61A Lecture 8

Wednesday, September 18

Announcements

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm–4pm and 4pm–6pm in 1 Pimentel

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm–4pm and 4pm–6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm-4pm and 4pm-6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB
  - Extra office hours over the weekend

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm-4pm and 4pm-6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB
  - Extra office hours over the weekend
  - Includes topics up to and including this lecture

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm-4pm and 4pm-6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB
  - Extra office hours over the weekend
  - Includes topics up to and including this lecture
  - Fill out the form on the website if you cannot attend

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm-4pm and 4pm-6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB
  - Extra office hours over the weekend
  - Includes topics up to and including this lecture
  - Fill out the form on the website if you cannot attend
- Homework 3 is due in two weeks: Tuesday 10/1 @ 11:59pm

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm-4pm and 4pm-6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB
  - Extra office hours over the weekend
  - Includes topics up to and including this lecture
  - Fill out the form on the website if you cannot attend
- Homework 3 is due in two weeks: Tuesday 10/1 @ 11:59pm
  - It contains lots of recursion problems, for practice!

Announcements

- Project 1 is due Thursday 9/19 @ 11:59pm
- Midterm 1 is on Monday 9/23 from 7pm to 9pm
  - 2 review sessions on Saturday 9/21 2pm-4pm and 4pm-6pm in 1 Pimentel
  - HKN review session on Sunday 9/22 from 4pm to 7pm in 2050 Valley LSB
  - Extra office hours over the weekend
  - Includes topics up to and including this lecture
  - Fill out the form on the website if you cannot attend
- Homework 3 is due in two weeks: Tuesday 10/1 @ 11:59pm
  - It contains lots of recursion problems, for practice!
- Optional Hog strategy contest ends Thursday 10/3 @ 11:59pm

Hog Contest Rules

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.
- To enter: *submit proj1contest* with a file hog.py that defines a final_strategy function by **Thursday 10/3 @ 11:59pm**

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.
- To enter: *submit proj1contest* with a file hog.py that defines a final_strategy function by **Thursday 10/3 @ 11:59pm**
- All winning entries will receive 2 points of extra credit

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.
- To enter: *submit proj1contest* with a file hog.py that defines a final_strategy function by **Thursday 10/3 @ 11:59pm**
- All winning entries will receive 2 points of extra credit
- The real prize: honor and glory

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.
- To enter: *submit proj1contest* with a file hog.py that defines a final_strategy function by **Thursday 10/3 @ 11:59pm**
- All winning entries will receive 2 points of extra credit
- The real prize: honor and glory

Fall 2011 Winners

Keegan Mann,
Yan Duan & Ziming Li,
Brian Prike & Zhenghao Qian,
Parker Schuh & Robert Chatham

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.
- To enter: *submit proj1contest* with a file hog.py that defines a final_strategy function by **Thursday 10/3 @ 11:59pm**
- All winning entries will receive 2 points of extra credit
- The real prize: honor and glory

Fall 2011 Winners	**Fall 2012 Winners**
Keegan Mann, Yan Duan & Ziming Li, Brian Prike & Zhenghao Qian, Parker Schuh & Robert Chatham	Chenyang Yuan, Joseph Hui

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Hog Contest Rules

- Up to two people submit one entry; Max of one entry per person.
- Your score is the number of entries against which you win more than 50% of the time.
- All strategies must be deterministic, pure functions of the current player scores! *Non-deterministic strategies will be disqualified*.
- One more special rule: ***Ham Hijinks.*** Choose -1 to swap the 4-sided and 6-sided dice.
- To enter: *submit proj1contest* with a file hog.py that defines a final_strategy function by **Thursday 10/3 @ 11:59pm**
- All winning entries will receive 2 points of extra credit
- The real prize: honor and glory

Fall 2011 Winners

Keegan Mann,
Yan Duan & Ziming Li,
Brian Prike & Zhenghao Qian,
Parker Schuh & Robert Chatham

Fall 2012 Winners

Chenyang Yuan,
Joseph Hui

Fall 2013 Winners

YOUR NAME COULD BE HERE...
FOREVER!

http://inst.eecs.berkeley.edu/~cs61a/fa13/proj/hog_contest/hog_contest.html

Order of Recursive Calls

The Cascade Function

(Demo)

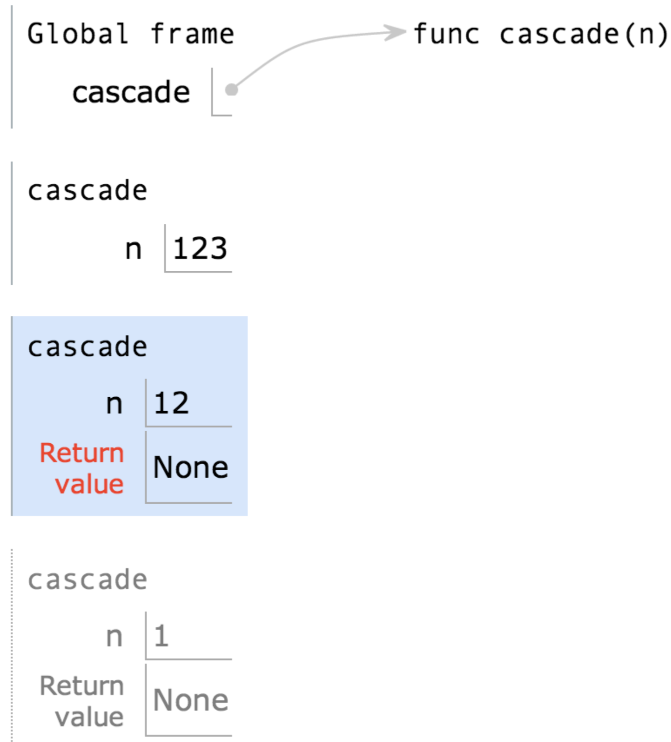
Example: http://goo.gl/O90qzK

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



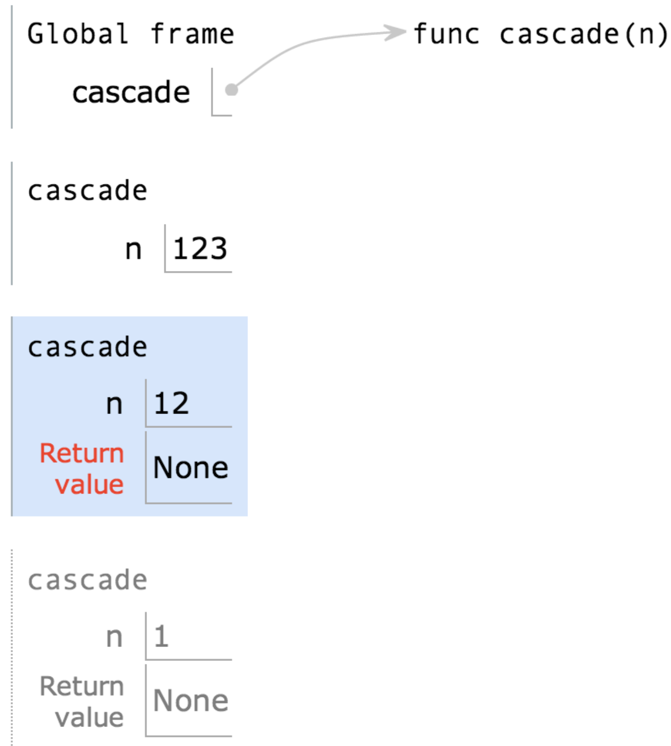
Example: http://goo.gl/O90qzK

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



Program output:

```
123
12
1
12
```

Example: http://goo.gl/O90qzK

The Cascade Function

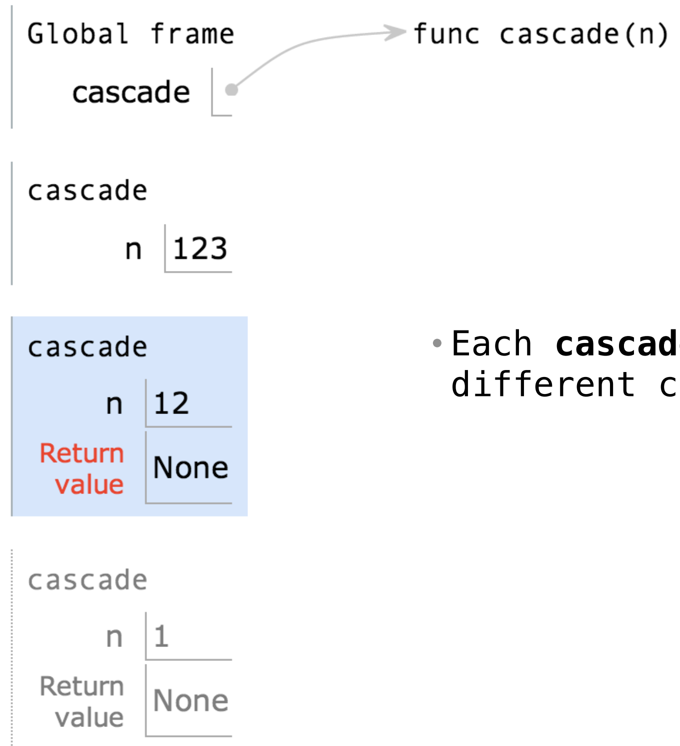
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each **cascade** frame is from a different call to **cascade.**

Example: http://goo.gl/O90qzK

The Cascade Function

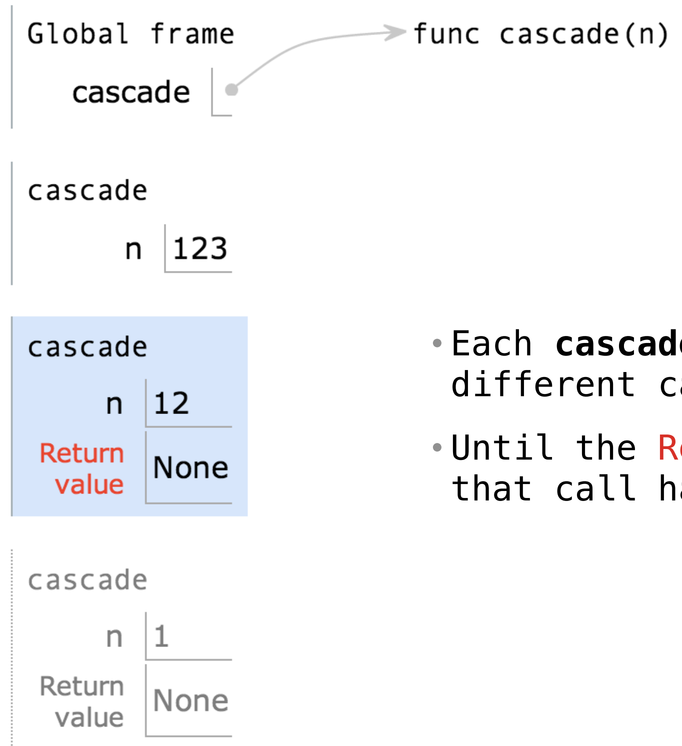
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each **cascade** frame is from a different call to **cascade.**
- Until the Return value appears, that call has not completed.

Example: http://goo.gl/O90qzK

The Cascade Function

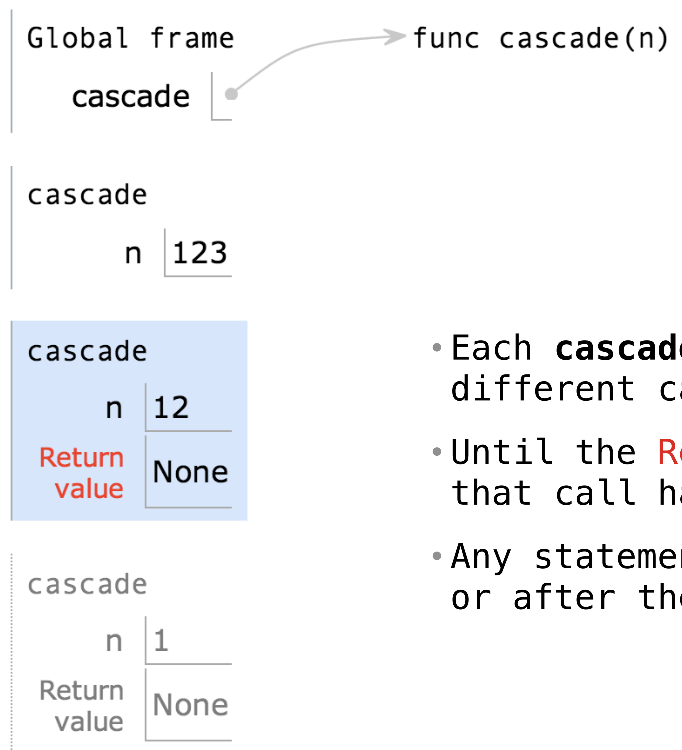
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each **cascade** frame is from a different call to **cascade.**
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Example: http://goo.gl/O90qzK

The Cascade Function

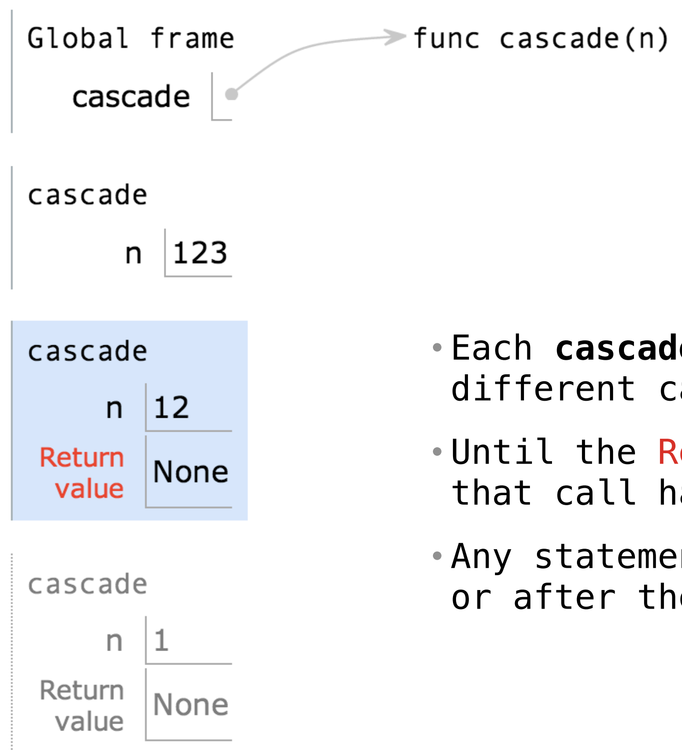
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each **cascade** frame is from a different call to **cascade.**
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Example: http://goo.gl/O90qzK

