

## 61A Lecture 20

---

Monday, October 21

## Announcements

---

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405



## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320
- Midterm 2 is on Monday 10/28 7pm–9pm

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320
- Midterm 2 is on Monday 10/28 7pm–9pm
  - Topics and locations are posted on the course website

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320
- Midterm 2 is on Monday 10/28 7pm–9pm
  - Topics and locations are posted on the course website
  - Have an unavoidable conflict? Fill out the conflict form by Friday 10/25 @ 11:59pm

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320
- Midterm 2 is on Monday 10/28 7pm–9pm
  - Topics and locations are posted on the course website
  - Have an unavoidable conflict? Fill out the conflict form by Friday 10/25 @ 11:59pm
  - Review session on Saturday 10/26 1pm–4pm in 1 Pimentel

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320
- Midterm 2 is on Monday 10/28 7pm–9pm
  - Topics and locations are posted on the course website
  - Have an unavoidable conflict? Fill out the conflict form by Friday 10/25 @ 11:59pm
  - Review session on Saturday 10/26 1pm–4pm in 1 Pimentel
  - Student-organized "engineering bowl" about midterm 2 on Tuesday 4pm–6pm in 240 Bechtel

## Announcements

---

- Homework 6 is due Tuesday 10/22 @ 11:59pm
  - Includes a mid-semester survey about the course so far
- Project 3 is due Thursday 10/24 @ 11:59pm
  - Extra reader office hours this week:
    - Tuesday 6–7:30 in Soda 405
    - Wednesday 5:30–7 in Soda 405
    - Thursday 5:30–7 in Soda 320
- Midterm 2 is on Monday 10/28 7pm–9pm
  - Topics and locations are posted on the course website
  - Have an unavoidable conflict? Fill out the conflict form by Friday 10/25 @ 11:59pm
  - Review session on Saturday 10/26 1pm–4pm in 1 Pimentel
  - Student-organized "engineering bowl" about midterm 2 on Tuesday 4pm–6pm in 240 Bechtel
- Homework 7 is due Tuesday 11/5 @ 11:59pm (Two weeks)

## Generic Functions



## Generic Functions

---

## Generic Functions

---

An abstraction might have more than one representation.

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

A function might want to operate on multiple data types.

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

A function might want to operate on multiple data types.

**Today's Topics:**

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

A function might want to operate on multiple data types.

### **Today's Topics:**

- Generic functions



## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

A function might want to operate on multiple data types.

### **Today's Topics:**

- Generic functions
- String representations of objects

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

A function might want to operate on multiple data types.

### **Today's Topics:**

- Generic functions
- String representations of objects
- Property methods

## Generic Functions

---

An abstraction might have more than one representation.

- Python has many sequence types: tuples, ranges, lists, etc.

An abstract data type might have multiple implementations.

- Some representations are better suited to some problems.

A function might want to operate on multiple data types.

### **Today's Topics:**

- Generic functions
- String representations of objects
- Property methods
- Multiple representations of data using the Python object system

## String Representations

## String Representations

---

## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

Strings are important: they represent *language* and *programs*.



## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

Strings are important: they represent *language* and *programs*.

In Python, all objects produce two string representations:

## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

Strings are important: they represent *language* and *programs*.

In Python, all objects produce two string representations:

- The "str" is legible to **humans**.

## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

Strings are important: they represent *language* and *programs*.

In Python, all objects produce two string representations:

- The "str" is legible to **humans**.
- The "repr" is legible to the **Python interpreter**.

## String Representations

---

An object value should **behave** like the kind of data it is meant to represent;

For instance, by **producing a string** representation of itself.

Strings are important: they represent *language* and *programs*.

In Python, all objects produce two string representations:

- The "str" is legible to **humans**.
- The "repr" is legible to the **Python interpreter**.

When the "str" and "repr" **strings are the same**, that's a sign that a programming language is legible to humans!

## The "repr" String for an Object

---

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.



## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.

```
>>> 12e12
```

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.

```
>>> 12e12  
12000000000000.0
```

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
```

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

Some objects don't have a simple Python-readable string.

## The "repr" String for an Object

---

The `repr` function returns a Python expression (as a string) that evaluates to an equal object.

```
repr(object) -> string
```

Return the canonical string representation of the object.  
For most object types, `eval(repr(object)) == object`.

The result of calling `repr` on the value of an expression is what Python prints in an interactive session.

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

Some objects don't have a simple Python-readable string.

```
>>> repr(min)
'<built-in function min>'
```

## The "str" String for an Object

---

Human interpretable strings are useful as well:

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
```



## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
```

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
>>> repr(today)
```

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
>>> repr(today)
'datetime.date(2013, 10, 21)'
```

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
>>> repr(today)
'datetime.date(2013, 10, 21)'
>>> str(today)
```

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
>>> repr(today)
'datetime.date(2013, 10, 21)'
>>> str(today)
'2013-10-21'
```

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
>>> repr(today)
'datetime.date(2013, 10, 21)'
>>> str(today)
'2013-10-21'
```

The result of calling `str` on the value of an expression is what Python prints using the `print` function.

## The "str" String for an Object

---

Human interpretable strings are useful as well:

```
>>> import datetime
>>> today = datetime.date(2013, 10, 21)
>>> repr(today)
'datetime.date(2013, 10, 21)'
>>> str(today)
'2013-10-21'
```

The result of calling `str` on the value of an expression is what Python prints using the `print` function.

(Demo)

Implementing str and repr



## Polymorphic Functions

---

## Polymorphic Functions

---

*Polymorphic* function: A function that can be applied to many (*poly*) different forms (*morph*) of data

## Polymorphic Functions

---

*Polymorphic* function: A function that can be applied to many (*poly*) different forms (*morph*) of data

`str` and `repr` are both polymorphic; they apply to anything.

## Polymorphic Functions

---

*Polymorphic* function: A function that can be applied to many (*poly*) different forms (*morph*) of data

`str` and `repr` are both polymorphic; they apply to anything.

`repr` invokes a zero-argument method `__repr__` on its argument.

## Polymorphic Functions

---

*Polymorphic* function: A function that can be applied to many (*poly*) different forms (*morph*) of data

`str` and `repr` are both polymorphic; they apply to anything.

`repr` invokes a zero-argument method `__repr__` on its argument.

```
>>> today.__repr__()
'datetime.date(2012, 10, 8)'
```

## Polymorphic Functions

---

*Polymorphic* function: A function that can be applied to many (*poly*) different forms (*morph*) of data

`str` and `repr` are both polymorphic; they apply to anything.

`repr` invokes a zero-argument method `__repr__` on its argument.

```
>>> today.__repr__()
'datetime.date(2012, 10, 8)'
```

`str` invokes a zero-argument method `__str__` on its argument.

## Polymorphic Functions

---

*Polymorphic* function: A function that can be applied to many (*poly*) different forms (*morph*) of data

`str` and `repr` are both polymorphic; they apply to anything.

`repr` invokes a zero-argument method `__repr__` on its argument.

```
>>> today.__repr__()
'datetime.date(2012, 10, 8)'
```

`str` invokes a zero-argument method `__str__` on its argument.

```
>>> today.__str__()
'2012-10-08'
```

## Implementing repr and str

---



## Implementing repr and str

---

The behavior of `repr` is slightly more complicated than invoking `__repr__` on its argument:

## Implementing repr and str

---

The behavior of `repr` is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)

## Implementing repr and str

---

The behavior of `repr` is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

## Implementing repr and str

---

The behavior of `repr` is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

## Implementing repr and str

---

The behavior of repr is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

The behavior of str:

## Implementing repr and str

---

The behavior of repr is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

The behavior of str:

- An instance attribute called `__str__` is ignored.

## Implementing repr and str

---

The behavior of repr is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

The behavior of str:

- An instance attribute called `__str__` is ignored.
- If no `__str__` attribute is found, uses repr string. (Demo)

## Implementing repr and str

---

The behavior of repr is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

The behavior of str:

- An instance attribute called `__str__` is ignored.
- If no `__str__` attribute is found, uses repr string. (Demo)
- **Question:** How would we implement this behavior?



