61A Lecture 10

Wednesday, September 25

Announcements

Announcements

- Homework 3 due Tuesday 10/1 @ 11:59pm
- Optional Hog Contest entries due Thursday 10/3 @ 11:59pm
- Composition scores will be assigned this week (perhaps by Monday).
  - 3/3 is very rare on the first project.
  - You can gain back any points you lose on the first project by revising it (November).

Data

Data Types

Every value has a type

(demo)

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Properties of native data types:

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Numeric types in Python:

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Represents integers exactly

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Represents integers exactly

```
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Represents real numbers approximately

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Objects

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All Programmers

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All Programmers

Great Programmer

Rational Numbers

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$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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Exact representation of fractions

Rational Numbers

$$\frac{\texttt{numerator}}{\texttt{denominator}}$$

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A pair of integers

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As soon as division occurs, the exact representation may be lost!

Rational Numbers

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Assume we can compose and decompose rational numbers:

Rational Numbers

numerator
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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

- `rational(n, d)` *returns a rational number* `x`

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numerator
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Selectors

- `numer(x)` *returns the numerator of* x
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Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy}$$

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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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Rational Number Arithmetic Implementation

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Rational Number Arithmetic Implementation

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def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
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```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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```
def equal_rational(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

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- rational(n, d) *returns a rational number* x
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These functions implement an *abstract data type* for rational numbers

Pairs

Pairs as Tuples

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>>> pair = (1, 2)
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>>> x, y = pair
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>>> x
```

Pairs as Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
```

Pairs as Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)


>>> x, y = pair
>>> x
1
>>> y
```

Pairs as Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2
```

Pairs as Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2
```

Pairs as Tuples

```
>>> pair = (1, 2)
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(1, 2)
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```
>>> x, y = pair
>>> x
1
>>> y
2
```

Pairs as Tuples

```
>>> pair = (1, 2)
>>> pair
(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
```

Pairs as Tuples

```
>>> pair = (1, 2)
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(1, 2)

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

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```
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>>> pair[1]
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>>> y
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```

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(1, 2)
```

```
>>> x, y = pair
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>>> y
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```
>>> pair[0]
1
>>> pair[1]
2
>>> from operator import getitem
```

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(1, 2)
```

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>>> pair[1]
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>>> from operator import getitem
>>> getitem(pair, 0)
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A tuple literal:
Comma-separated expression

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"Unpacking" a tuple

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Element selection

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Element selection

More tuples next lecture

Representing Rational Numbers

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    """Construct a rational number x that represents n/d."""
    return (n, d)
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Construct a tuple

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from operator import getitem

def numer(x):
    """Return the numerator of rational number x."""
    return getitem(x, 0)
```

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    """Construct a rational number x that represents n/d."""
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Construct a tuple

```
from operator import getitem

def numer(x):
    """Return the numerator of rational number x."""
    return getitem(x, 0)

def denom(x):
    """Return the denominator of rational number x."""
    return getitem(x, 1)
```

Representing Rational Numbers

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def rational(n, d):
    """Construct a rational number x that represents n/d."""
    return (n, d)
```

Construct a tuple

```
from operator import getitem

def numer(x):
    """Return the numerator of rational number x."""
    return getitem(x, 0)

def denom(x):
    """Return the denominator of rational number x."""
    return getitem(x, 1)
```

Select from a tuple

Reducing to Lowest Terms

Example:

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$$\frac{3}{2} * \frac{5}{3}$$

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Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10}$$

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```
from fractions import gcd
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from fractions import gcd

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from fractions import gcd   # Greatest common divisor

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```

Abstraction Barriers

Abstraction Barriers

Rational numbers as whole data values

`add_rational  mul_rational  equal_rational`

Rational numbers as numerators & denominators

`rational  numer  denom`

Rational numbers as tuples

`tuple  getitem`

However tuples are implemented in Python

Violating Abstraction Barriers

```
add_rational( (1, 2), (1, 4) )

def divide_rational(x, y):
    return (x[0] * y[1], x[1] * y[0])
```

Violating Abstraction Barriers

Does not use constructors

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No selectors!

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Violating Abstraction Barriers

Data Representations

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You can recognize abstract data types by their behavior, not by their class

Behavior Conditions of a Pair

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(Demo)

Functional Pair Implementation

Example: http://goo.gl/9hVt8f

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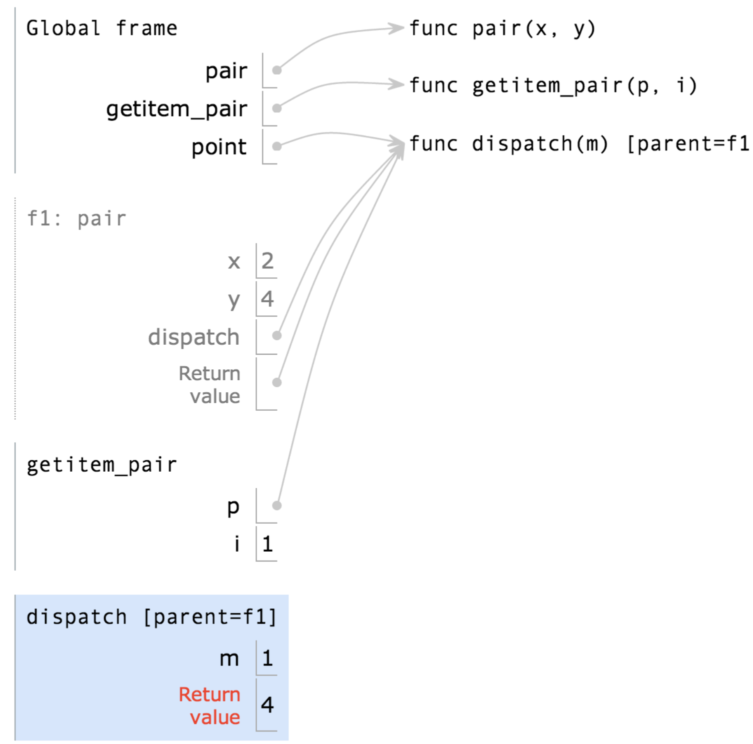

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Using a Functionally Implemented Pair

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This pair representation is valid!