The Cascade Function

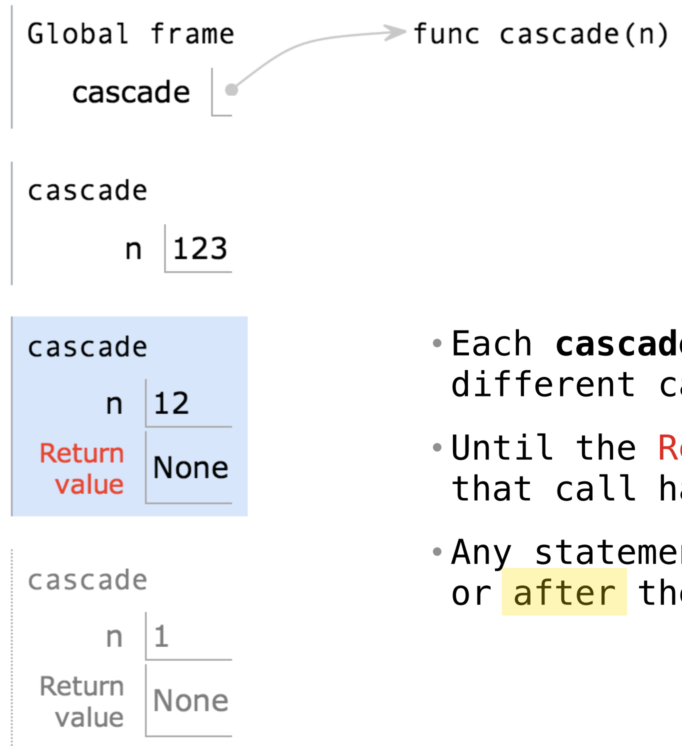
(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```

Program output:

```
123
12
1
12
```



- Each **cascade** frame is from a different call to **cascade.**
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

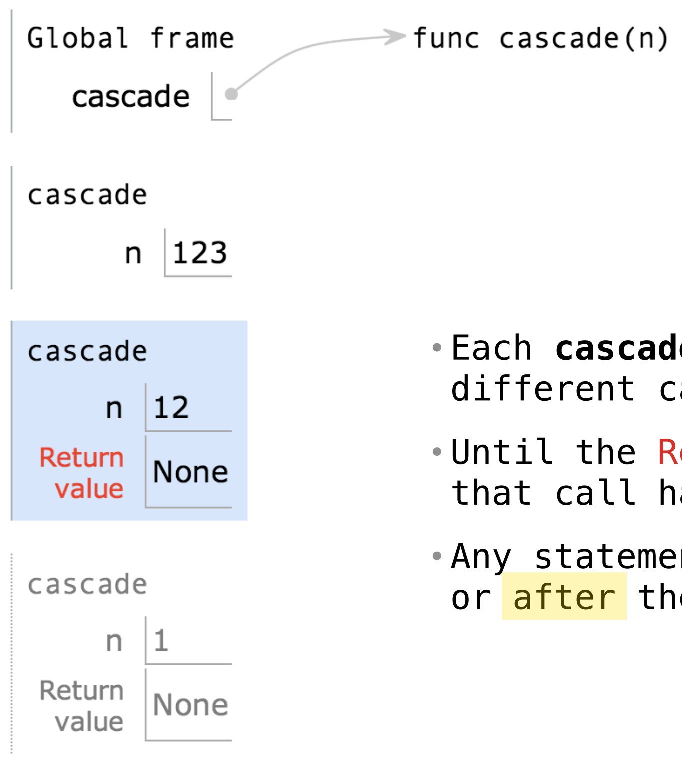
Example: http://goo.gl/O90qzK

The Cascade Function

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```



Program output:

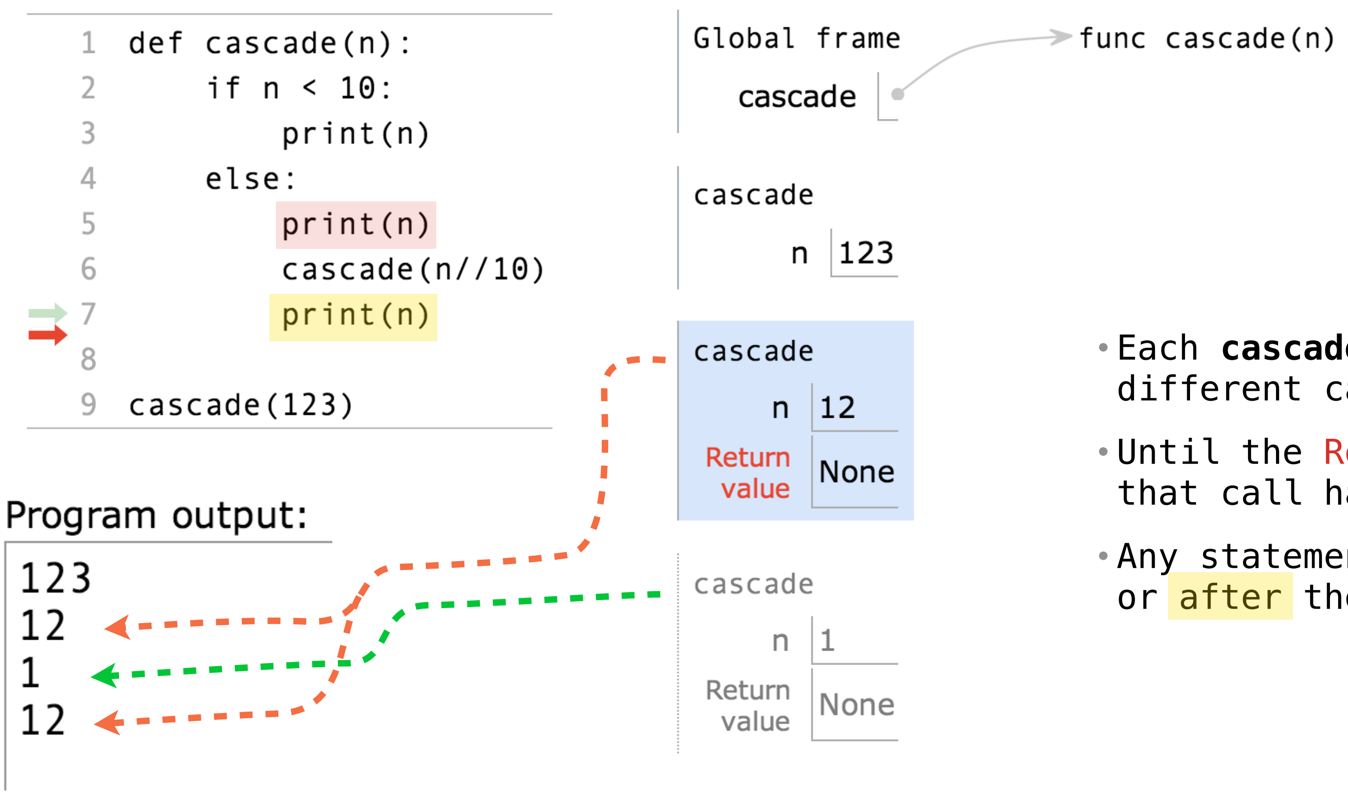
```
123
12
1
12
```

- Each **cascade** frame is from a different call to **cascade.**
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Example: http://goo.gl/O90qzK

The Cascade Function

(Demo)



- Each **cascade** frame is from a different call to **cascade.**
- Until the Return value appears, that call has not completed.
- Any statement can appear before or after the recursive call.

Example: http://goo.gl/O90qzK

Two Definitions of Cascade

(Demo)

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better.

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better.
- In this case, the longer implementation is more clear (at least to me).

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better.
- In this case, the longer implementation is more clear (at least to me).
- When learning to write recursive functions, put the base cases first.

Two Definitions of Cascade

(Demo)

```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)
```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n//10)
        print(n)
```

- If two implementations are equally clear, then shorter is usually better.
- In this case, the longer implementation is more clear (at least to me).
- When learning to write recursive functions, put the base cases first.
- Both are recursive functions, even though only the first has typical structure.

Tree Recursion

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n: 1, 2, 3, 4, 5, 6, 7, 8, 9,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1,	2,	3,	4,	5,	6,	7,	8,	9,
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n: 1, 2, 3, 4, 5, 6, 7, 8, 9, ... , 35

fib(n): 0, 1, 1, 2, 3, 5, 8, 13, 21,

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1,	2,	3,	4,	5,	6,	7,	8,	9,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	5,702,887

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1, 2, 3, 4, 5, 6, 7, 8, 9, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 5,702,887

```
def fib(n):
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1, 2, 3, 4, 5, 6, 7, 8, 9, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 5,702,887

```
def fib(n):
    if n == 1:
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1,	2,	3,	4,	5,	6,	7,	8,	9,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	5,702,887

```
def fib(n):
    if n == 1:
        return 0
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1, 2, 3, 4, 5, 6, 7, 8, 9, ... , 35
fib(n):	0, 1, 1, 2, 3, 5, 8, 13, 21, ... , 5,702,887

```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1,	2,	3,	4,	5,	6,	7,	8,	9,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	5,702,887

```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1,	2,	3,	4,	5,	6,	7,	8,	9,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	5,702,887

```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes **more than one** call to that function.

n:	1,	2,	3,	4,	5,	6,	7,	8,	9,	... ,	35
fib(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	... ,	5,702,887

```
def fib(n):
    if n == 1:
        return 0
    elif n == 2:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure

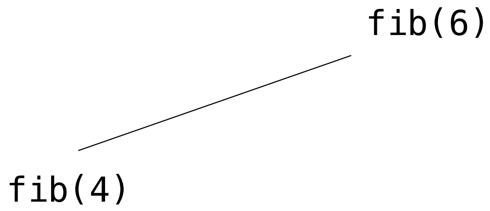
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure

fib(6)

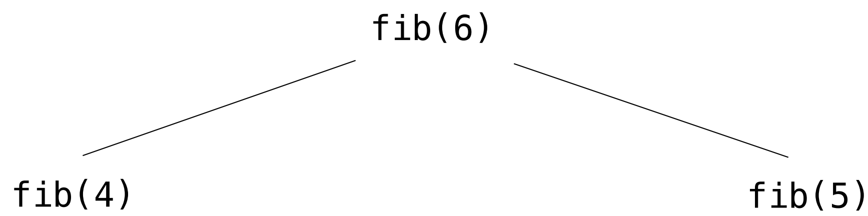
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



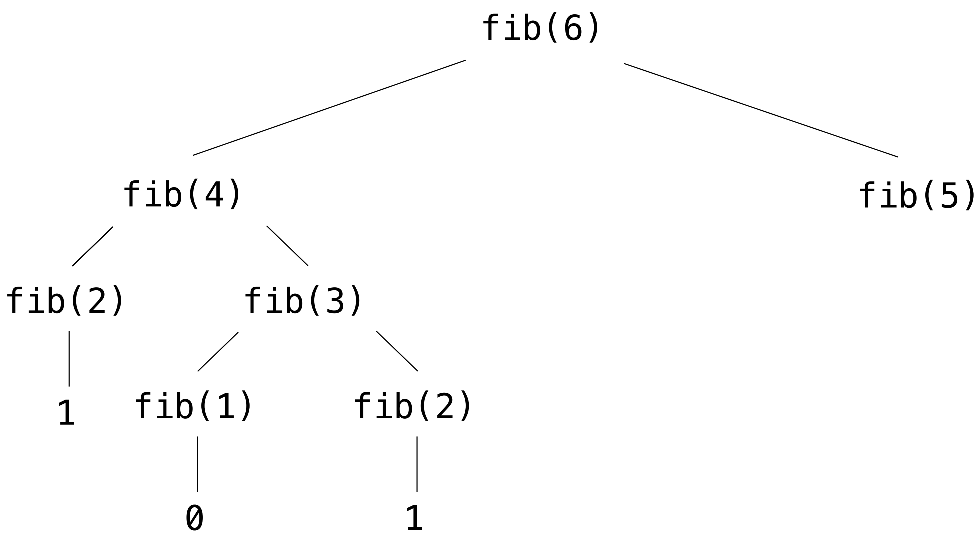
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



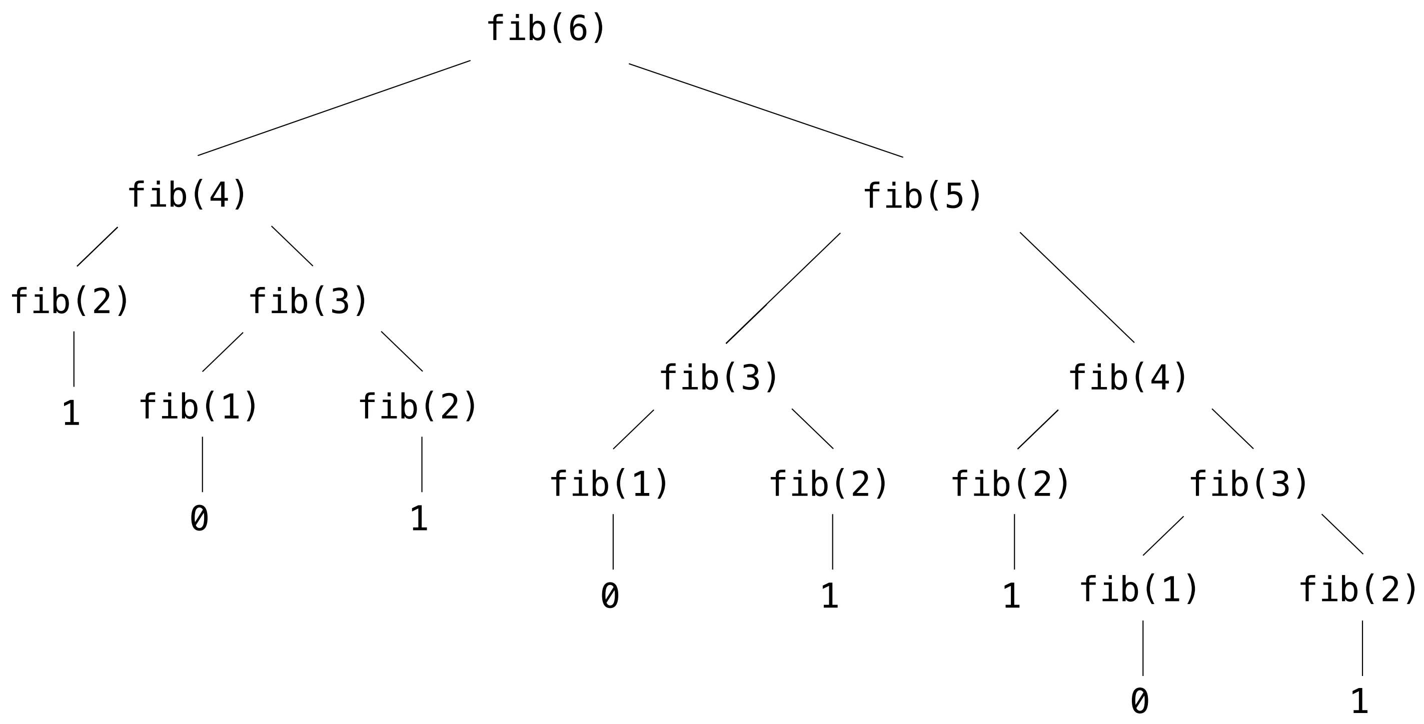
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