## Implementing repr and str

---

The behavior of repr is slightly more complicated than invoking `__repr__` on its argument:

- An instance attribute called `__repr__` is ignored. (Demo)
- **Question:** How would we implement this behavior?

The behavior of str:

- An instance attribute called `__str__` is ignored.
- If no `__str__` attribute is found, uses repr string. (Demo)
- **Question:** How would we implement this behavior?
- `str` is a class, not a function

# Interfaces

# Interfaces

---

## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

Message passing allows **different data types** to respond to the **same message**.

## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

Message passing allows **different data types** to respond to the **same message**.

A shared message that elicits similar behavior from different object classes is a powerful method of abstraction.

## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

Message passing allows **different data types** to respond to the **same message**.

A shared message that elicits similar behavior from different object classes is a powerful method of abstraction.

An *interface* is a **set of shared messages**, along with a specification of **what they mean**.

## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

Message passing allows **different data types** to respond to the **same message**.

A shared message that elicits similar behavior from different object classes is a powerful method of abstraction.

An *interface* is a **set of shared messages**, along with a specification of **what they mean**.

**Examples:**



## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

Message passing allows **different data types** to respond to the **same message**.

A shared message that elicits similar behavior from different object classes is a powerful method of abstraction.

An *interface* is a **set of shared messages**, along with a specification of **what they mean**.

### Examples:

Classes that implement `__repr__` and `__str__` methods *that return Python and human readable strings* thereby **implement an interface** for producing Python string representations.

## Interfaces

---

**Message passing:** Objects interact by passing messages, such as attribute names.

Message passing allows **different data types** to respond to the **same message**.

A shared message that elicits similar behavior from different object classes is a powerful method of abstraction.

An *interface* is a **set of shared messages**, along with a specification of **what they mean**.

### Examples:

Classes that implement `__repr__` and `__str__` methods *that return Python and human readable strings* thereby **implement an interface** for producing Python string representations.

Classes that implement `__len__` and `__getitem__` are sequences.

## Property Methods

## Property Methods

---

Often, we want the value of instance attributes to be linked.

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
```



## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
>>> f.denom -= 3
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
>>> f.denom -= 3
>>> f.float_value
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
>>> f.denom -= 3
>>> f.float_value
2.0
```

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
>>> f.denom -= 3
>>> f.float_value
2.0
```

The `@property` decorator on a method designates that it will be called whenever it is *looked up* on an instance.

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
>>> f.denom -= 3
>>> f.float_value
2.0
```

The `@property` decorator on a method designates that it will be called whenever it is *looked up* on an instance.

It allows zero-argument methods to be called without an explicit call expression.

## Property Methods

---

Often, we want the value of instance attributes to be linked.

```
>>> f = Rational(3, 5)
>>> f.float_value
0.6
>>> f.numer = 4
>>> f.float_value
0.8
>>> f.denom -= 3
>>> f.float_value
2.0
```

The `@property` decorator on a method designates that it will be called whenever it is *looked up* on an instance.

It allows zero-argument methods to be called without an explicit call expression.

(Demo)



