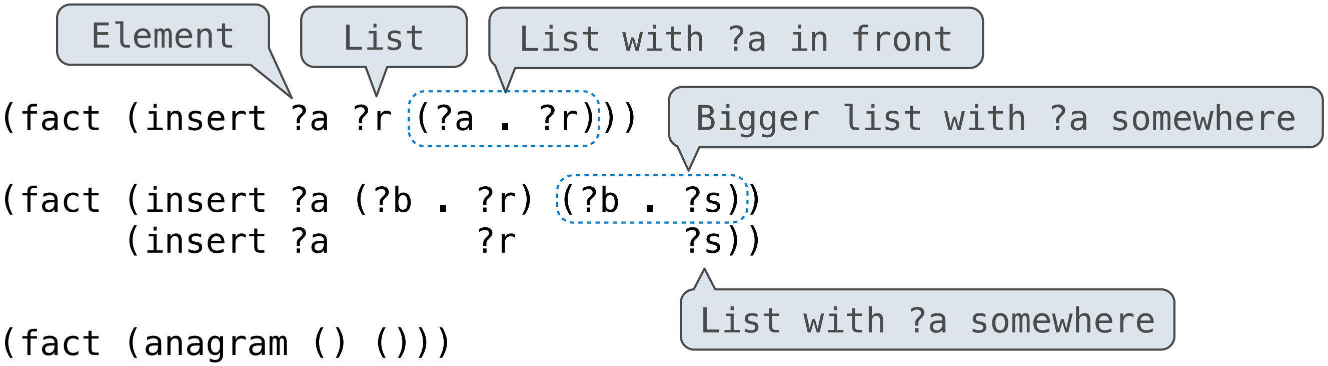

Element

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List with ?a in front

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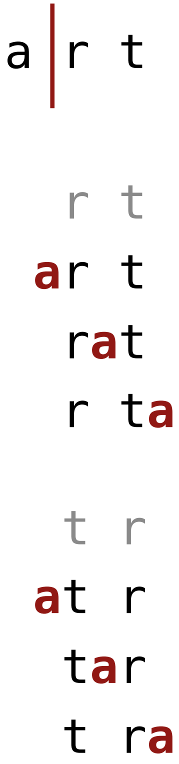

(Demo)

Unification

Pattern Matching

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

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```
( (a  b) c  (a  b) )
(   ?x   c    ?x   )
```

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
                         True, {x: (a b)}
(   ?x   c    ?x   )
```

Pattern Matching

`The basic operation of the Logic interpreter is to attempt to *unify* two relations.`

`Unification is finding an assignment to variables that makes two relations the same.`

```
( (a  b) c  (a  b) )
                      ▷   True, {x: (a b)}
(   ?x   c    ?x   )


( (a  b) c  (a  b) )
```

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a   b) c   (a   b) )  ▷  True, {x: (a b)}
(    ?x   c      ?x   )
```

```
( (a   b) c   (a   b) )
( (a ?y) ?z   (a   b) )
```

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
(   ?x   c    ?x   )      True, {x: (a b)}
```

```
( (a  b) c  (a  b) )
( (a ?y) ?z (a  b) )      True, {y: b, z: c}
```

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a  b) c  (a  b) )
                        ▷   True, {x: (a b)}
(   ?x   c    ?x    )


( (a  b) c  (a  b) )
                        ▷   True, {y: b, z: c}
( (a ?y) ?z (a  b) )


( (a  b) c  (a  b) )

(   ?x  ?x    ?x    )
```

Pattern Matching

The basic operation of the Logic interpreter is to attempt to *unify* two relations.

Unification is finding an assignment to variables that makes two relations the same.

```
( (a   b) c   (a   b) )
(    ?x   c     ?x    )     True, {x: (a b)}

( (a   b) c   (a   b) )
( (a ?y) ?z (a   b) )       True, {y: b, z: c}

( (a   b) c   (a   b) )
(    ?x  ?x     ?x    )     False
```

Unification

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Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

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1. Look up variables in the current environment.

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```
( (a  b) c (a  b) )

(   ?x   c   ?x   )
```

```
 {              }
```

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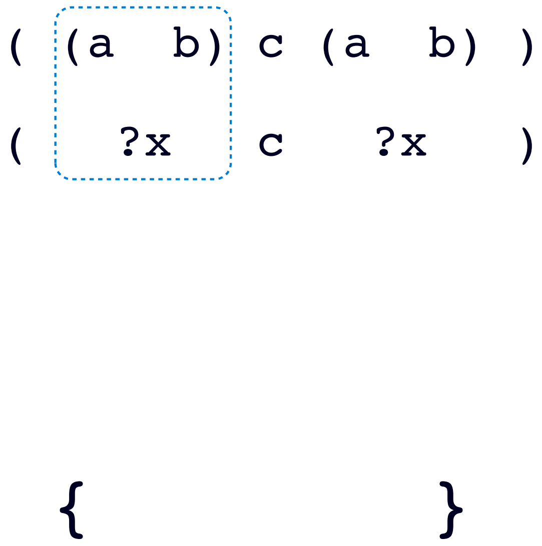

```
( (a  b) c (a  b) )

(   ?x   c   ?x    )


   {              }
```

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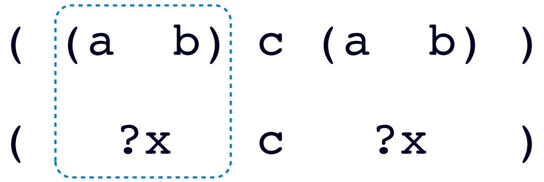

```
( (a   b) c (a   b) )

(   ?x    c    ?x    )
```

```
{  x: (a b)  }
```

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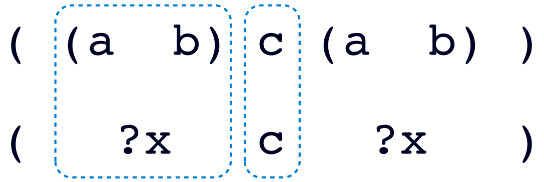

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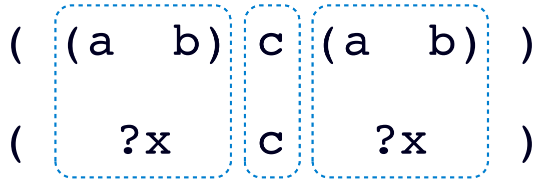

((a b) c (a b))

(?x c ?x)

{ x: (a b) }

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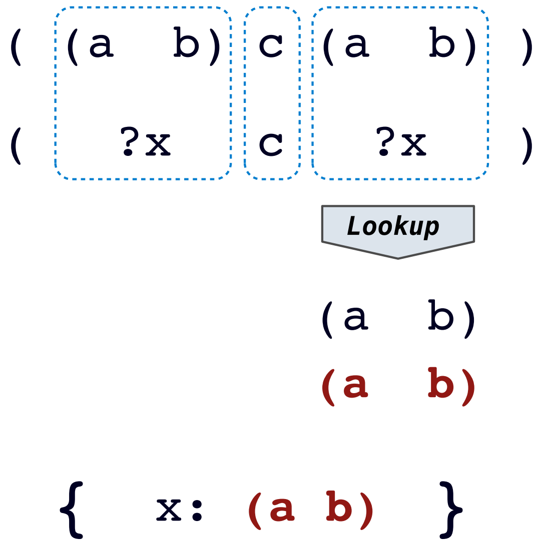

((a b) c (a b))

(?x c ?x)

Lookup

(a b)

(a b)