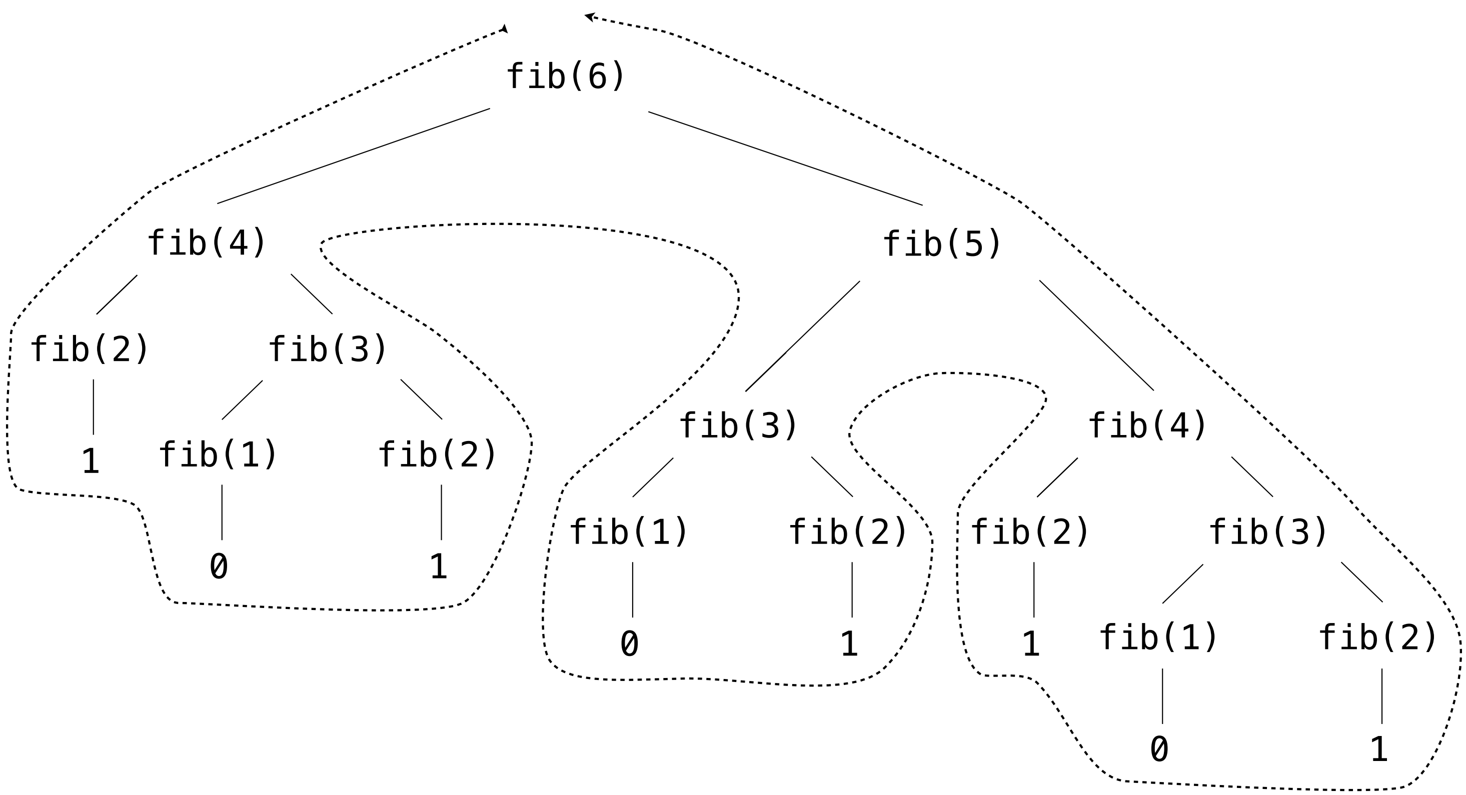
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



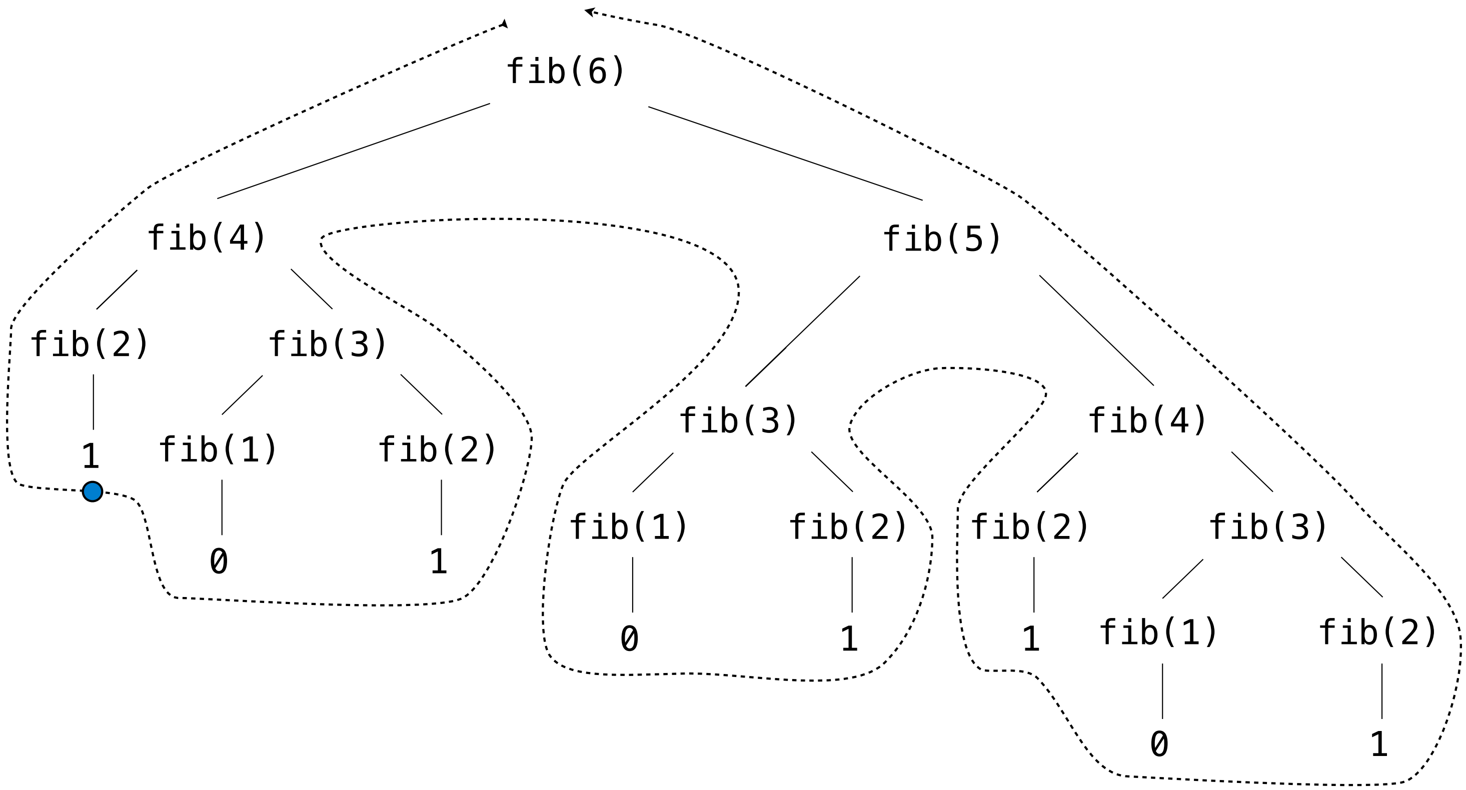
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



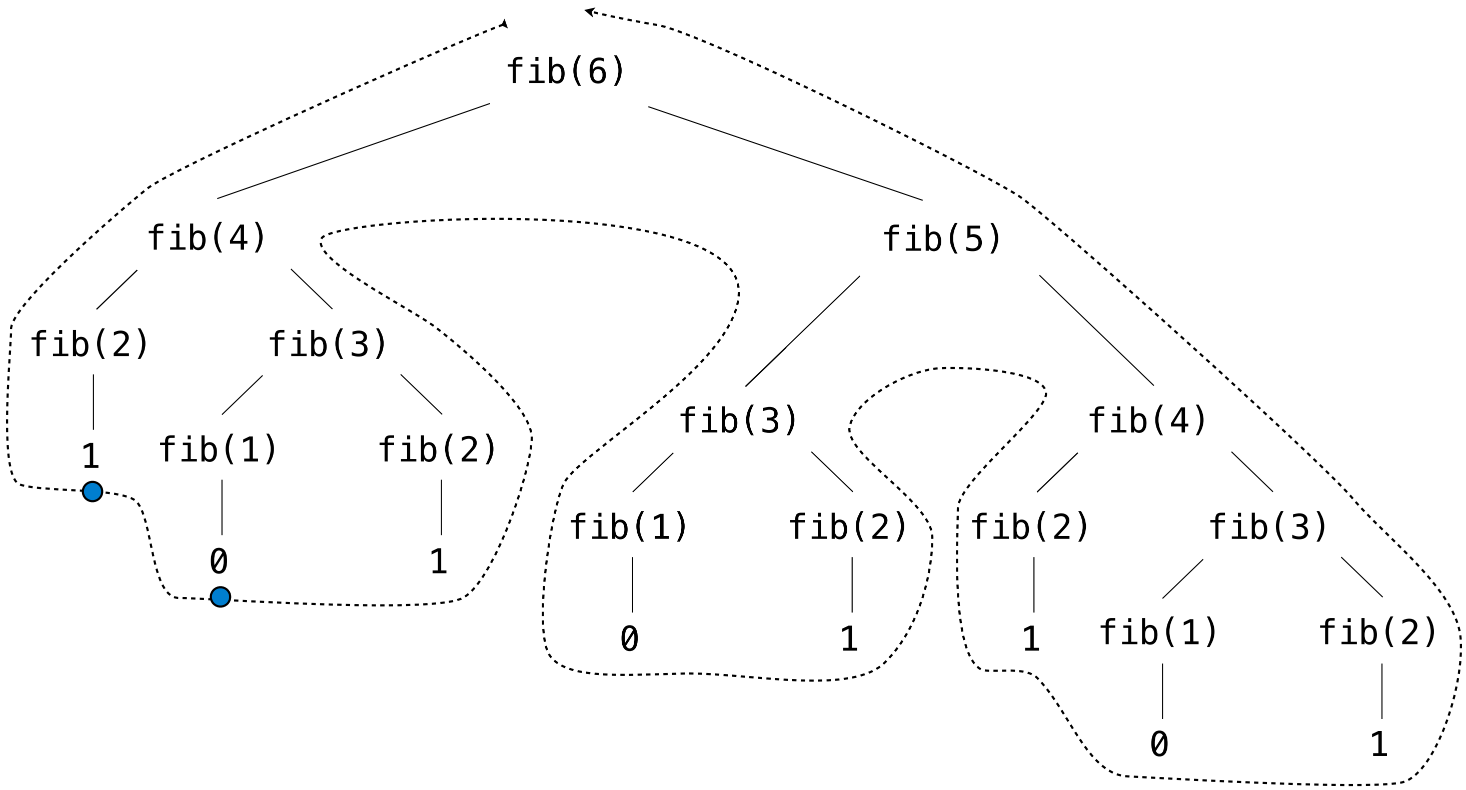
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



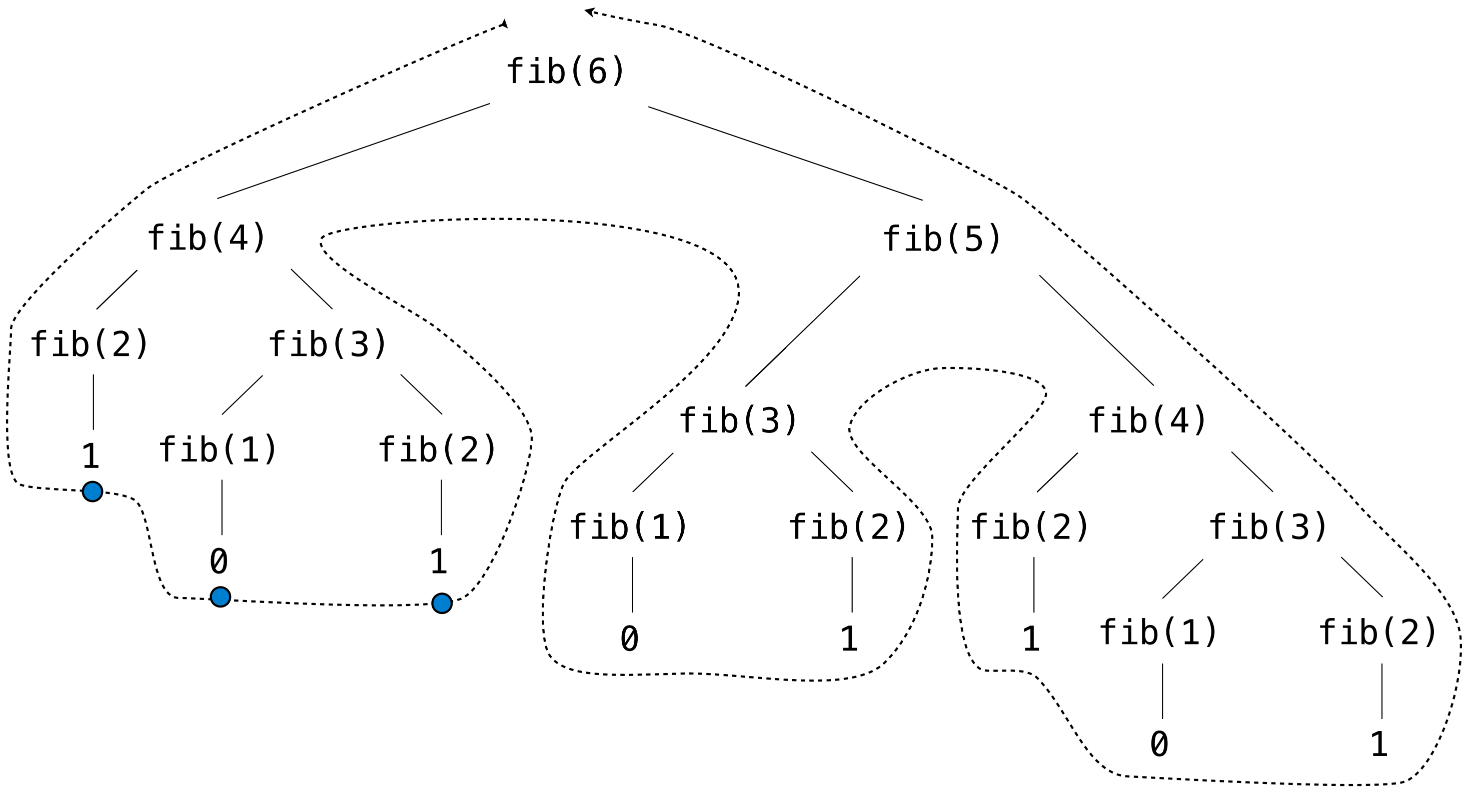
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