Example: Complex Numbers

## Multiple Representations of Abstract Data

---

## Multiple Representations of Abstract Data

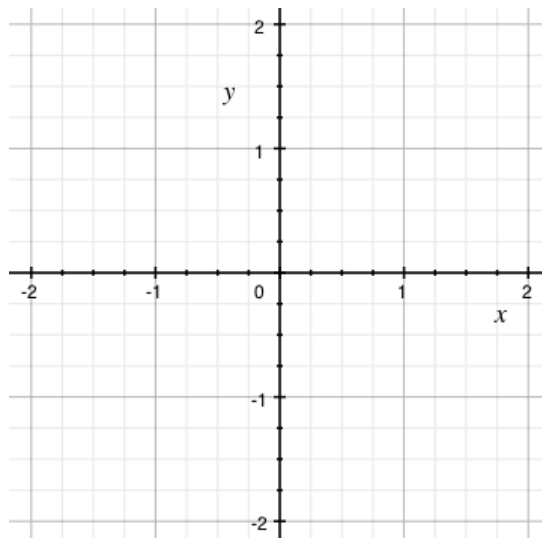
---

Rectangular and polar representations for complex numbers

## Multiple Representations of Abstract Data

---

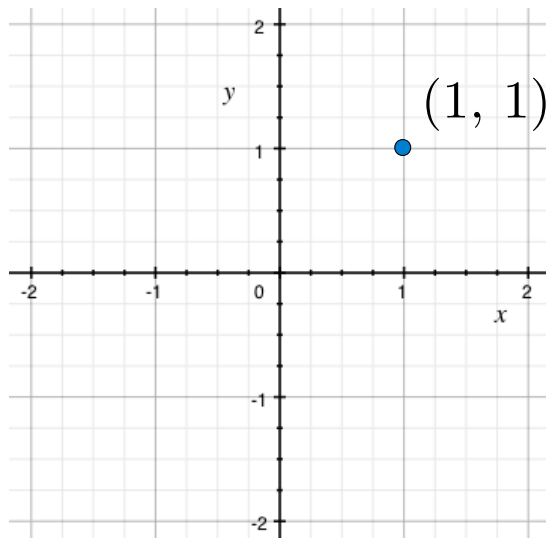
Rectangular and polar representations for complex numbers



## Multiple Representations of Abstract Data

---

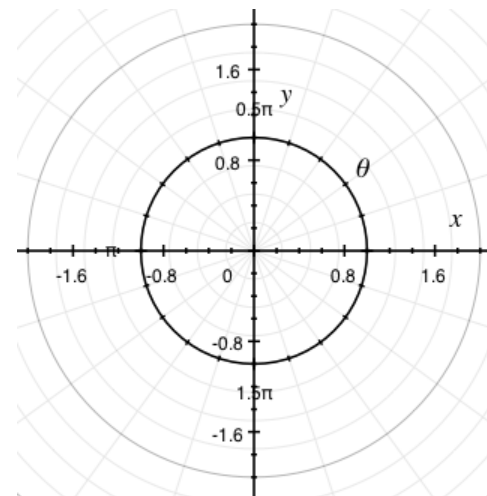
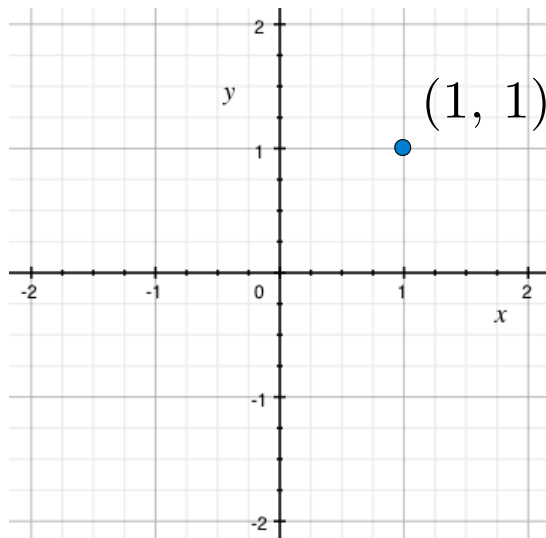
Rectangular and polar representations for complex numbers