{ x: **(a b)** }

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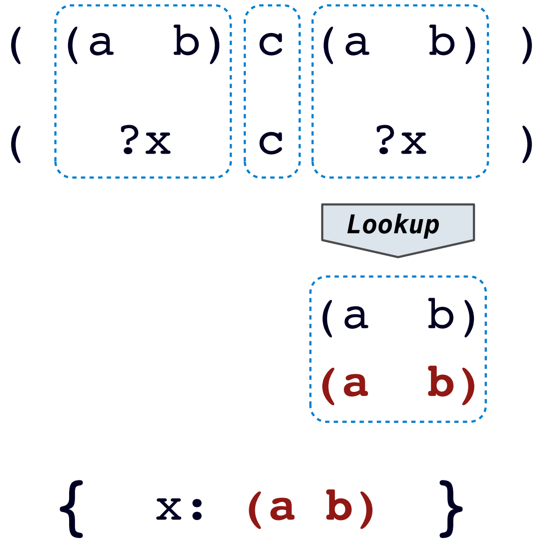

((a b) c (a b))

(?x c ?x)

Lookup

(a b)

(a b)

{ x: (a b) }

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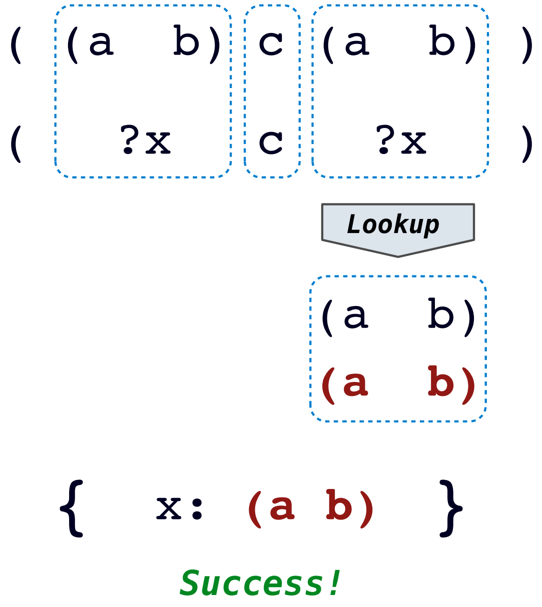

((a b) c (a b))

(?x c ?x)

Lookup

(a b)

(a b)

{ x: (a b) }

Success!

Unification

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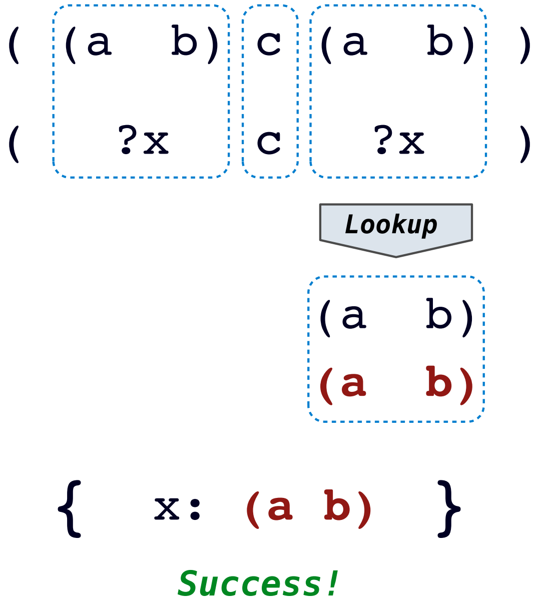

```
( (a  b) c (a  b) )
(   ?x   c   ?x   )
```

```
(a  b)
(a  b)
```

```
{  x: (a b)  }
```

Success!

```
( (a  b) c (a  b) )
(   ?x   ?x   ?x  )
```

```
{             }
```

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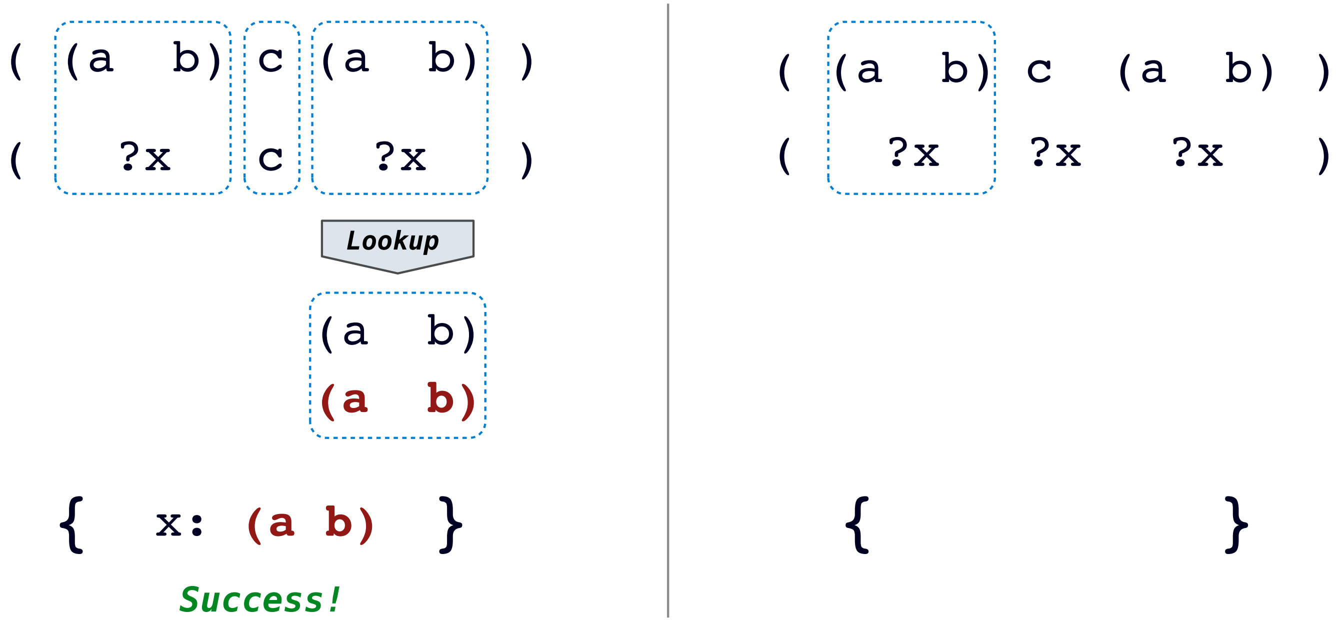


```
( (a   b) c (a   b) )
(    ?x   c    ?x   )
```

Lookup

```
(a   b)
(a   b)
```

```
{  x: (a b)  }
```

Success!

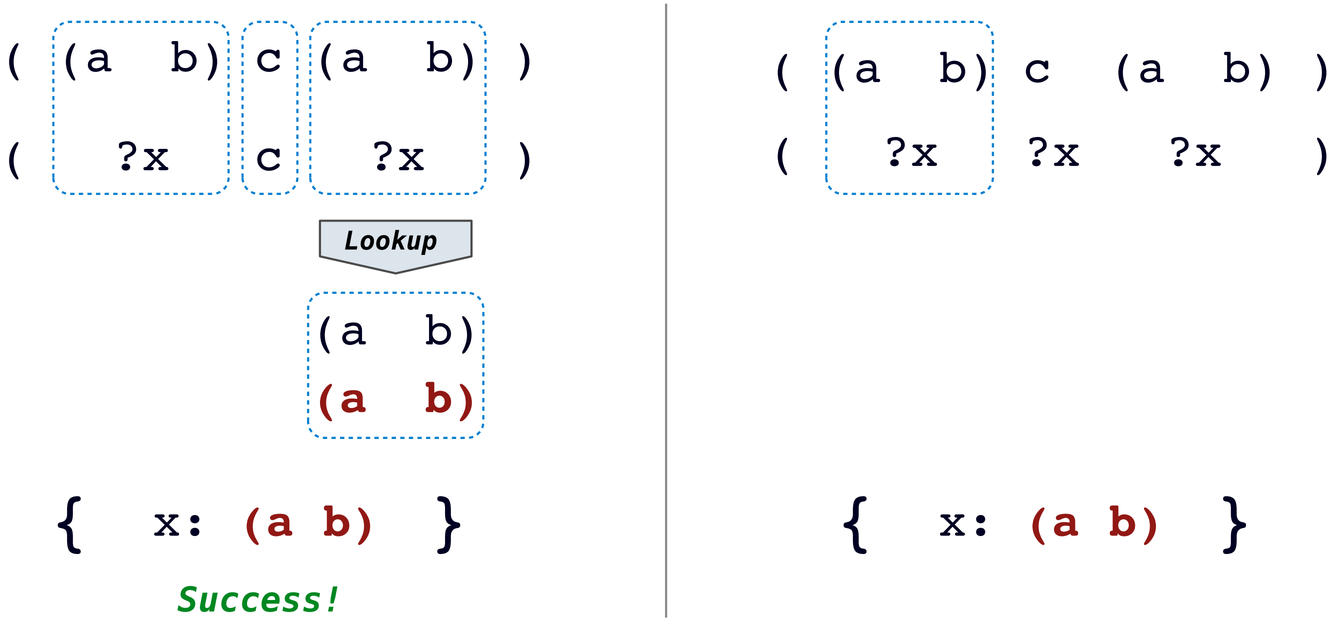
```
( (a   b) c   (a   b) )
(    ?x   ?x    ?x    )
```

```
{            }
```