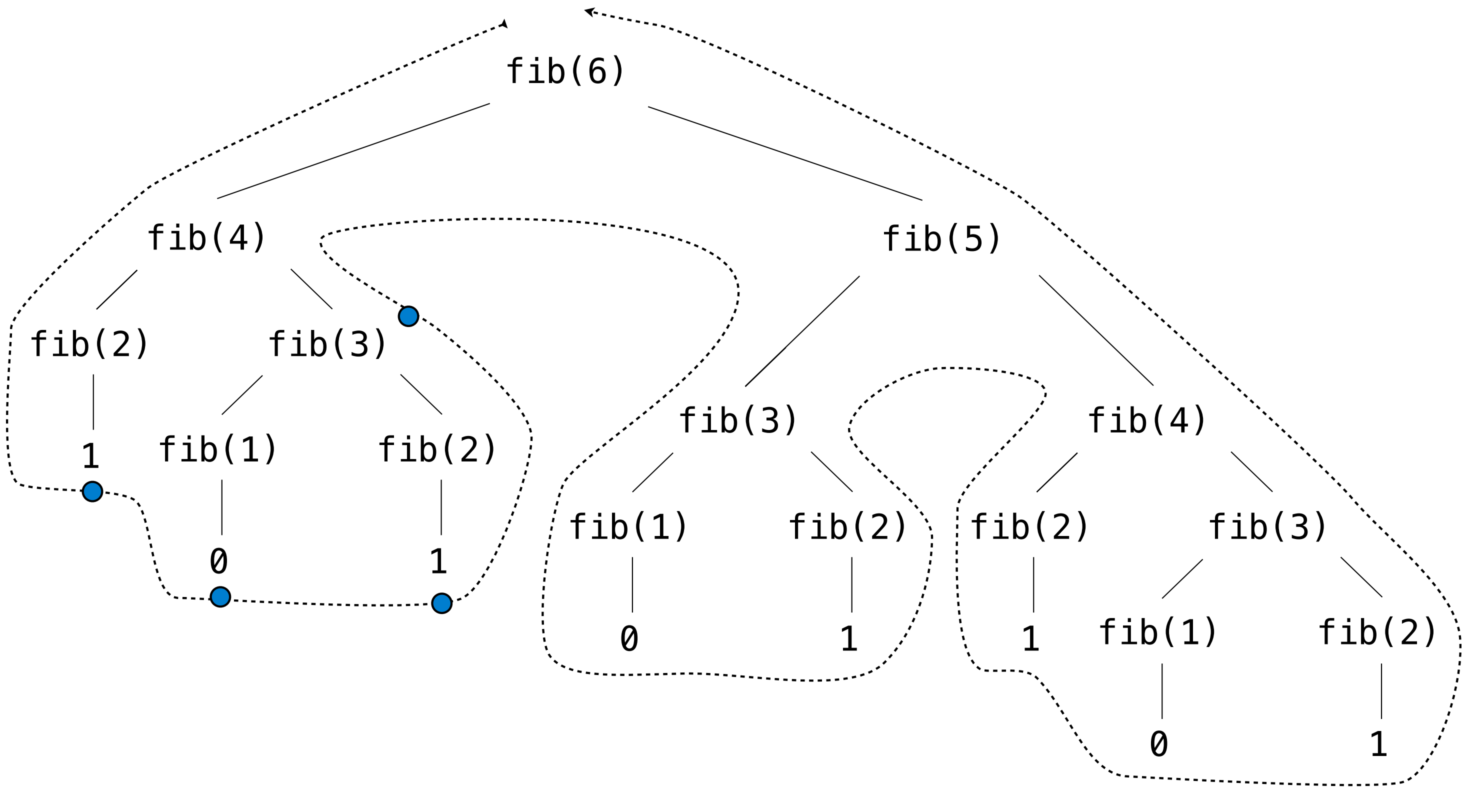
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



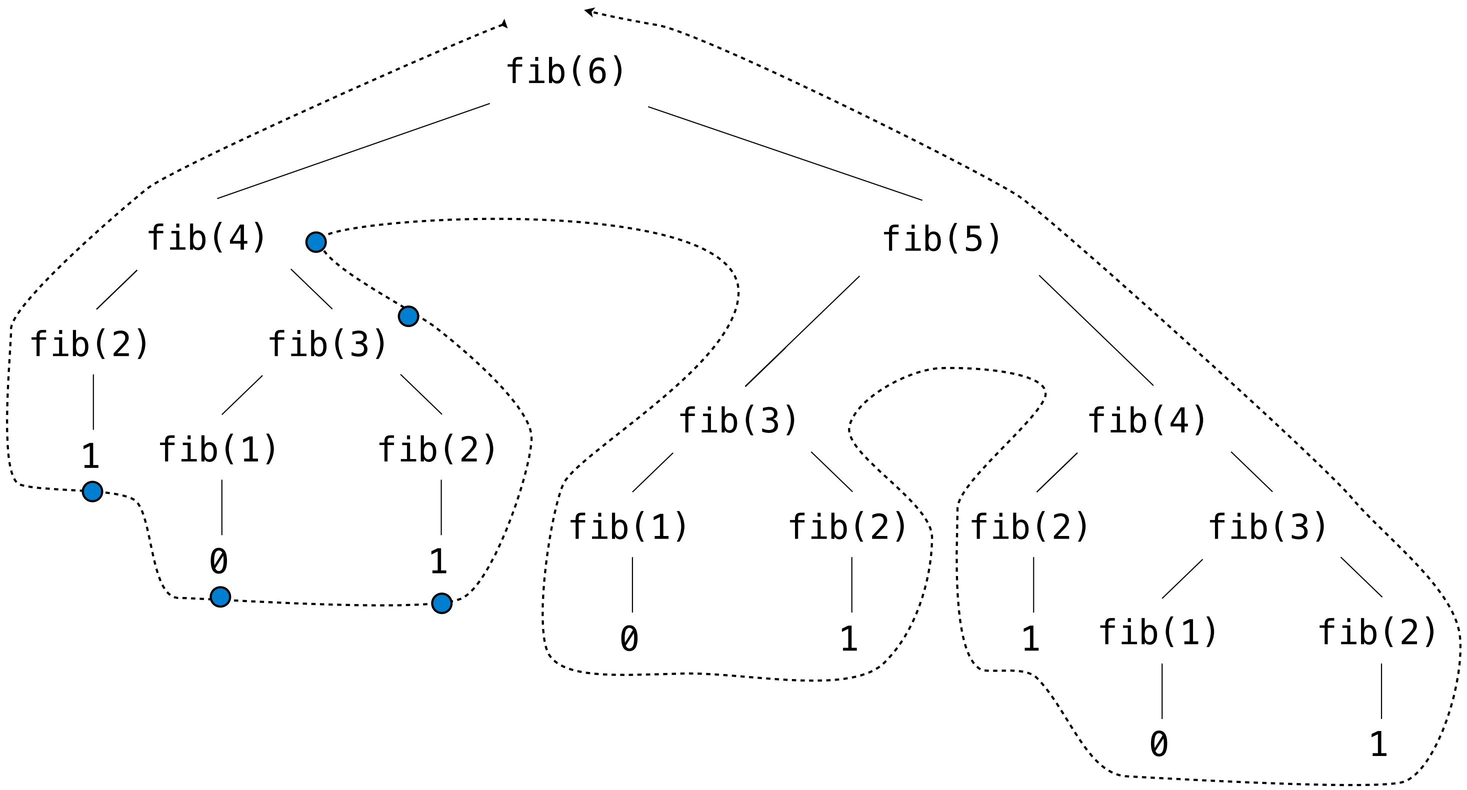
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



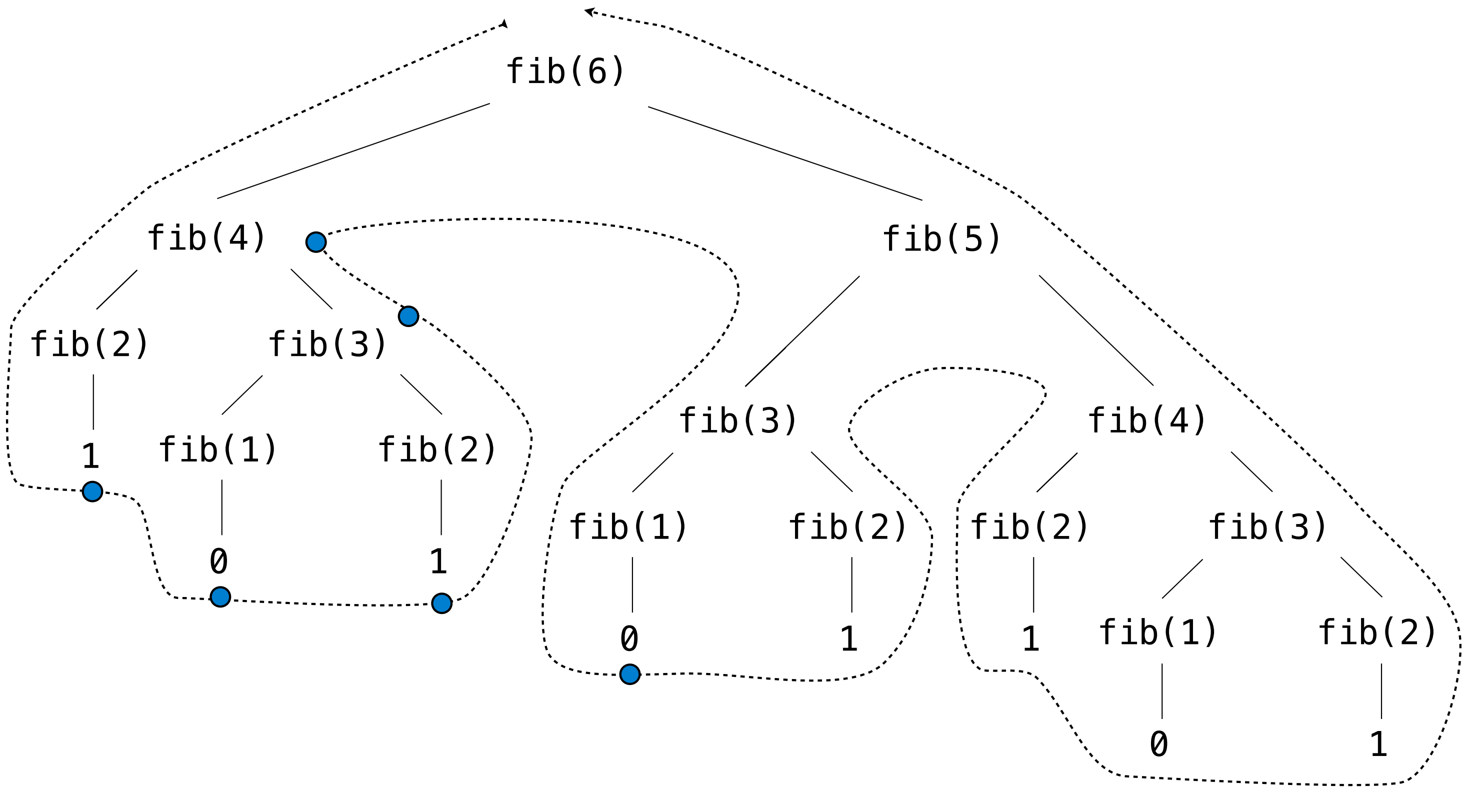
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



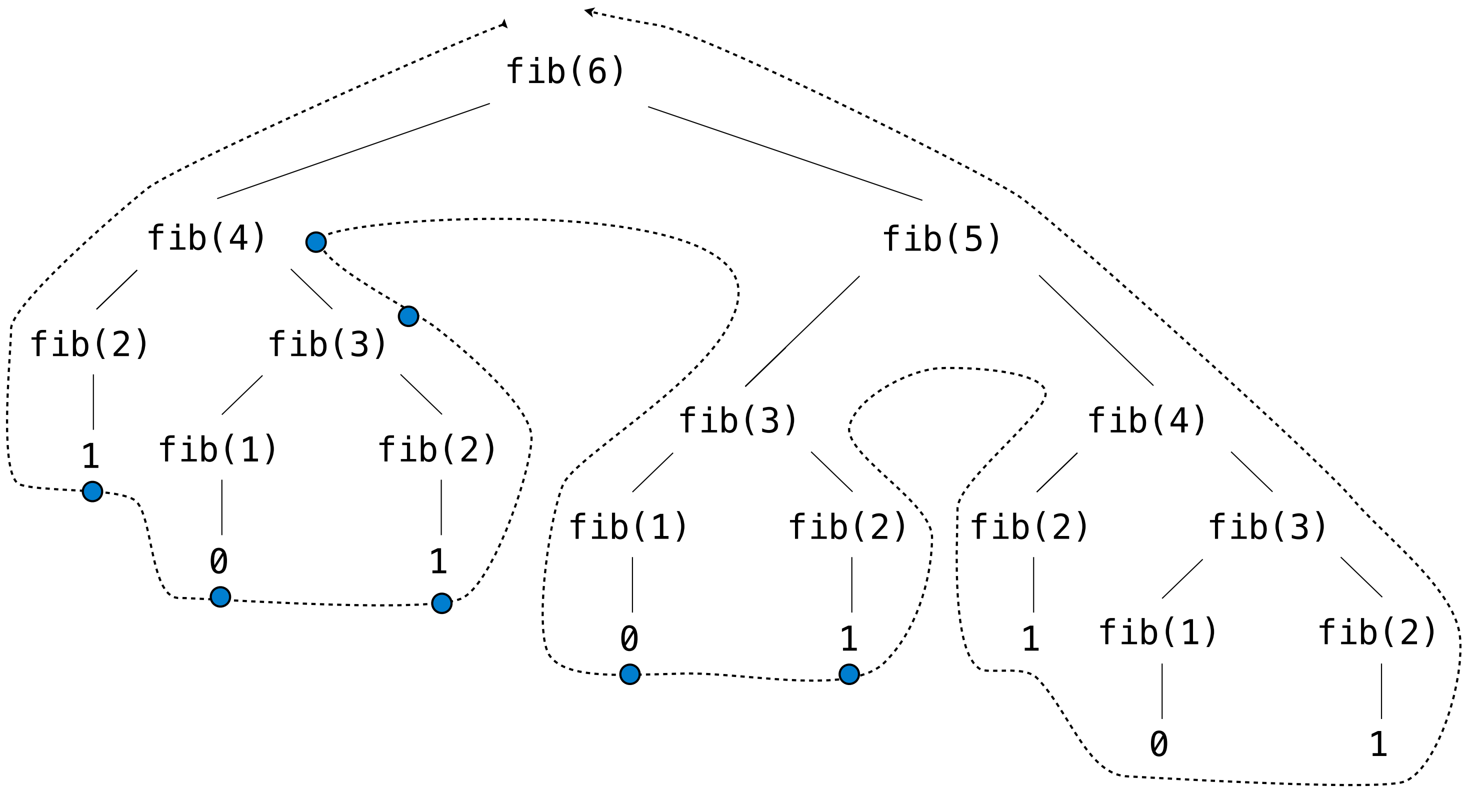
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