## Multiple Representations of Abstract Data

---

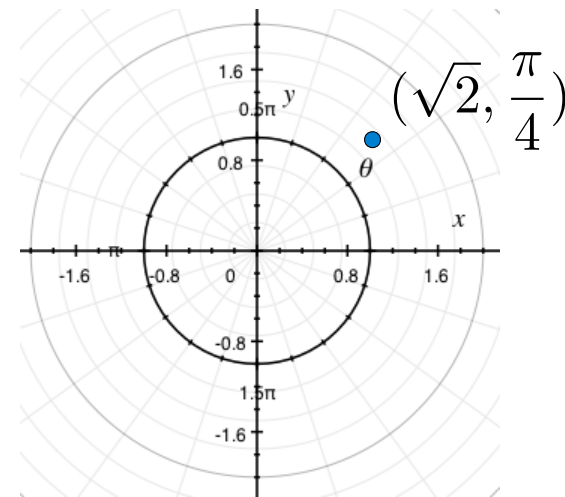
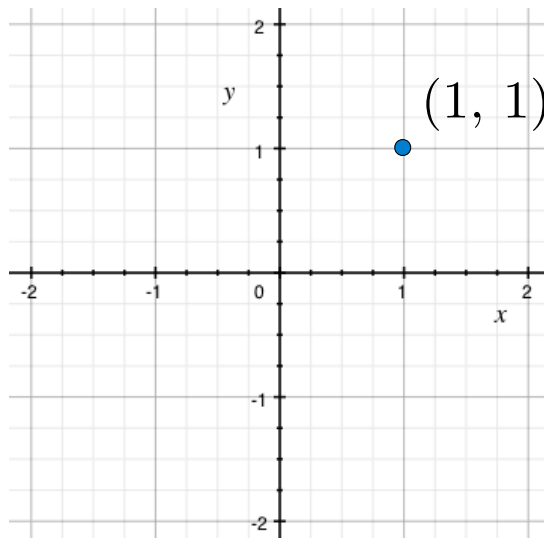
Rectangular and polar representations for complex numbers



## Multiple Representations of Abstract Data

---

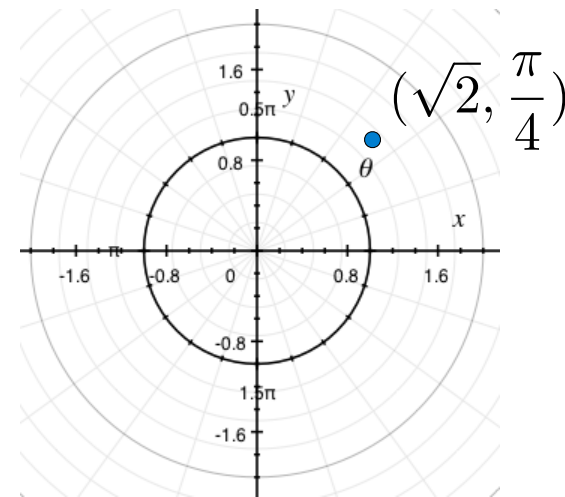
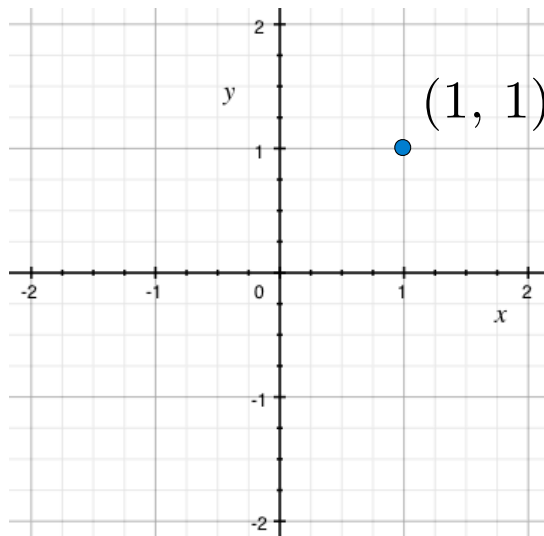
Rectangular and polar representations for complex numbers