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.



```
( (a  b) c (a  b) )
(   ?x   c   ?x   )
```

Lookup

```
(a  b)
(a  b)
```

{ x: (a b) }

Success!

```
( (a  b) c  (a  b) )
(   ?x   ?x   ?x   )
```

{ x: (a b) }

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

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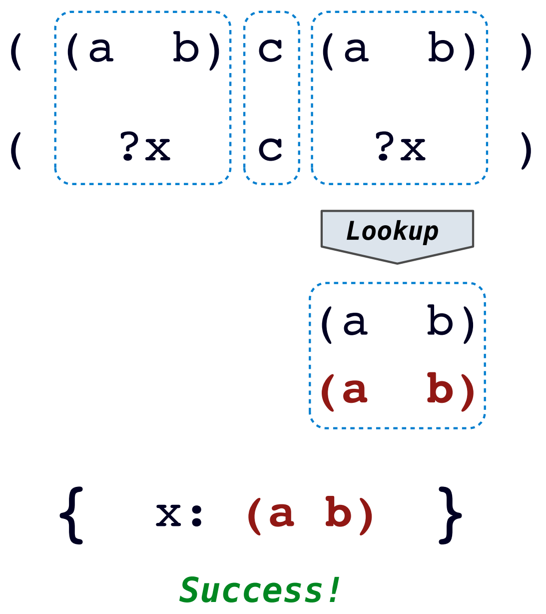


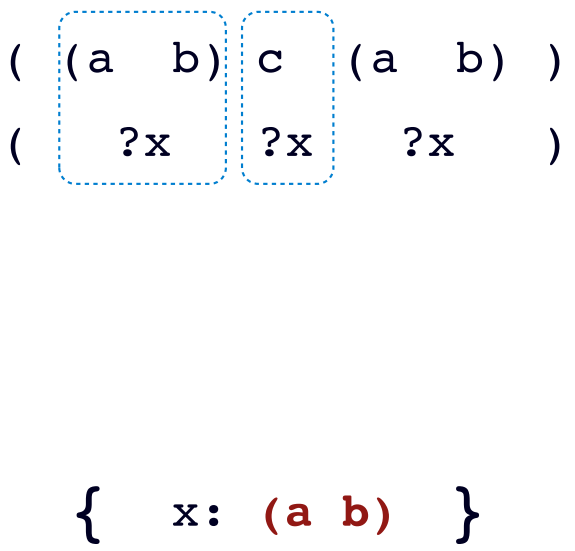
```
( (a   b)  c  (a   b)  )
(    ?x    c     ?x    )
```

Lookup

```
(a   b)
(a   b)
```

```
{   x:  (a b)   }
```

Success!



```
( (a   b)  c  (a   b)  )
(    ?x    ?x    ?x    )
```

```
{   x:  (a b)   }
```

Unification

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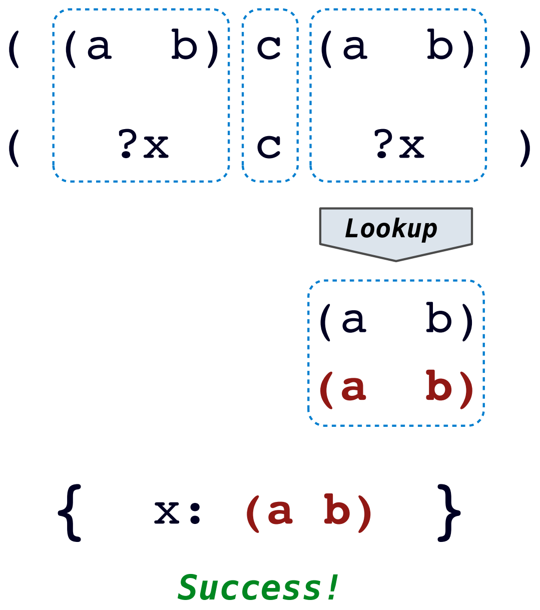
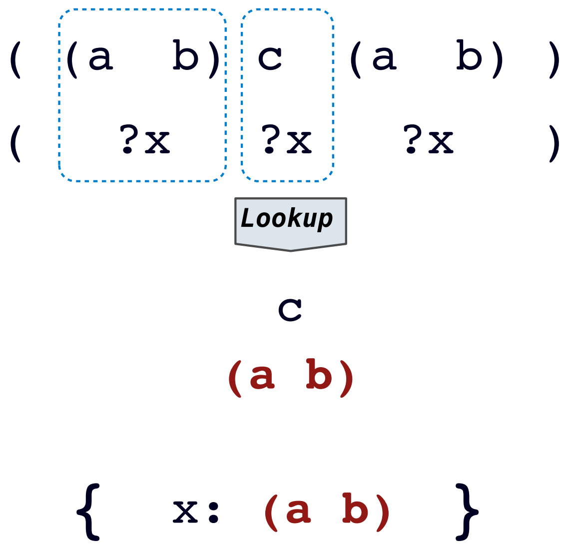
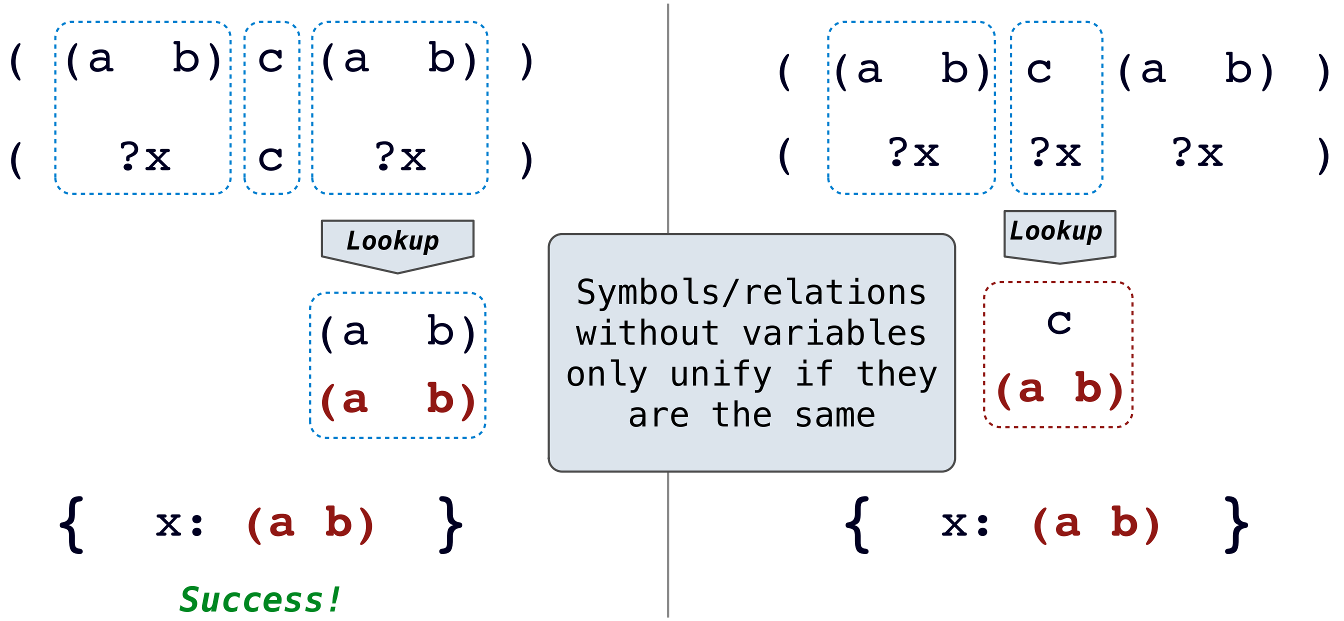




Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.