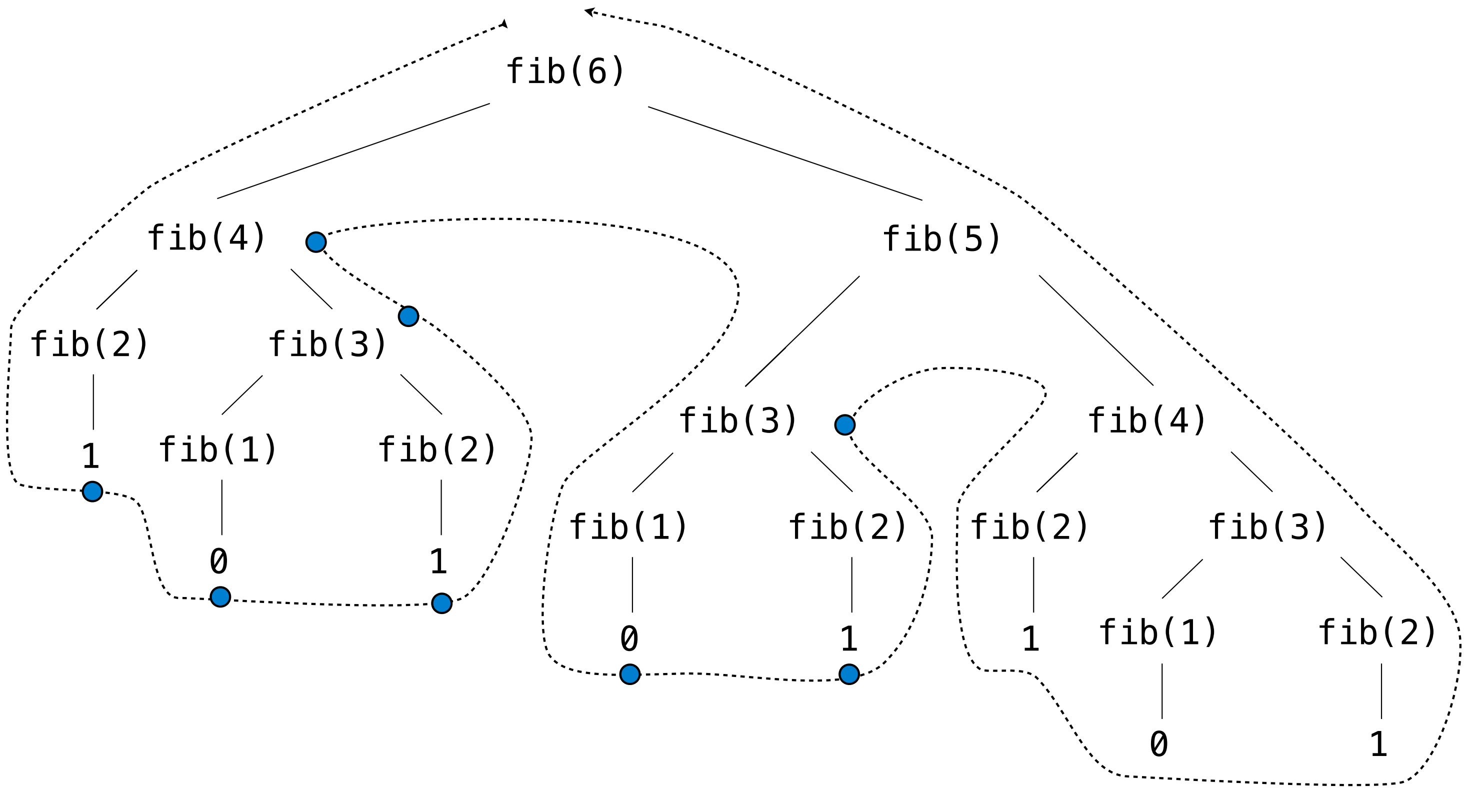
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



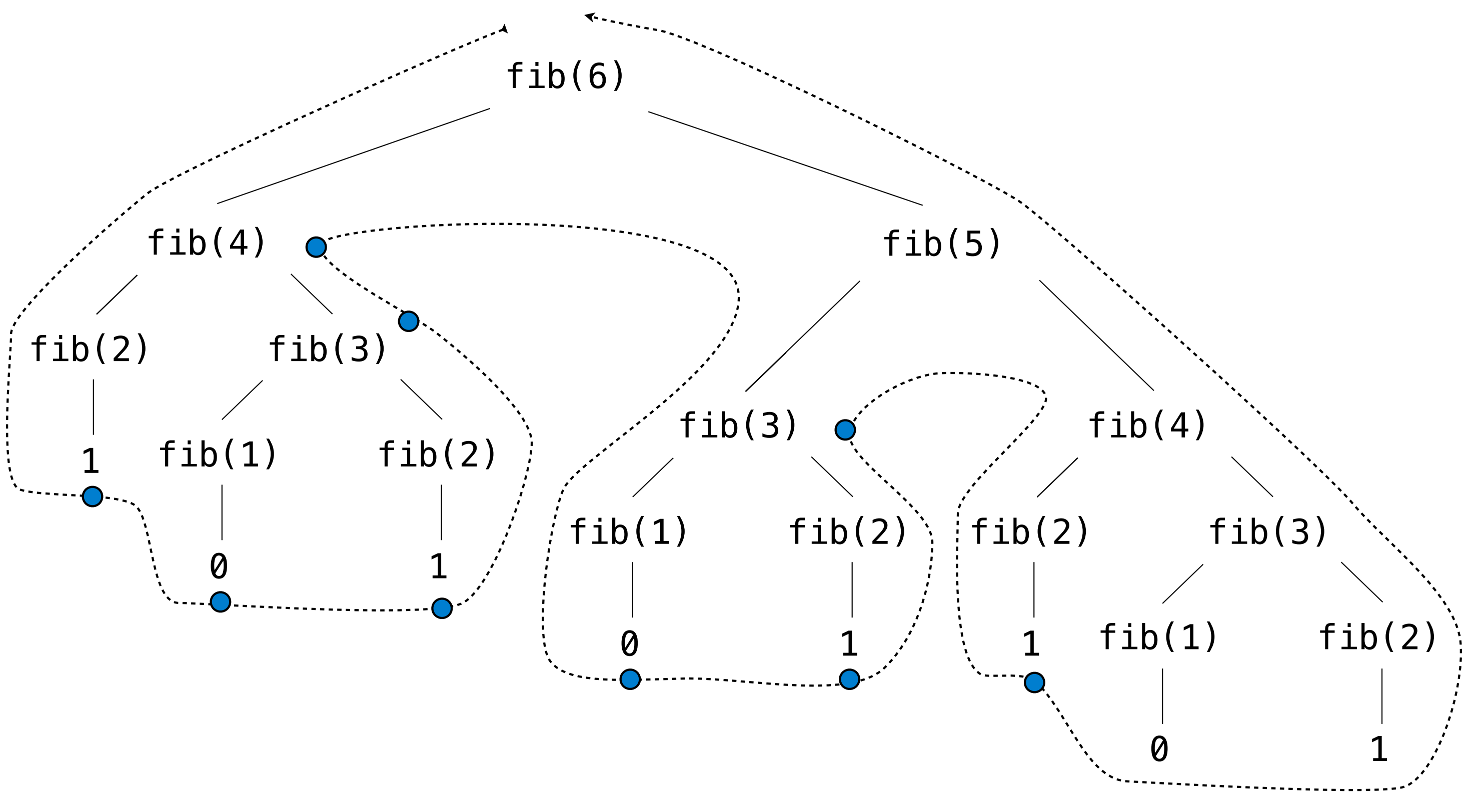
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



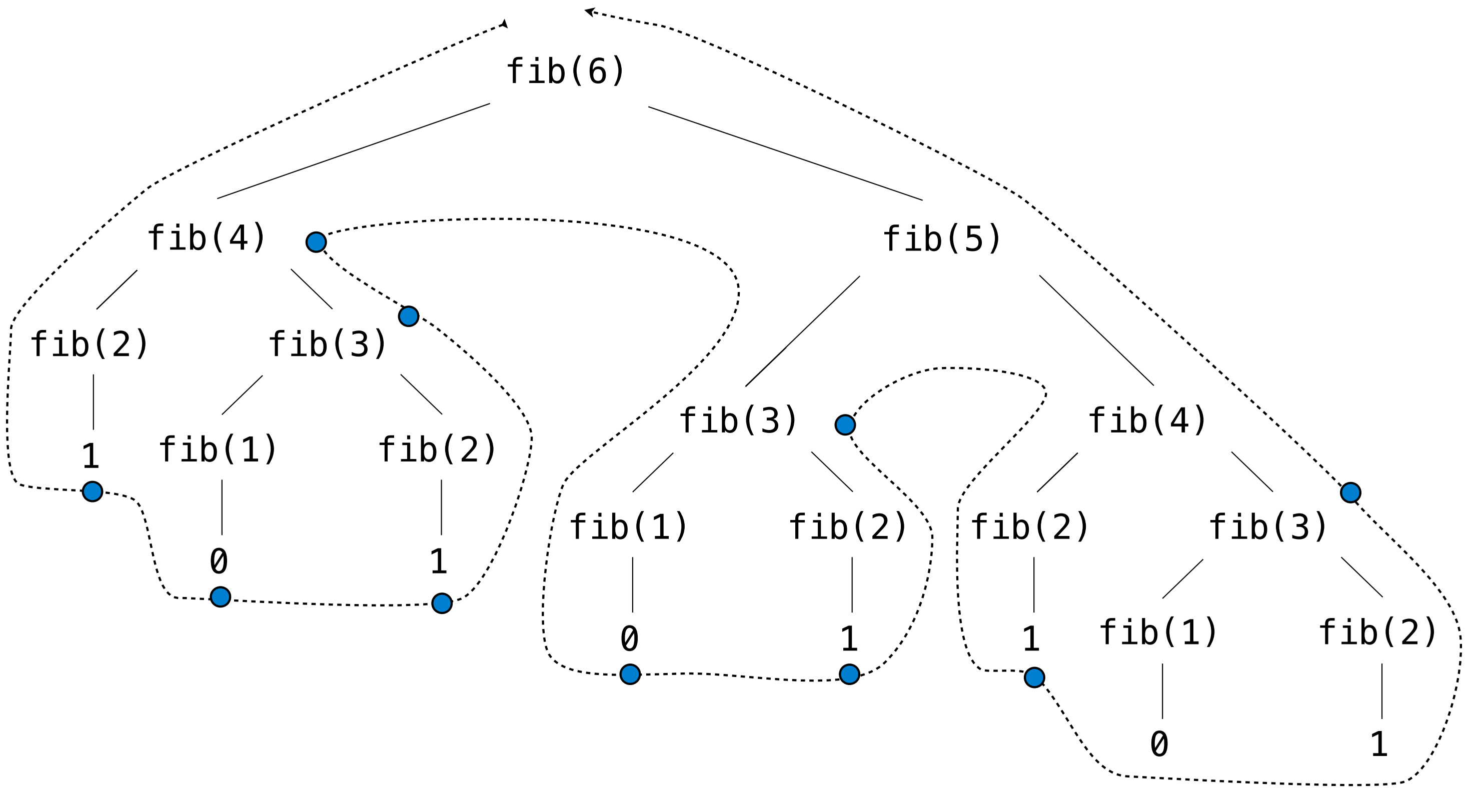
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



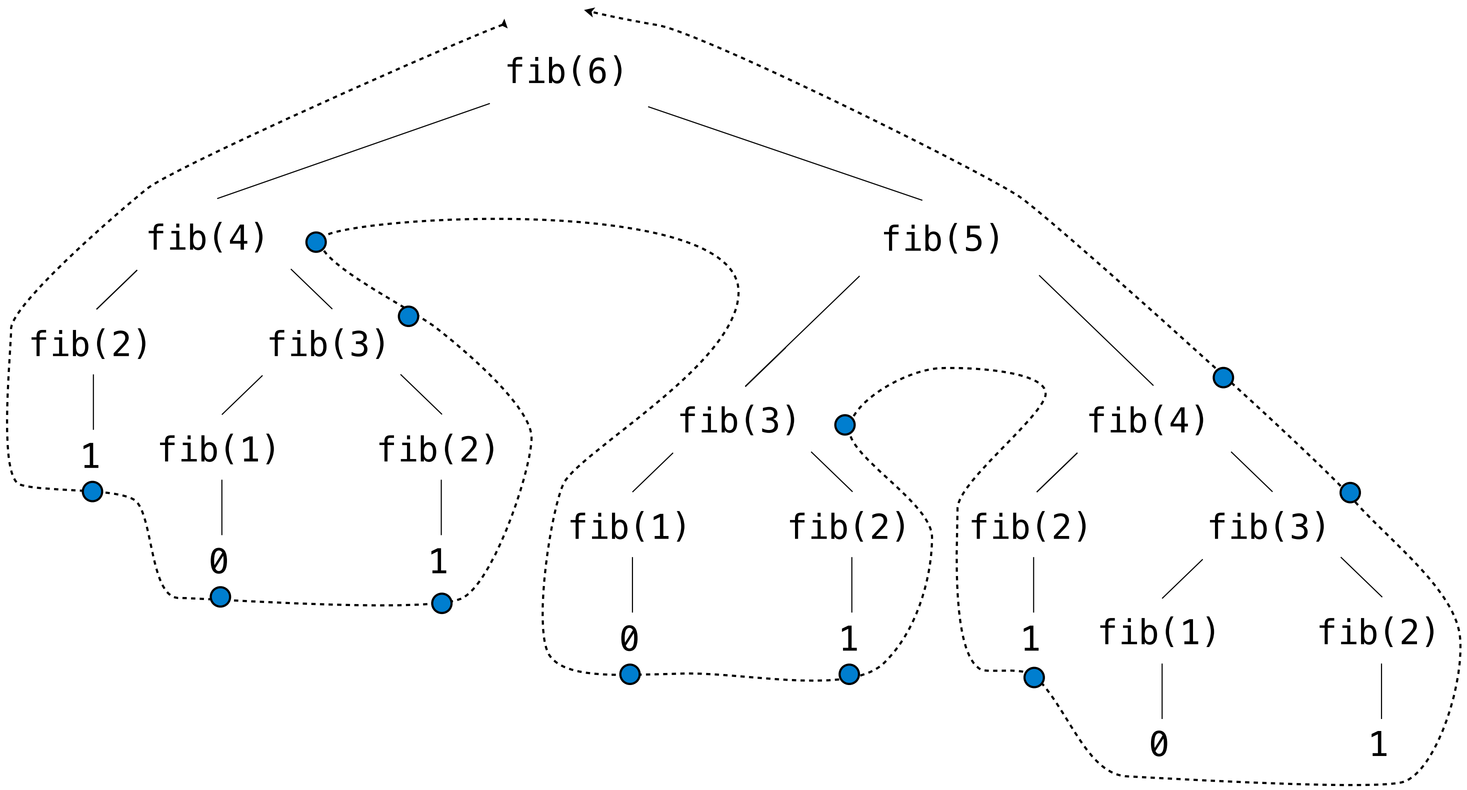
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