## Multiple Representations of Abstract Data

---

Rectangular and polar representations for complex numbers



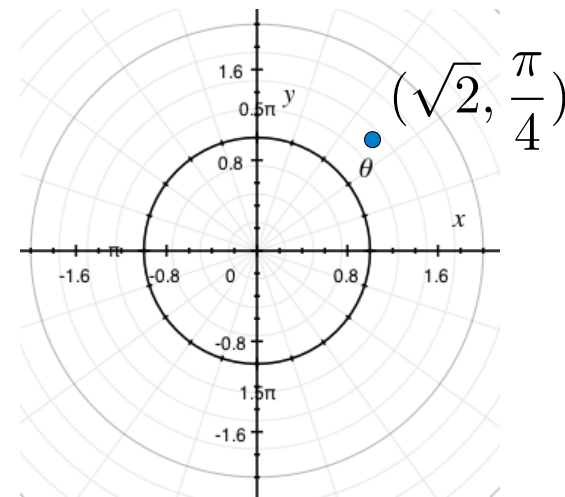
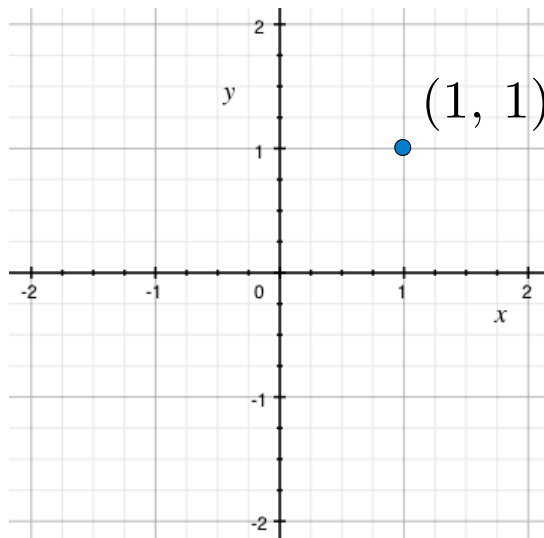
Most operations don't care about the representation.



## Multiple Representations of Abstract Data

---

Rectangular and polar representations for complex numbers



Most operations don't care about the representation.

Some mathematical operations are easier on one than the other.