((a b) c (a b))

(?x c ?x)

Lookup

(a b)

(a b)

{ x: **(a b)** }

Success!

Symbols/relations without variables only unify if they are the same

((a b) c (a b))

(?x ?x ?x)

Lookup

c

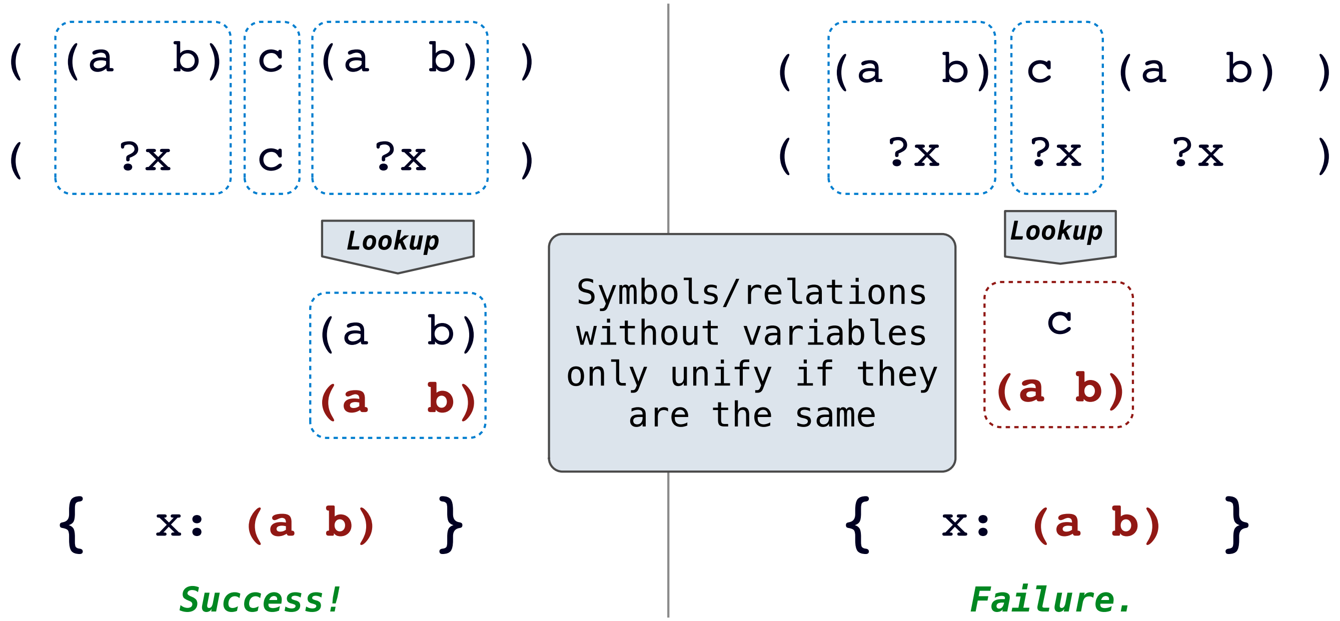
(a b)

{ x: **(a b)** }

Unification

Unification recursively unifies each pair of corresponding elements in two relations, accumulating an assignment.

1. Look up variables in the current environment.
2. Establish new bindings to unify elements.



```
( (a   b) c (a   b) )
(   ?x    c   ?x    )
```

Lookup

```
(a   b)
(a   b)
```

{ x: (a b) }

Success!

Symbols/relations without variables only unify if they are the same

```
( (a   b) c  (a   b) )
(   ?x    ?x   ?x    )
```

Lookup

```
  c
(a b)
```

{ x: (a b) }

Failure.

Unifying Variables

Unifying Variables

Two relations that contain variables can be unified as well.

Unifying Variables

Two relations that contain variables can be unified as well.

```
(    ?x        ?x    )

((a ?y c) (a b ?z))
```

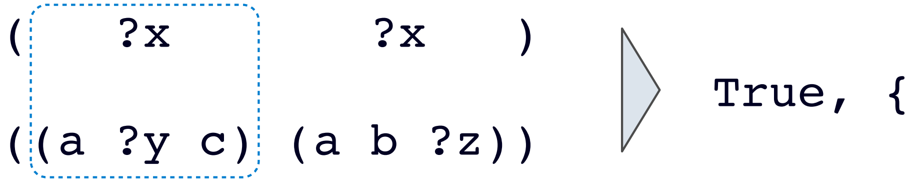
Unifying Variables

Two relations that contain variables can be unified as well.

```
(   ?x         ?x )
                        ▷   True, {
((a ?y c) (a b ?z))
```

Unifying Variables

Two relations that contain variables can be unified as well.



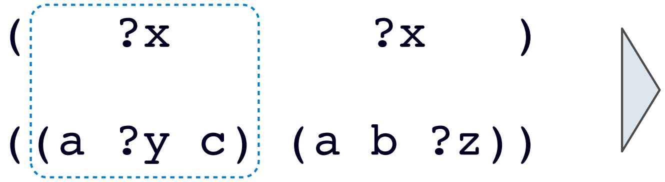
```
(    ?x           ?x     )

((a ?y c)  (a b ?z))
```

`True, {`

Unifying Variables

Two relations that contain variables can be unified as well.



```
(    ?x          ?x    )

((a ?y c)  (a b ?z))
```

```
True, {x: (a ?y c),
```

Unifying Variables

Two relations that contain variables can be unified as well.

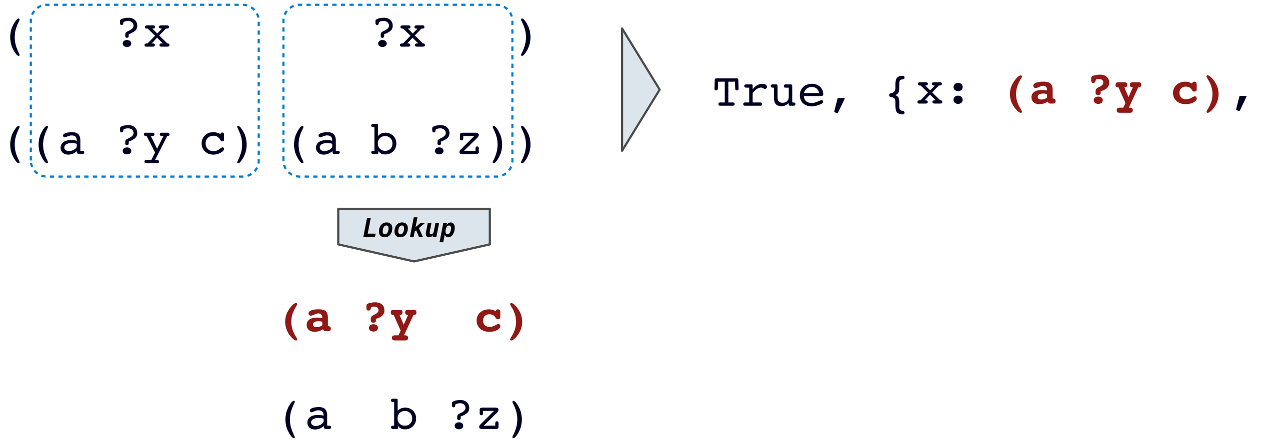
```
(    ?x          ?x     )

((a ?y c)  (a b ?z))
```