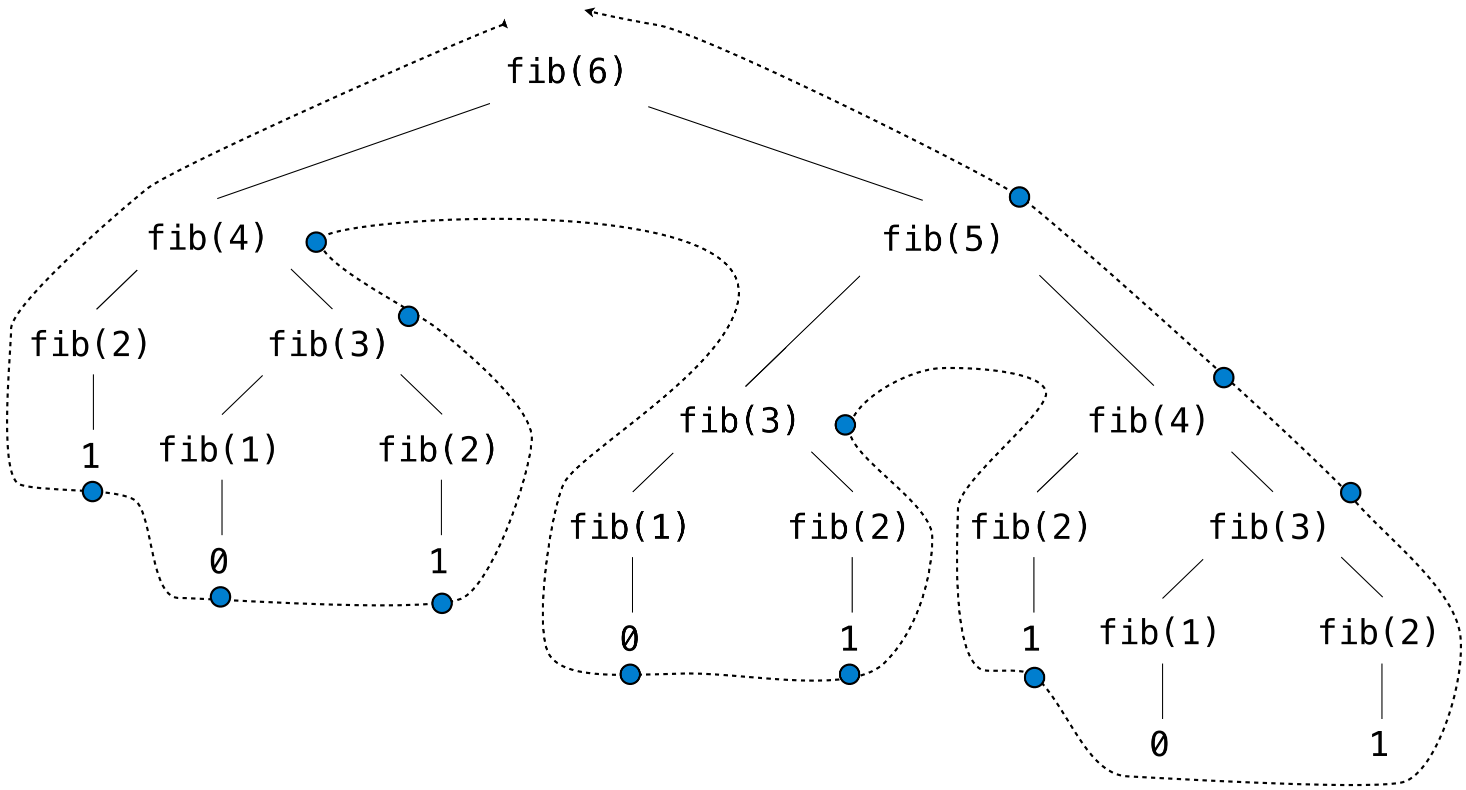
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



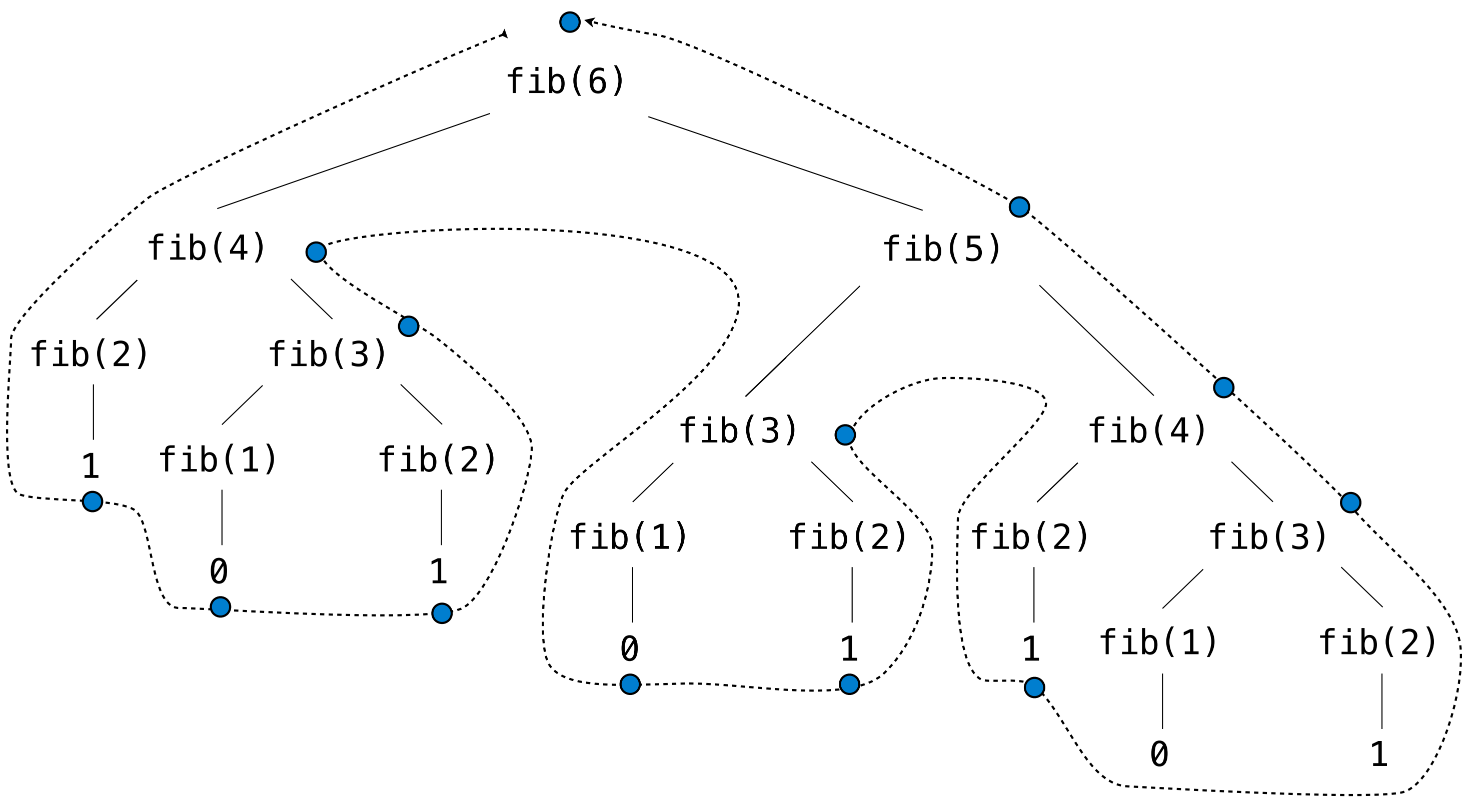
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



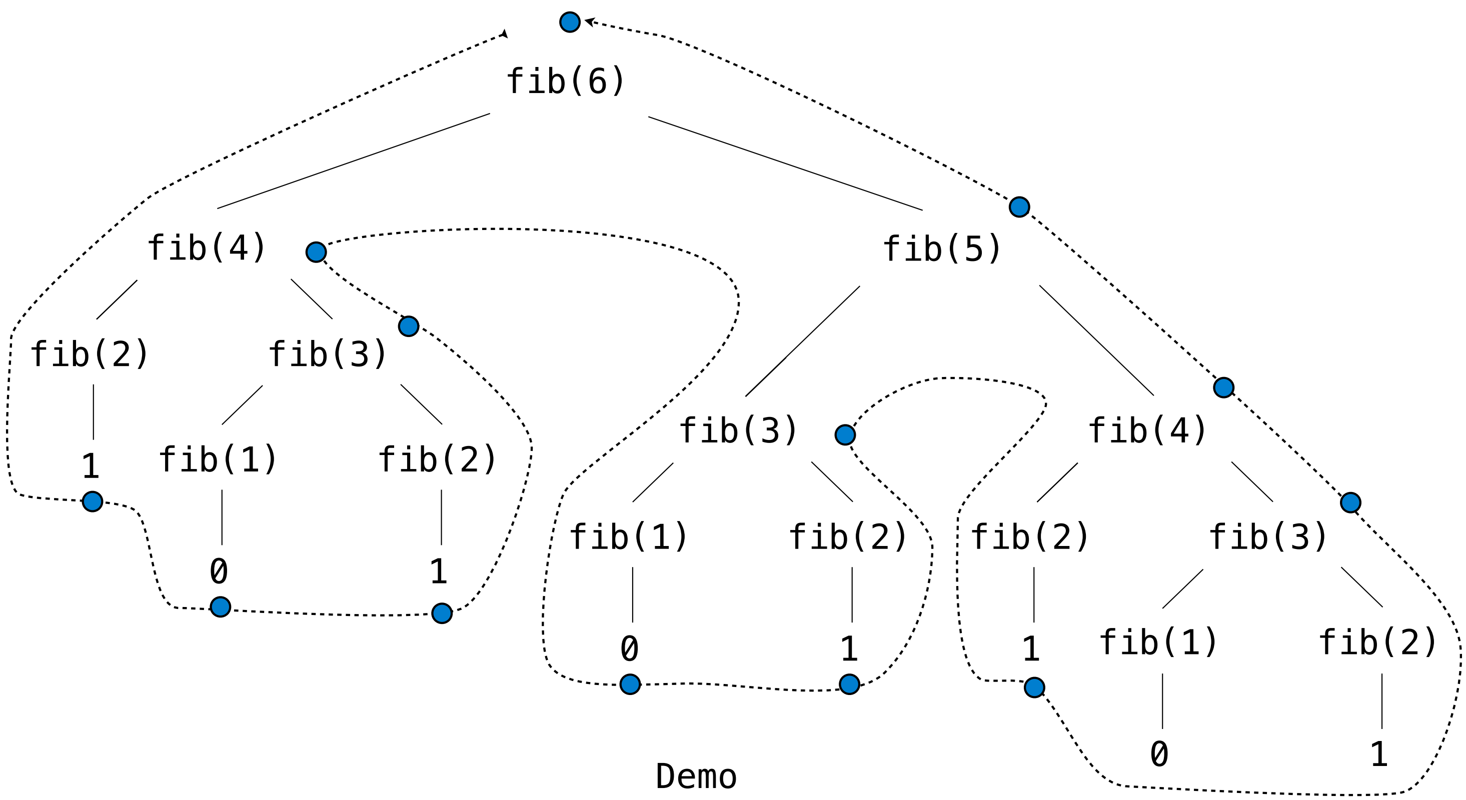
A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



A Tree-Recursive Process

The computational process of **fib** evolves into a tree structure



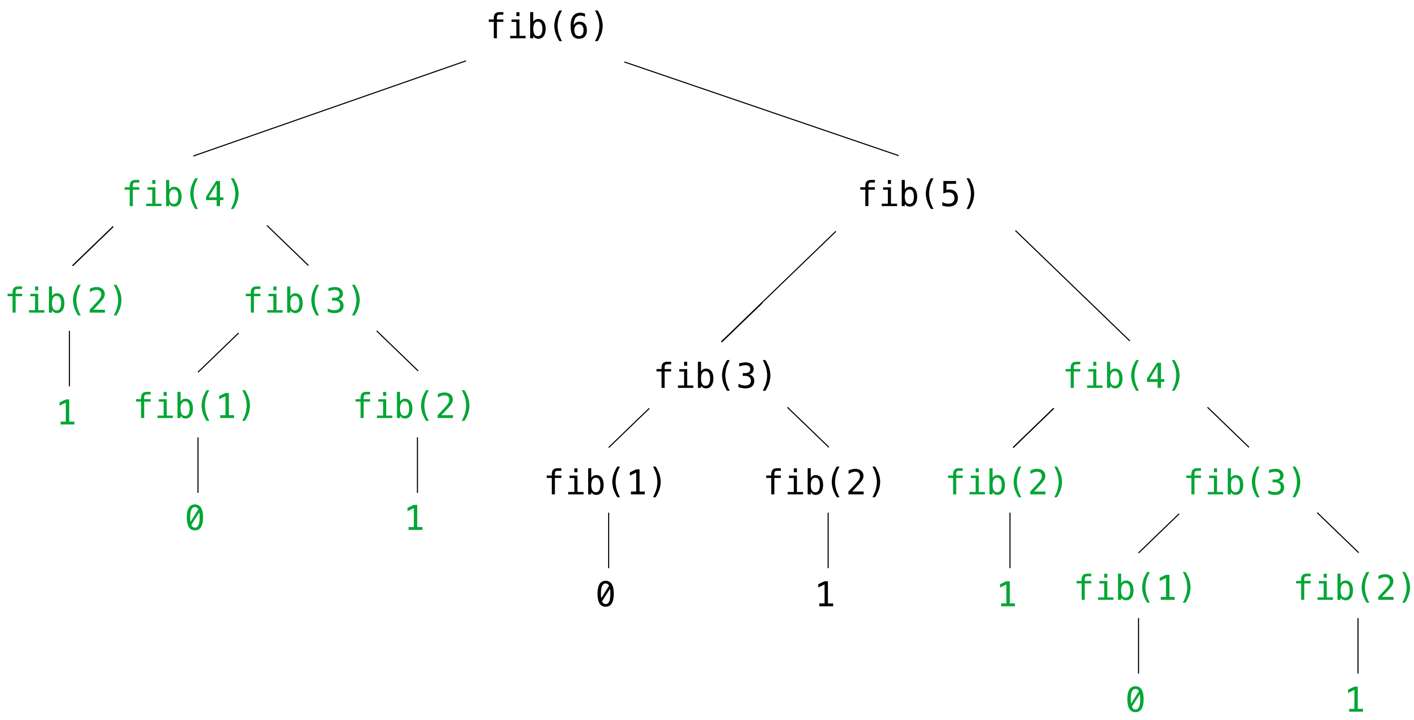
Repetition in Tree-Recursive Computation

Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times.