## Arithmetic Abstraction Barriers

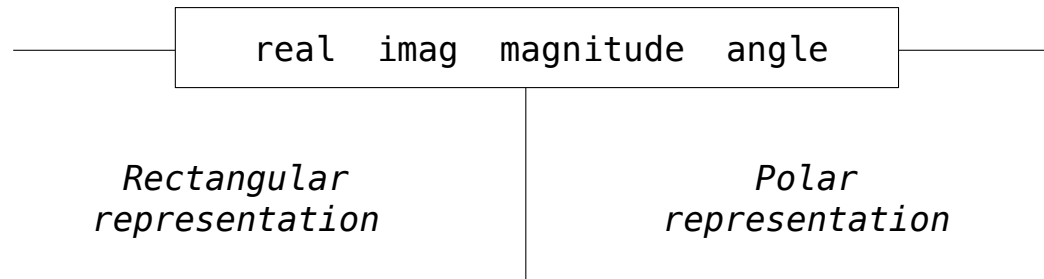
---

*Rectangular  
representation*

*Polar  
representation*

## Arithmetic Abstraction Barriers

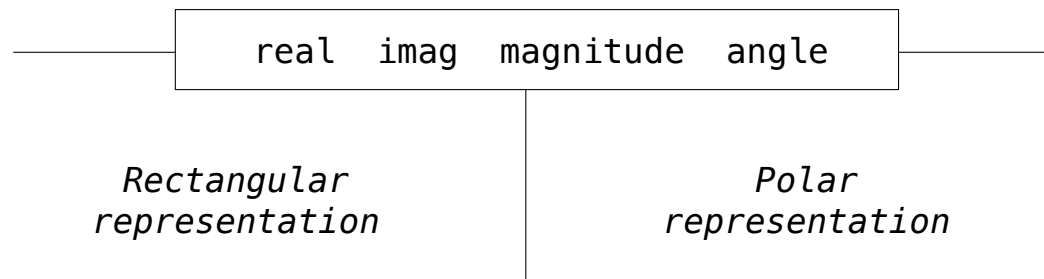
---



## Arithmetic Abstraction Barriers

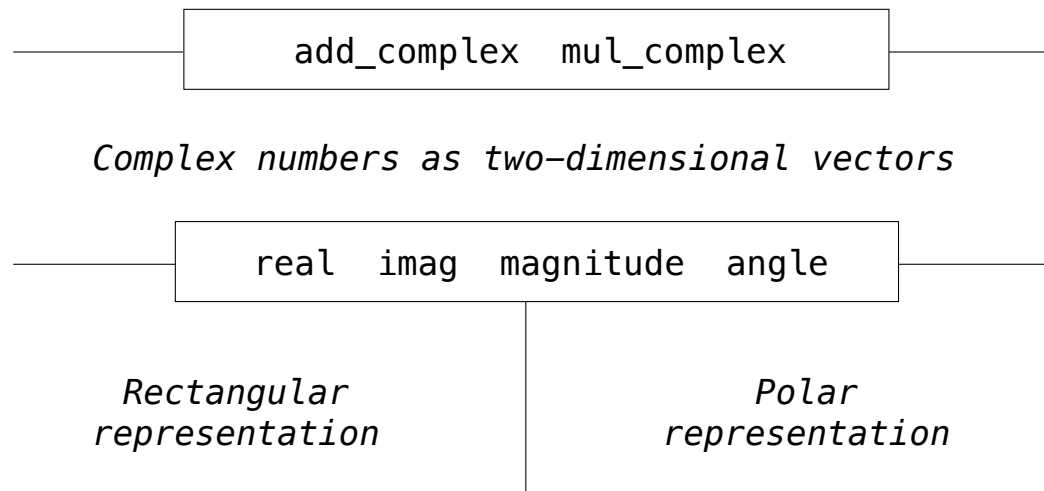
---

*Complex numbers as two-dimensional vectors*



## Arithmetic Abstraction Barriers

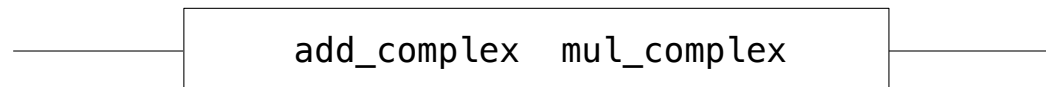
---



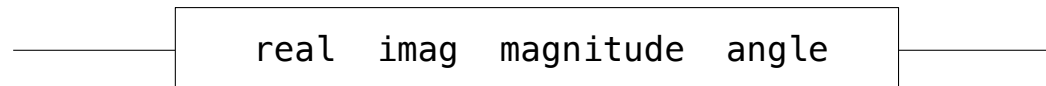
## Arithmetic Abstraction Barriers

---

*Complex numbers as whole data values*



*Complex numbers as two-dimensional vectors*



*Rectangular  
representation*

*Polar  
representation*

## Implementing Complex Numbers

## An Interface for Complex Numbers

---



## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):
```

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):  
    return ComplexRI(z1.real + z2.real,
```

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):  
    return ComplexRI(z1.real + z2.real,  
                    z1.imag + z2.imag)
```

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):  
    return ComplexRI(z1.real + z2.real,  
                    z1.imag + z2.imag)  
  
def mul_complex(z1, z2):
```



## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):  
    return ComplexRI(z1.real + z2.real,  
                    z1.imag + z2.imag)  
  
def mul_complex(z1, z2):  
    return ComplexMA(z1.magnitude * z2.magnitude,
```

## An Interface for Complex Numbers

---

All complex numbers should have real and imag components.

All complex numbers should have a magnitude and angle.

(Demo)

Using this interface, we can implement complex arithmetic:

```
def add_complex(z1, z2):  
    return ComplexRI(z1.real + z2.real,  
                    z1.imag + z2.imag)  
  
def mul_complex(z1, z2):  
    return ComplexMA(z1.magnitude * z2.magnitude,  
                    z1.angle + z2.angle)
```

## The Rectangular Representation

---

## The Rectangular Representation

---

```
class ComplexRI:
```

## The Rectangular Representation

---

```
class ComplexRI:  
    def __init__(self, real, imag):  
        self.real = real  
        self.imag = imag
```

## The Rectangular Representation

---

```
class ComplexRI:

    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5
```

## The Rectangular Representation

---

```
class ComplexRI:
```

```
    def __init__(self, real, imag):  
        self.real = real  
        self.imag = imag
```