`True, {x: (a ?y c),`

Unifying Variables

Two relations that contain variables can be unified as well.



```
(   ?x          ?x     )
((a ?y c)  (a b ?z))
```

True, {x: **(a ?y c)**,

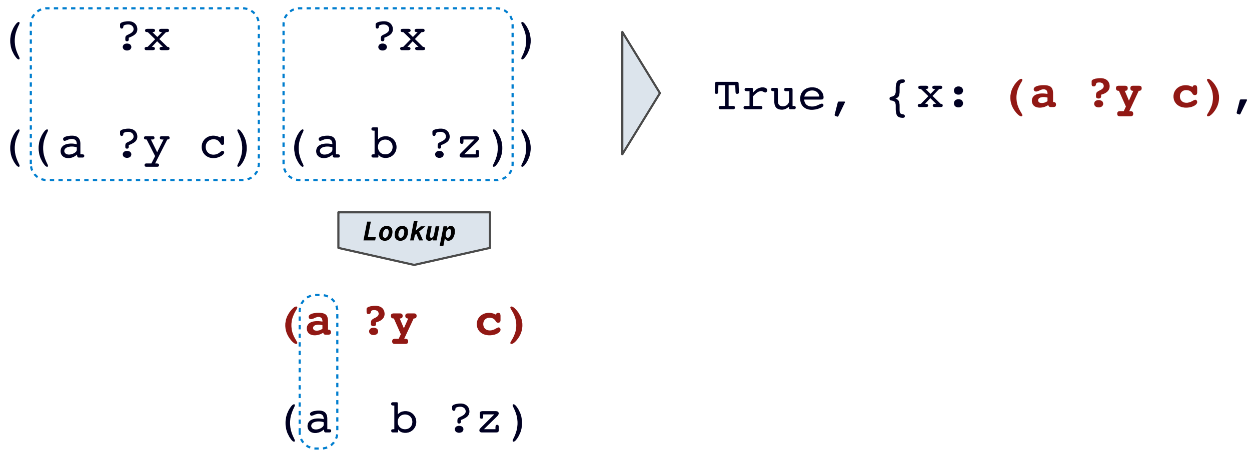
Lookup

(a ?y c)

(a b ?z)

Unifying Variables

Two relations that contain variables can be unified as well.



```
(    ?x          ?x     )

((a ?y c)  (a b ?z))
```

Lookup

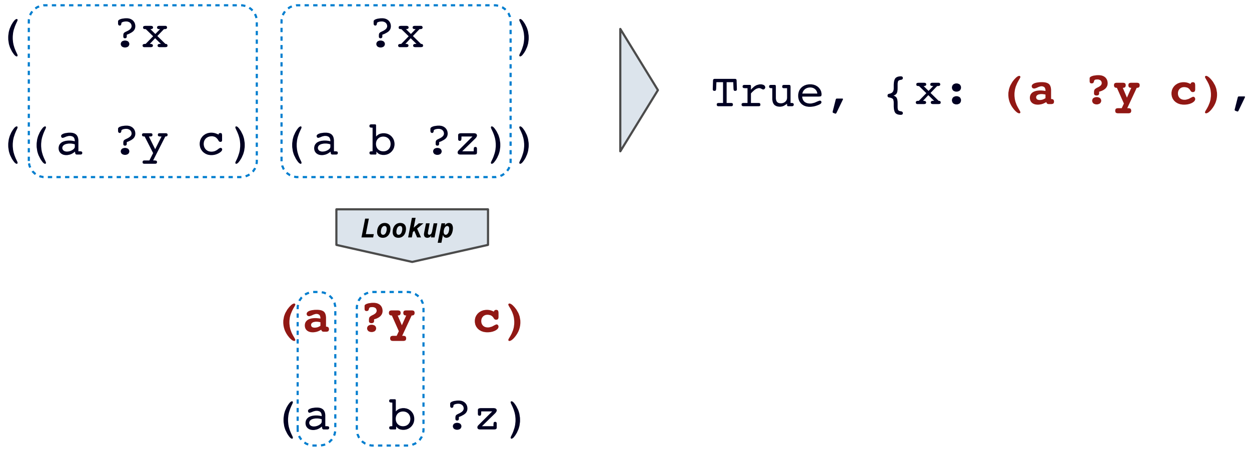
```
(a ?y  c)

(a   b ?z)
```

True, {x: (a ?y c),

Unifying Variables

Two relations that contain variables can be unified as well.



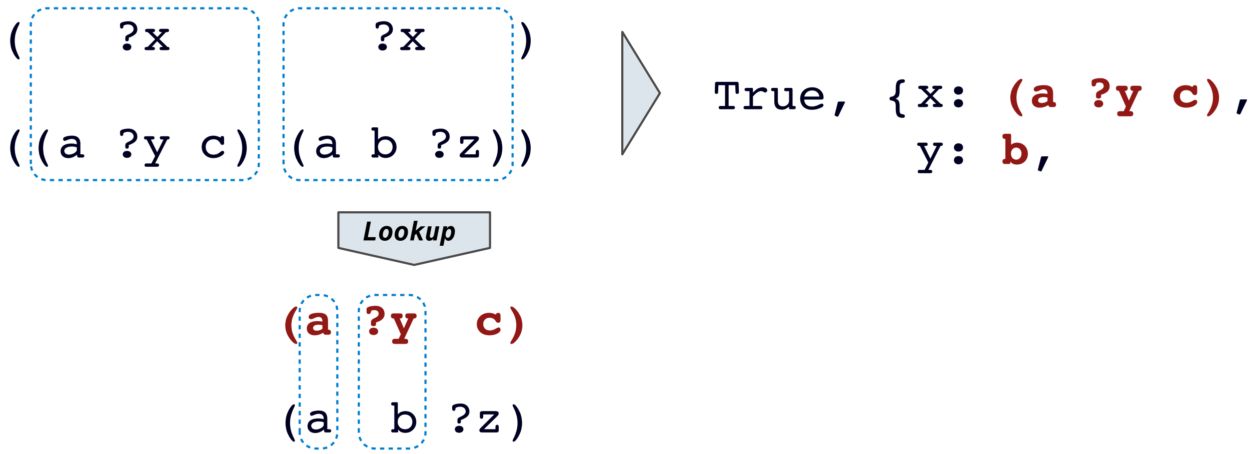
```
(      ?x           ?x     )
((a ?y c)    (a b ?z))
```

```
(a ?y  c)
(a  b ?z)
```

```
True, {x: (a ?y c),
```

Unifying Variables

`Two relations that contain variables can be unified as well.`



```
(    ?x          ?x      )

((a ?y c)   (a b ?z))
```

Lookup

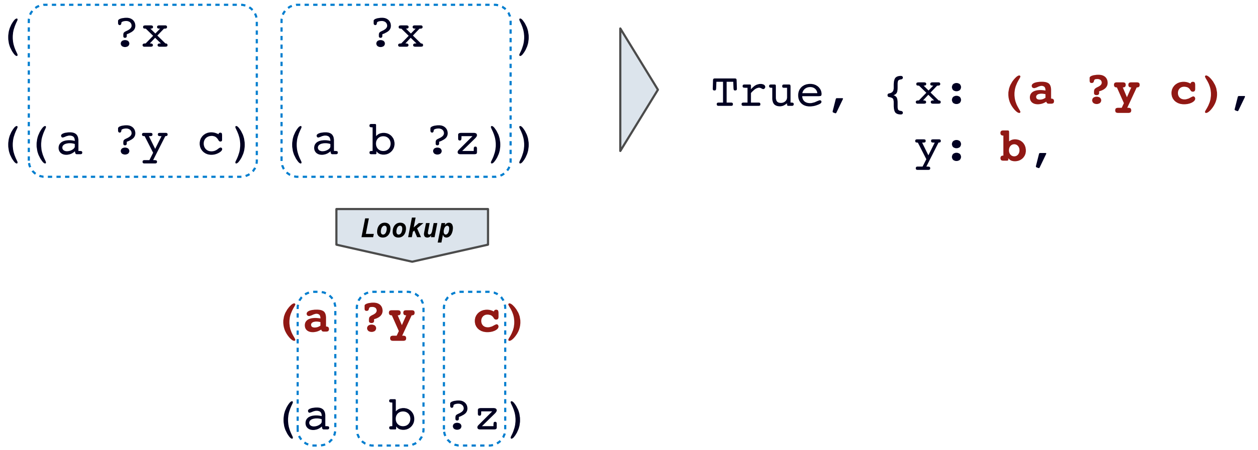
```
(a ?y c)

(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
```

Unifying Variables

Two relations that contain variables can be unified as well.



```
(    ?x            ?x      )

((a ?y c)    (a b ?z))
```

Lookup

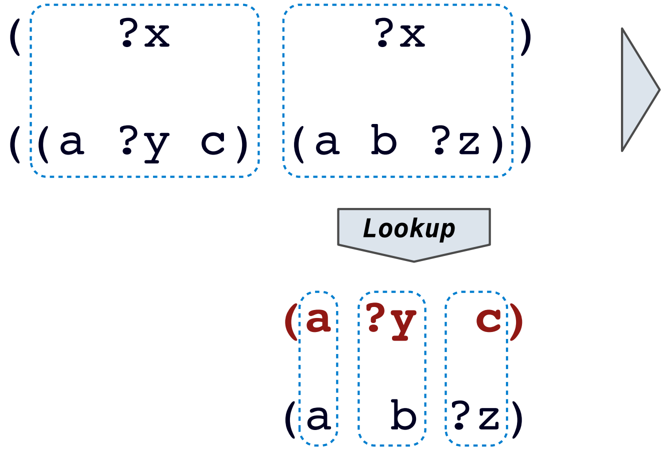
```
(a  ?y   c)

(a   b  ?z)
```

```
True, {x: (a ?y c),
       y: b,
```

Unifying Variables

Two relations that contain variables can be unified as well.



```
(   ?x          ?x      )

( (a ?y c)  (a b ?z) )
```

Lookup

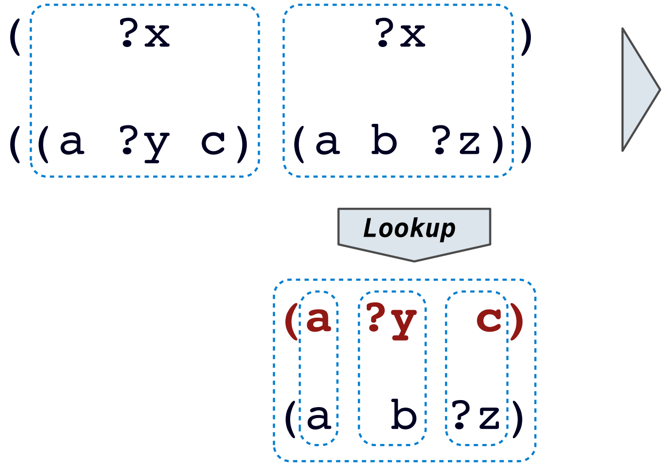
```
(a ?y  c)

(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Unifying Variables

Two relations that contain variables can be unified as well.



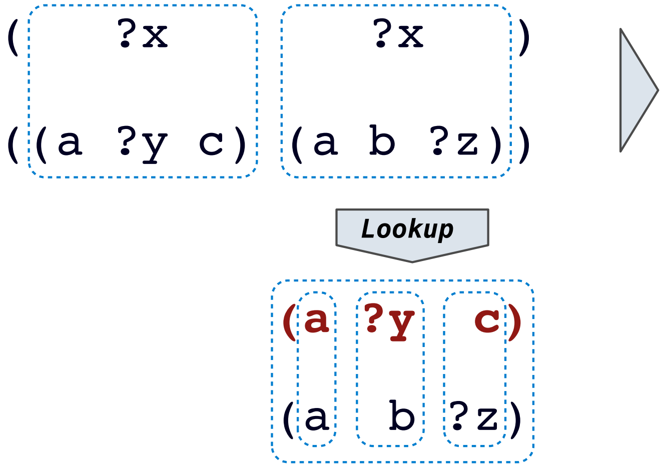
```
(    ?x         ?x    )
((a ?y c) (a b ?z))
```

```
(a ?y  c)
(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Unifying Variables

Two relations that contain variables can be unified as well.



```
( ?x       ?x       )
((a ?y c) (a b ?z))
```

Lookup

```
(a ?y c)
(a  b ?z)
```

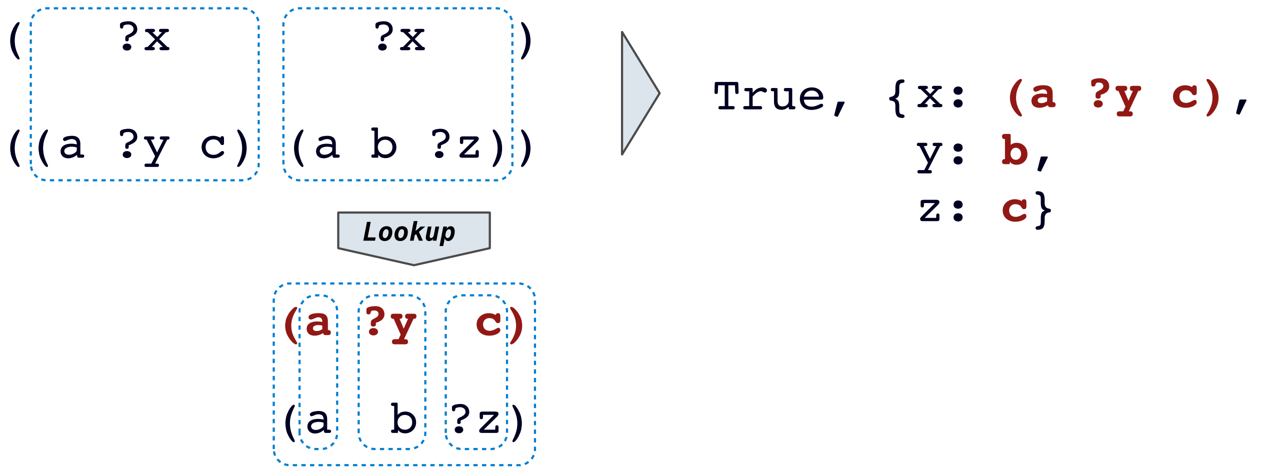
```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

Unifying Variables

Two relations that contain variables can be unified as well.