```
    @property
```

```
    def magnitude(self):  
        return (self.real ** 2 + self.imag ** 2) ** 0.5
```

Property decorator: "Call this function on attribute look-up"

## The Rectangular Representation

---

```
class ComplexRI:
```

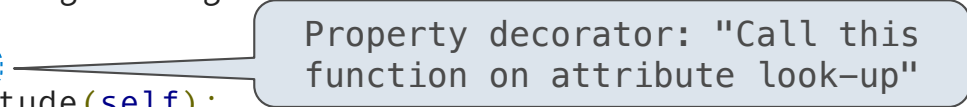
```
    def __init__(self, real, imag):  
        self.real = real  
        self.imag = imag
```

```
    @property
```

```
    def magnitude(self):  
        return (self.real ** 2 + self.imag ** 2) ** 0.5
```

```
    @property
```

```
    def angle(self):  
        return atan2(self.imag, self.real)
```



Property decorator: "Call this function on attribute look-up"



## The Rectangular Representation

---

```
class ComplexRI:
```

```
    def __init__(self, real, imag):  
        self.real = real  
        self.imag = imag
```

```
    @property
```

```
    def magnitude(self):  
        return (self.real ** 2 + self.imag ** 2) ** 0.5
```

```
    @property
```

```
    def angle(self):  
        return atan2(self.imag, self.real)
```

Property decorator: "Call this function on attribute look-up"

math.atan2(y,x): Angle between x-axis and the point (x,y)

## The Rectangular Representation

---

```
class ComplexRI:
```

```
    def __init__(self, real, imag):  
        self.real = real  
        self.imag = imag
```

```
    @property
```

```
    def magnitude(self):  
        return (self.real ** 2 + self.imag ** 2) ** 0.5
```

```
    @property
```

```
    def angle(self):  
        return atan2(self.imag, self.real)
```

```
    def __repr__(self):  
        return 'ComplexRI({0}, {1})'.format(self.real,  
                                            self.imag)
```

Property decorator: "Call this function on attribute look-up"

math.atan2(y,x): Angle between x-axis and the point (x,y)

## The Polar Representation

---

## The Polar Representation

---

```
class ComplexMA:
```

## The Polar Representation

---

```
class ComplexMA:  
    def __init__(self, magnitude, angle):  
        self.magnitude = magnitude  
        self.angle = angle
```

## The Polar Representation

---

```
class ComplexMA:

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)
```

## The Polar Representation

---

```
class ComplexMA:

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)
```

## The Polar Representation

---

```
class ComplexMA:

    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)

    def __repr__(self):
        return 'ComplexMA({0}, {1})'.format(self.magnitude,
                                           self.angle)
```



## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

```
>>> def add_complex(z1, z2):  
    return ComplexRI(z1.real + z2.real,  
                    z1.imag + z2.imag)  
  
>>> def mul_complex(z1, z2):  
    return ComplexMA(z1.magnitude * z2.magnitude,  
                    z1.angle + z2.angle)
```

## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

```
>>> def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

>>> def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)

>>> from math import pi
```

## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

```
>>> def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

>>> def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)

>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
```

## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

```
>>> def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

>>> def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)

>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
```

## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

```
>>> def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                     z1.imag + z2.imag)

>>> def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                     z1.angle + z2.angle)

>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
>>> mul_complex(ComplexRI(0, 1), ComplexRI(0, 1))
```

## Using Complex Numbers

---

Either type of complex number can be passed as either argument to `add_complex` or `mul_complex`:

```
>>> def add_complex(z1, z2):
    return ComplexRI(z1.real + z2.real,
                    z1.imag + z2.imag)

>>> def mul_complex(z1, z2):
    return ComplexMA(z1.magnitude * z2.magnitude,
                    z1.angle + z2.angle)

>>> from math import pi
>>> add_complex(ComplexRI(1, 2), ComplexMA(2, pi/2))
ComplexRI(1.0000000000000002, 4.0)
>>> mul_complex(ComplexRI(0, 1), ComplexRI(0, 1))
ComplexMA(1.0, 3.141592653589793)
```