```
(   ?x          ?x     )
((a ?y c)   (a b ?z))
```

Lookup

```
(a ?y c)
(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

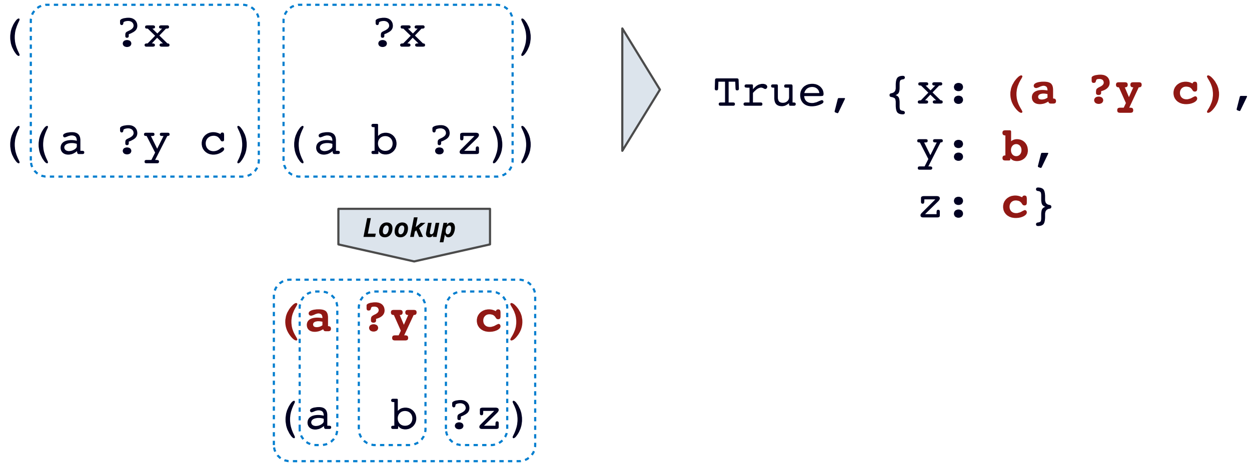
Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

`lookup('?x')`

Unifying Variables

Two relations that contain variables can be unified as well.



```
(    ?x          ?x     )
((a ?y c)   (a b ?z))
```

True, {x: **(a ?y c)**,
y: **b**,
z: **c**}

Lookup

```
(a ?y  c)
(a  b ?z)
```

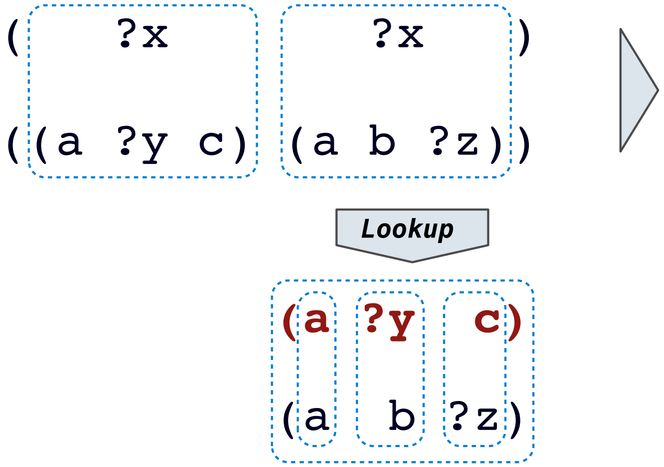
Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ **(a ?y c)**

Unifying Variables

Two relations that contain variables can be unified as well.



```
(   ?x          ?x     )
((a ?y c)   (a b ?z))
```

Lookup

```
(a ?y c)
(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

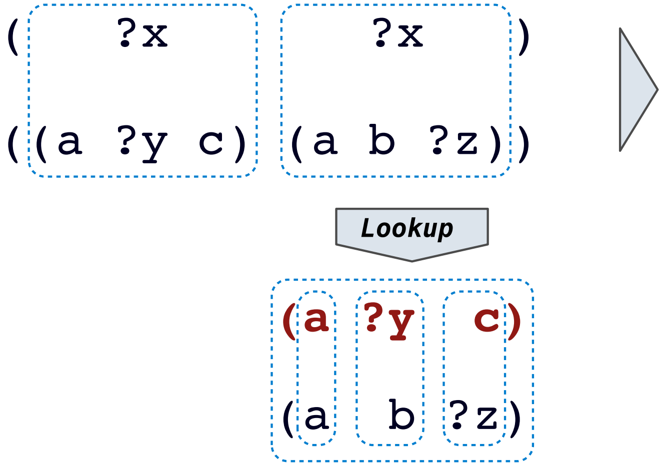
Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

`lookup('?x')` ⇨ **`(a ?y c)`**   **`lookup('?y')`**

Unifying Variables

Two relations that contain variables can be unified as well.



```
(    ?x        ?x    )
((a ?y c) (a b ?z))
```

Lookup

```
(a ?y c)
(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

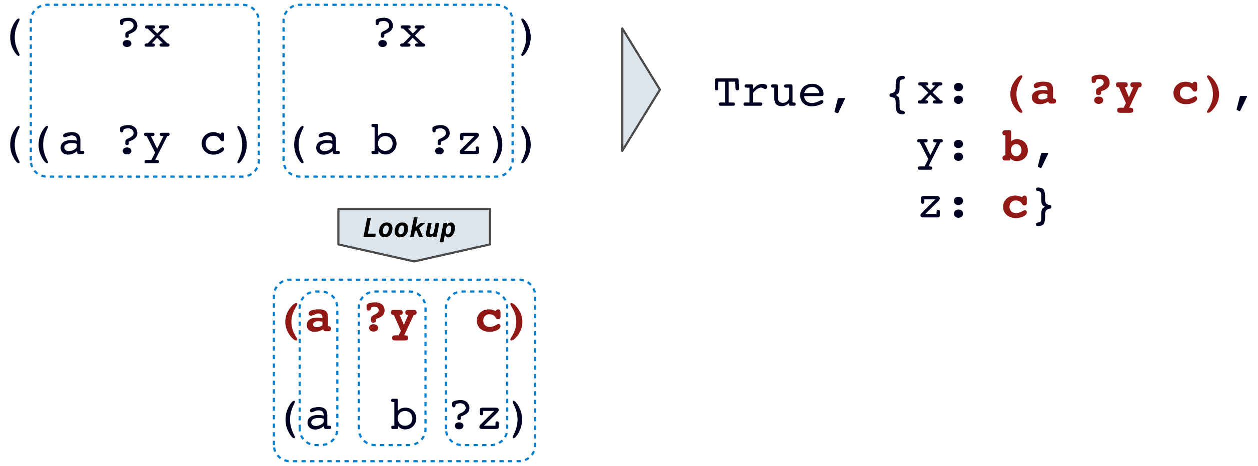
Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

`lookup('?x')` ⇨ `(a ?y c)`   **`lookup('?y')`** ⇨ `b`

Unifying Variables

Two relations that contain variables can be unified as well.



```
(   ?x        ?x     )

((a ?y c) (a b ?z))
```

Lookup

```
(a ?y  c)

(a  b ?z)
```

```
True, {x: (a ?y c),
       y: b,
       z: c}
```

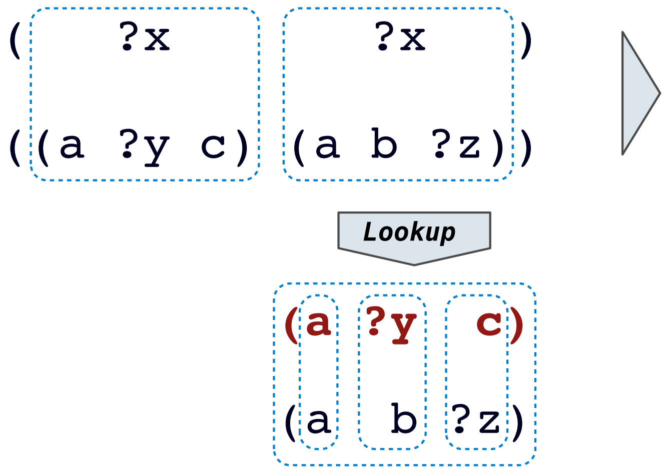
Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ (a ?y c)   **lookup('?y')** ⇨ b   **ground('?x')**

Unifying Variables

Two relations that contain variables can be unified as well.



```
True, {x: (a ?y c),
       y: b,
       z: c}
```

Substituting values for variables may require multiple steps.

This process is called *grounding*. Two unified expressions have the same grounded form.

lookup('?x') ⇨ **(a ?y c)**   **lookup('?y')** ⇨ **b**   **ground('?x')** ⇨ **(a b c)**

Implementing Unification

```
def unify(e, f, env):
  e = lookup(e, env)
  f = lookup(f, env)
  if e == f:
      return True
  elif isvar(e):
      env.define(e, f)
      return True
  elif isvar(f):
      env.define(f, e)
      return True
  elif scheme_atomp(e) or scheme_atomp(f):
      return False
  else:
      return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Implementing Unification

```
def unify(e, f, env):
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        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

2. Establish new bindings to unify elements.

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
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    elif isvar(e):
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        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
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        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a  b) c (a  b) )

(    ?x    c    ?x    )
```

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
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        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
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    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a  b) c (a  b) )
(   ?x   c   ?x   )
```

env: { }

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a   b) c (a   b) )
(    ?x   c    ?x   )
```

env: { }

Implementing Unification

```
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        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
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    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a   b) c (a   b) )
(   ?x    c   ?x    )
```

env: { }

Implementing Unification

```
def unify(e, f, env):
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        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
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    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a   b) c (a   b) )
(   ?x    c   ?x    )
```

env: { x: (a b) }

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
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    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.

```
( (a   b) c (a   b) )
(   ?x    c   ?x    )
```

env: { x: **(a b)** }

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
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        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
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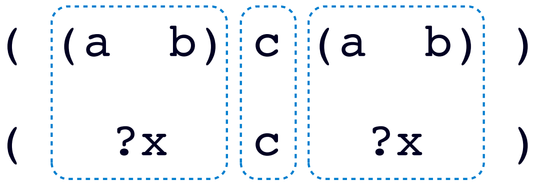
2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.



((a b) c (a b))

(?x c ?x)

env: { x: **(a b)** }

Implementing Unification

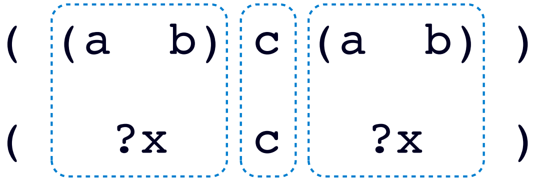
```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

1. Look up variables in the current environment

Symbols/relations without variables only unify if they are the same

2. Establish new bindings to unify elements.

Recursively unify the first and rest of any lists.



`( (a b) c (a b) )`

`( ?x c ?x )`

env: { x: **(a b)** }

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
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        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
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```

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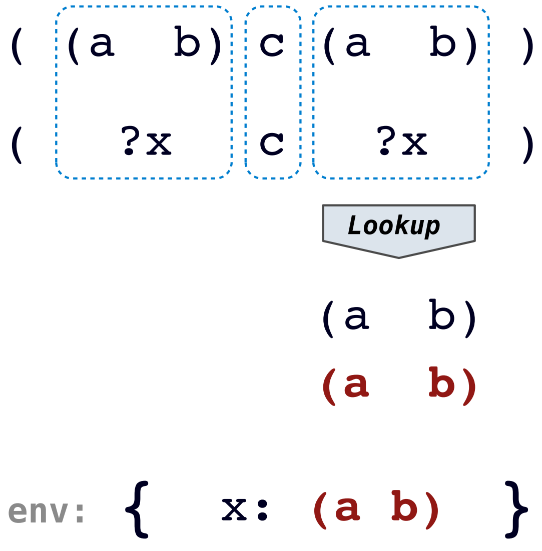

((a b) c (a b))

(?x c ?x)

Lookup

(a b)

(a b)

env: { x: (a b) }

Implementing Unification

```
def unify(e, f, env):
    e = lookup(e, env)
    f = lookup(f, env)
    if e == f:
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first,  f.first,  env) and unify(e.second, f.second, env)
```

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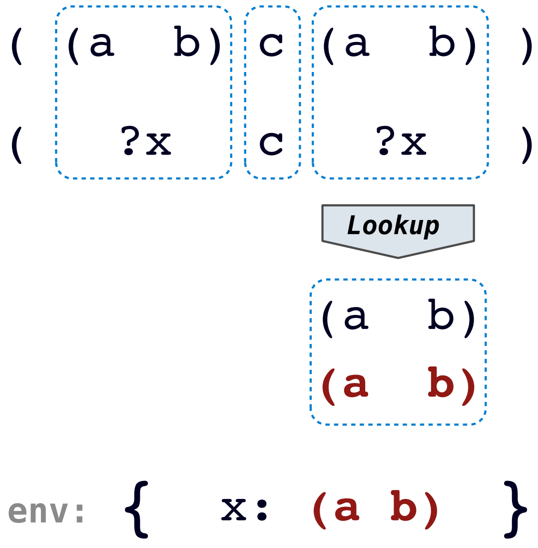

Search

Searching for Proofs

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

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```

Searching for Proofs

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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
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```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

```
(app (e . ?r) (c d) (e b c d))
```

Searching for Proofs

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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
```

```
(app (e . ?r) (c d) (e b c d))
```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))


(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

```
(app (e . ?r) (c d) (e b c d))
```

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

Variables are local to facts & queries

```
(app (e . ?r) (c d) (e b c d))
```

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```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

`(app ?left (c d) (e b c d))`

{a: **e**, y: **(c d)**, z: **(b c d)**, left: **(?a . ?r)**}

`(app (?a . ?r) ?y (?a . ?z))`

conclusion <- hypothesis

`(app ?r (c d) (b c d)))`

{a2: **b**, y2: **(c d)**, z2: **(c d)**, r: **(?a2 . ?r2)**}

`(app (?a2 . ?r2) ?y2 (?a2 . ?z2))`

`(app (e . ?r) (c d) (e b c d))`

`(app (b . ?r2) (c d) (b c d))`

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(fact  (app (?a . ?r) ?y (?a . ?z))
       (app        ?r  ?y        ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}        (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}        (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
```

Variables are local to facts & queries

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))

(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

Variables are local to facts & queries

Searching for Proofs

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```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

`(app (e . ?r) (c d) (e b c d))`

`(app (b . ?r2) (c d) (b c d))`

Variables are local to facts & queries

Searching for Proofs

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(fact  (app () ?x ?x))

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(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
```

(app (e . ?r) (c d) (e b c d))

```
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
```

(app (b . ?r2) (c d) (b c d))

Variables are local to facts & queries

```
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

(app () (c d) (c d))

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

Variables are local to facts & queries

```
(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app () (c d) (c d))
```

?left:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

Variables are local to facts & queries

```
(app () (c d) (c d))
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

?left:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

Variables are local to facts & queries

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

```
(app () (c d) (c d))
```

?left:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}        ▷  (app (e . ?r) (c d) (e b c d))
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}        ▷  (app (b . ?r2) (c d) (b c d))
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}   ▷  (app () (c d) (c d))
(app () ?x ?x)
```

Variables are local to facts & queries

?left: **(e .**

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

Variables are local to facts & queries

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

```
(app () (c d) (c d))
```

?left: **(e .**

?r:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))

(app (b . ?r2) (c d) (b c d))

(app () (c d) (c d))
```

Variables are local to facts & queries

?left: **(e .**

?r:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

```
(app () (c d) (c d))
```

Variables are local to facts & queries

?left: **(e .**

?r:

Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
    conclusion <- hypothesis
(app ?r (c d) (b c d)))
    {a2: b, y2: (c d), z2: (c d), r: (?a2 . ?r2)}
(app (?a2 . ?r2) ?y2 (?a2 . ?z2))
    conclusion <- hypothesis
(app ?r2 (c d) (c d))
    {r2: (), x: (c d)}
(app () ?x ?x)
```

```
(app (e . ?r) (c d) (e b c d))
```

```
(app (b . ?r2) (c d) (b c d))
```

Variables are local to facts & queries

```
(app () (c d) (c d))
```

?left: **(e .**

?r: **(b .**

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```
(fact  (app () ?x ?x))

(fact  (app (?a . ?r) ?y (?a . ?z))
       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

```
(app ?left (c d) (e b c d))
    {a: e, y: (c d), z: (b c d), left: (?a . ?r)}
(app (?a . ?r) ?y (?a . ?z))
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Variables are local to facts & queries

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```

?left: **(e .**

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Searching for Proofs

The Logic interpreter searches the space of facts to find unifying facts and an env that prove the query to be true.

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       (app       ?r  ?y       ?z ))

(query (app ?left (c d) (e b c d)))
```

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(Demo)

Addition

`(Demo)`