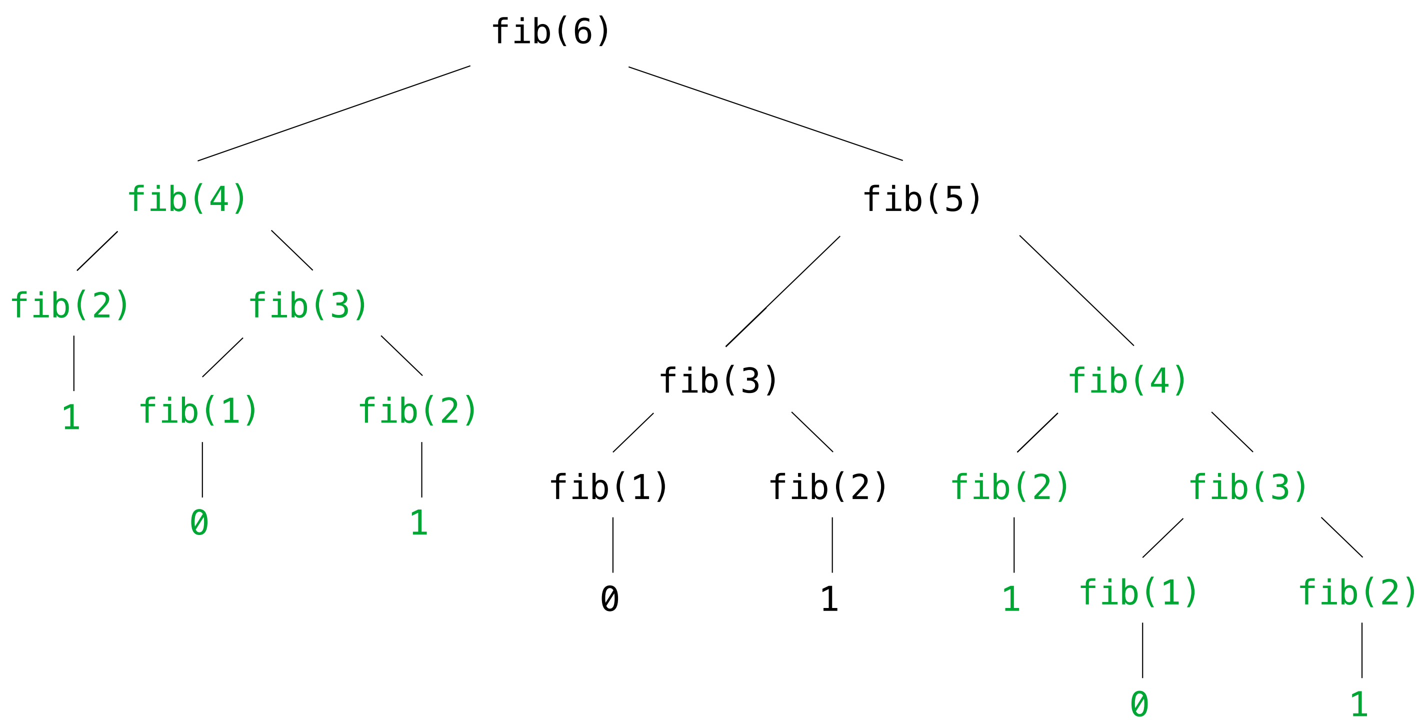
Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times.



Repetition in Tree-Recursive Computation

This process is highly repetitive; fib is called on the same argument multiple times.



We can speed up this computation dramatically in a few weeks by remembering results.

Example: Counting Partitions

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order**.**

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

```
partition(6, 4)
```

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

1 + 1 + 1 + 1 + 1 + 1 = 6

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

`partition(6, 4)`

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

1 + 1 + 1 + 1 + 1 + 1 = 6

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

1 + 1 + 1 + 1 + 1 + 1 = 6

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

2 + 4 = 6

1 + 1 + 4 = 6

3 + 3 = 6

1 + 2 + 3 = 6

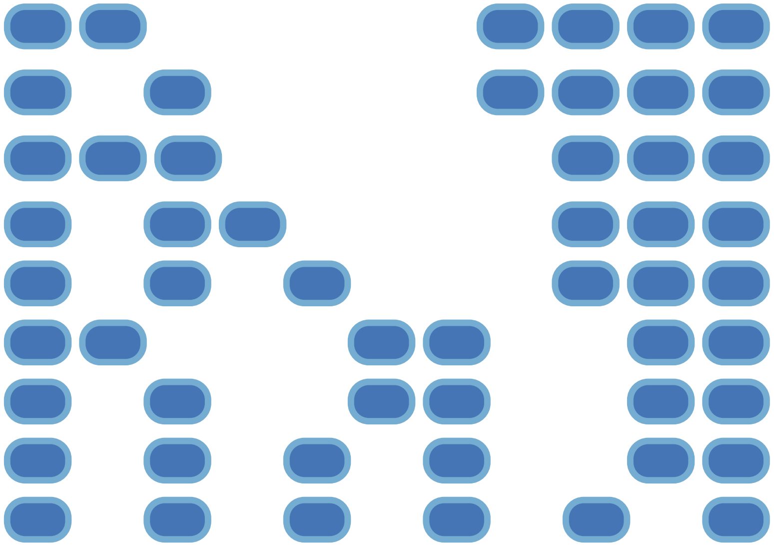
1 + 1 + 1 + 3 = 6

2 + 2 + 2 = 6

1 + 1 + 2 + 2 = 6

1 + 1 + 1 + 1 + 2 = 6

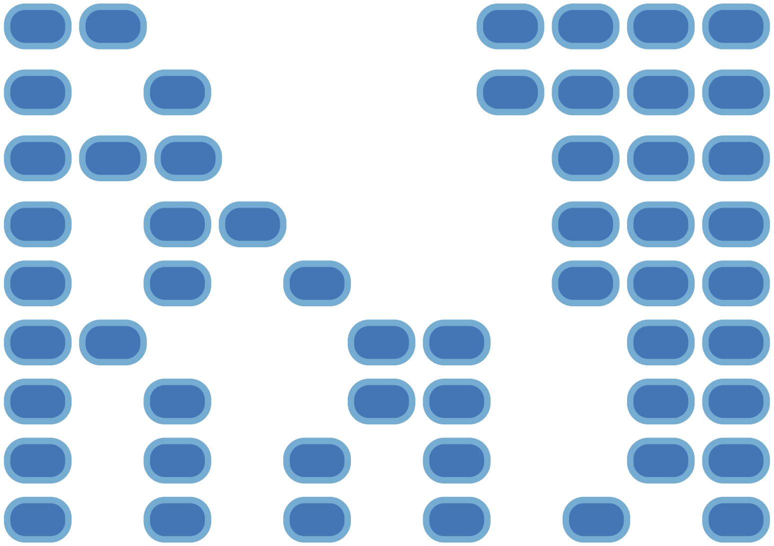
1 + 1 + 1 + 1 + 1 + 1 = 6



Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

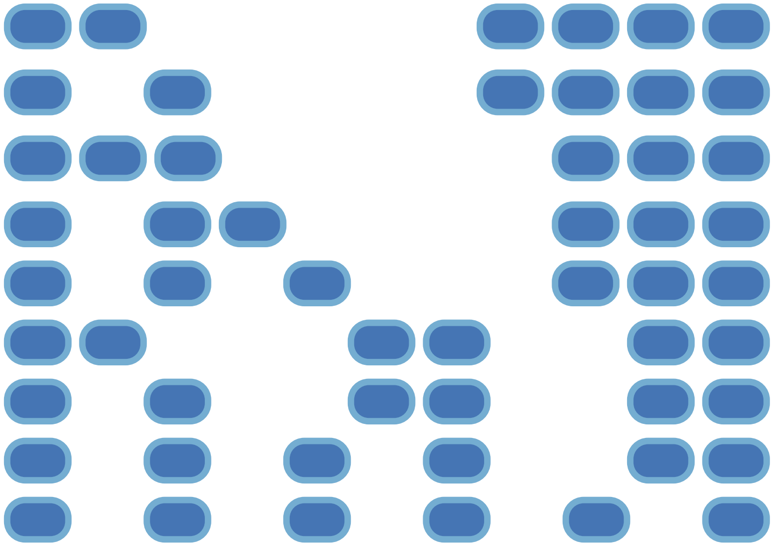


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.

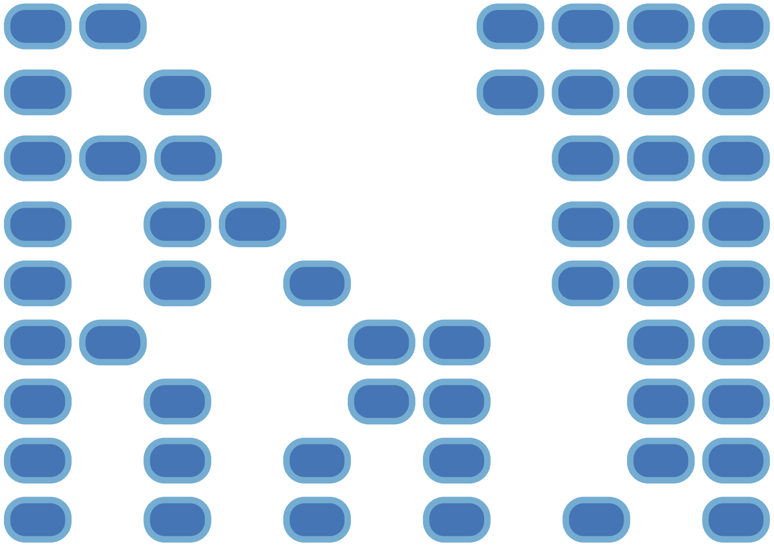


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:

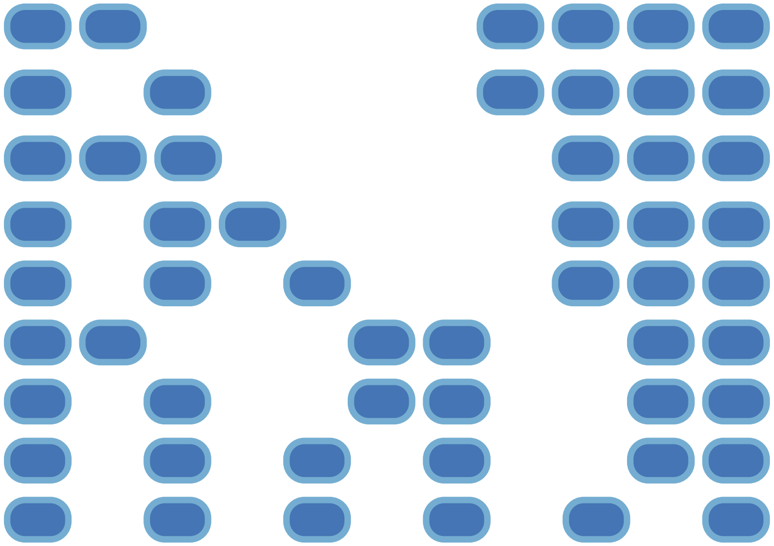


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4

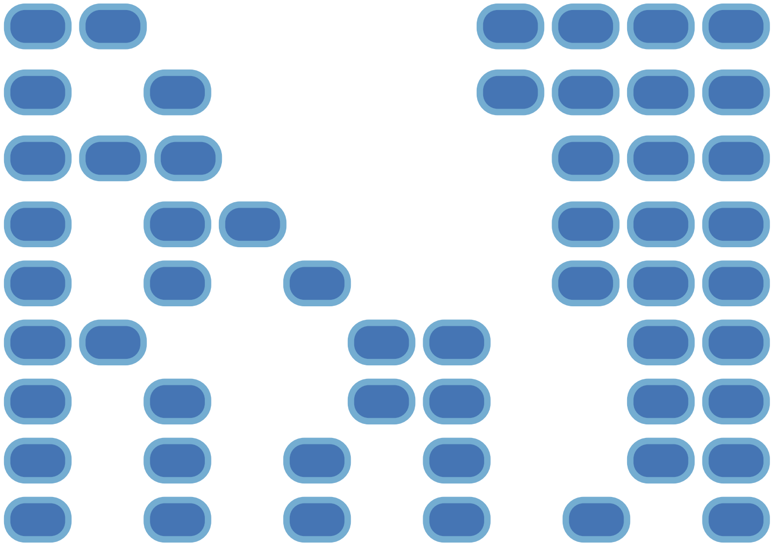


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

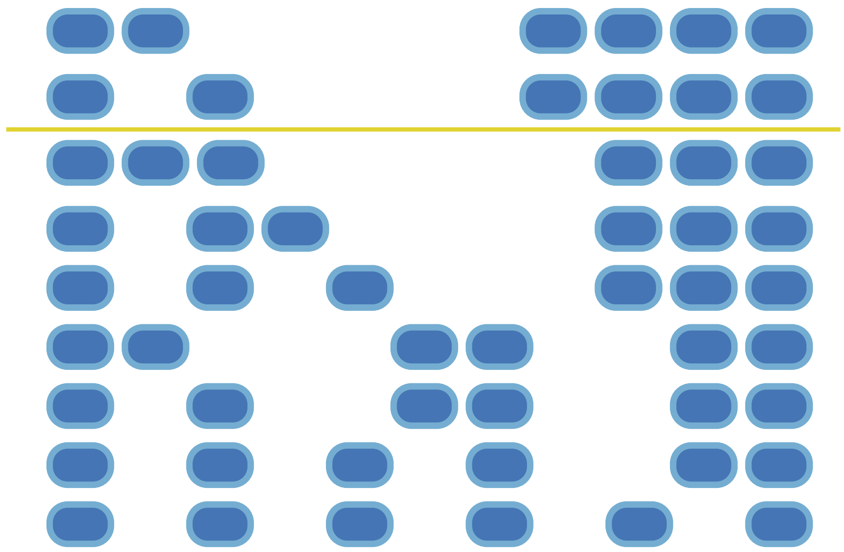


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

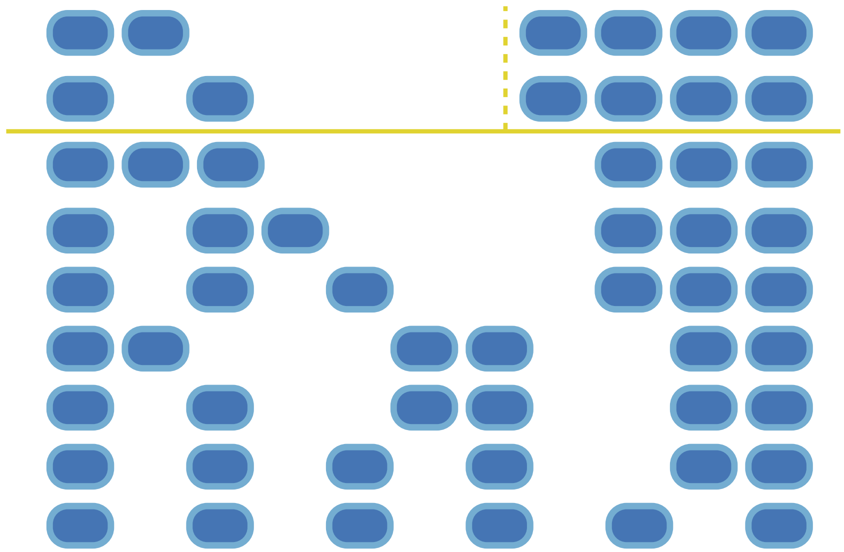


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4

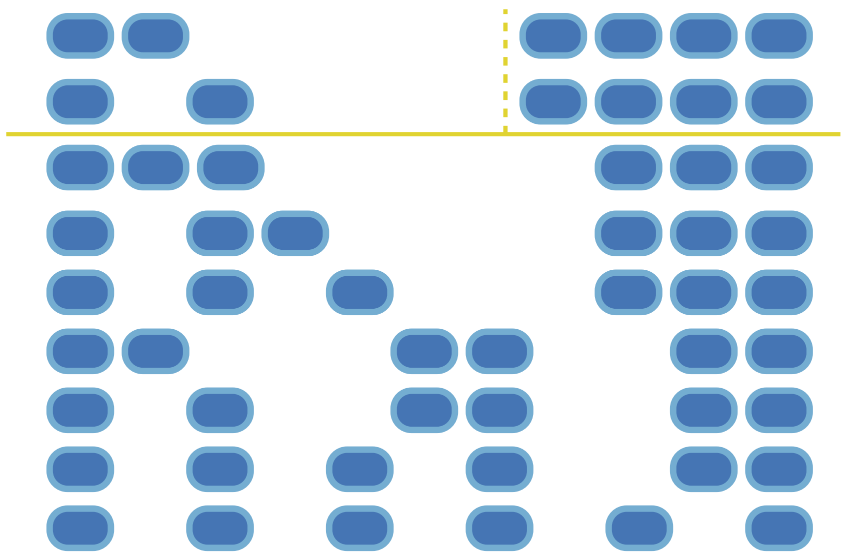


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:

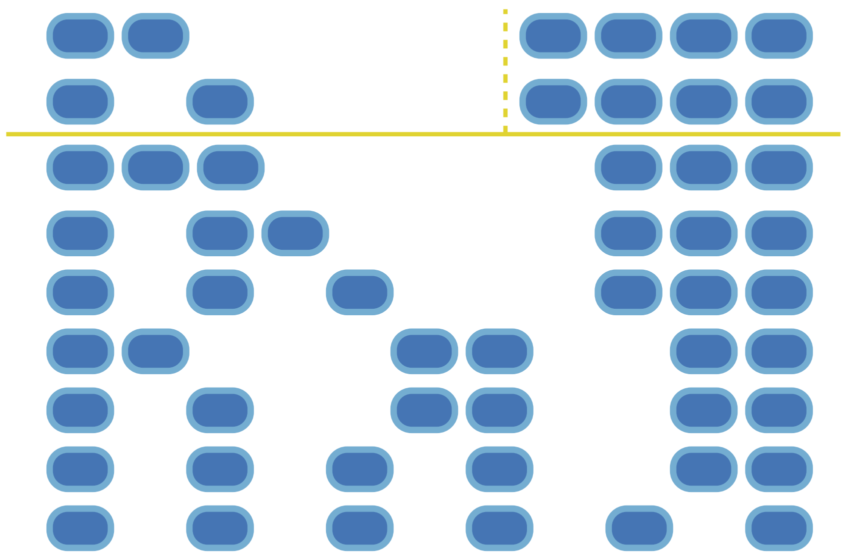


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)

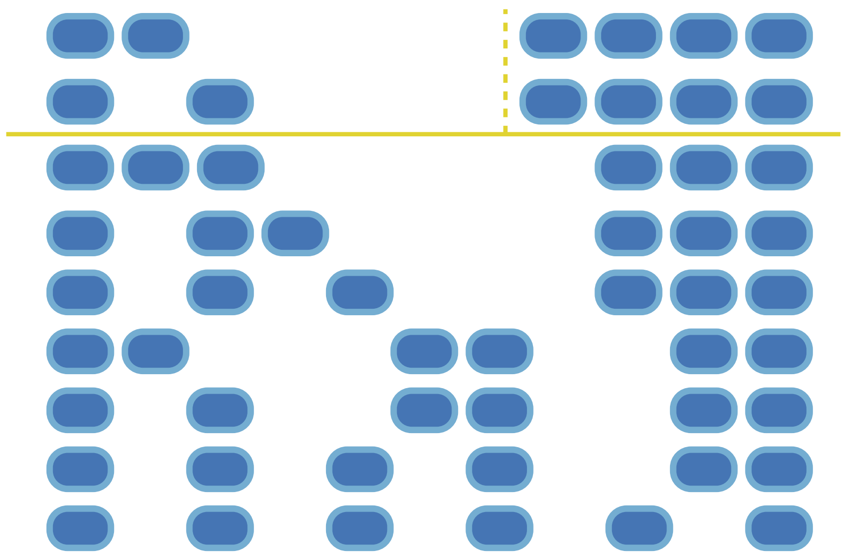


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)

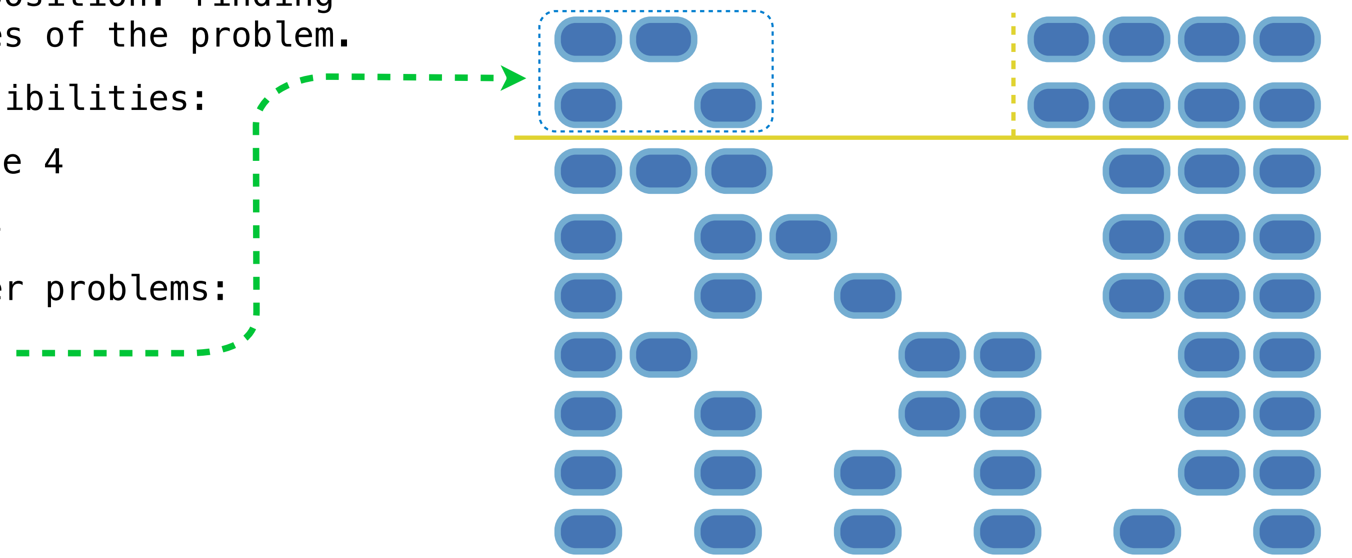


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)

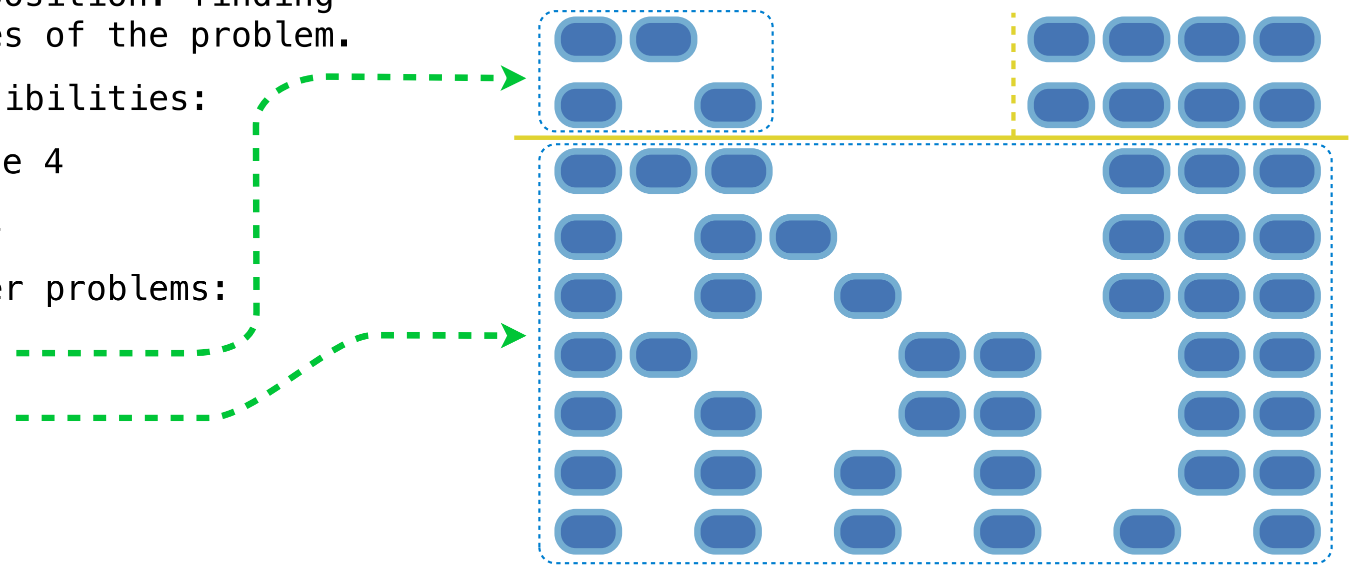


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)

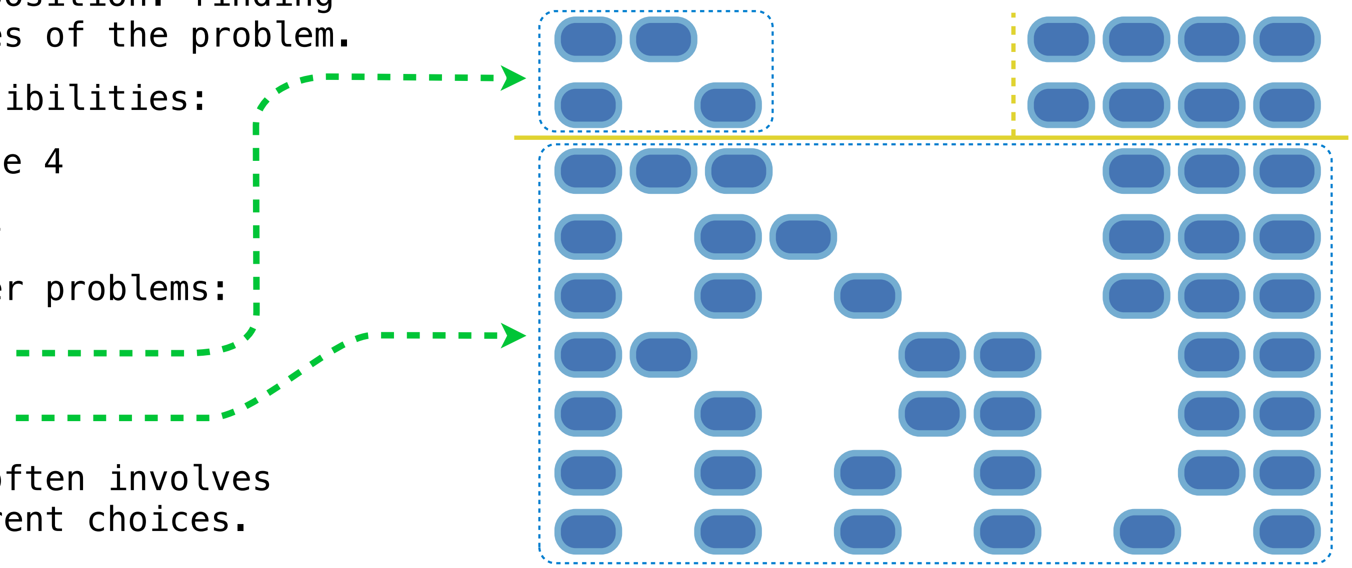


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

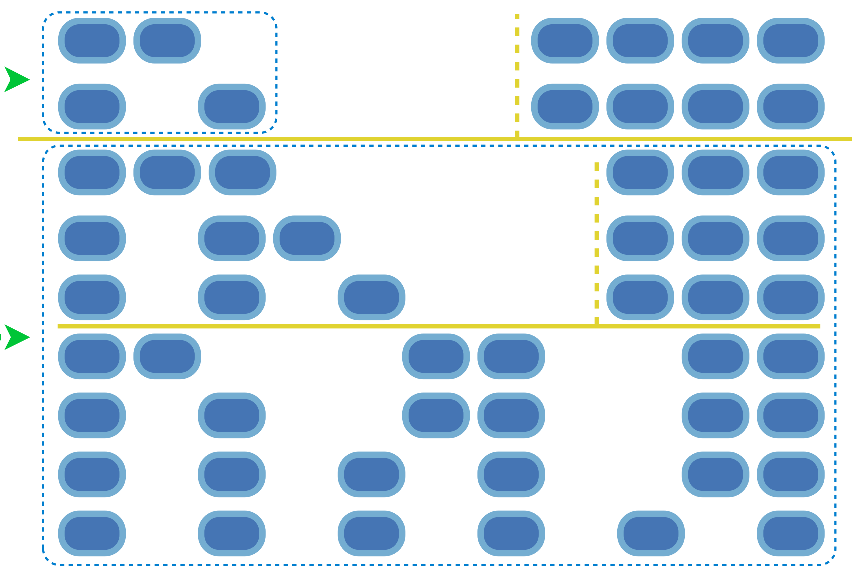


Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

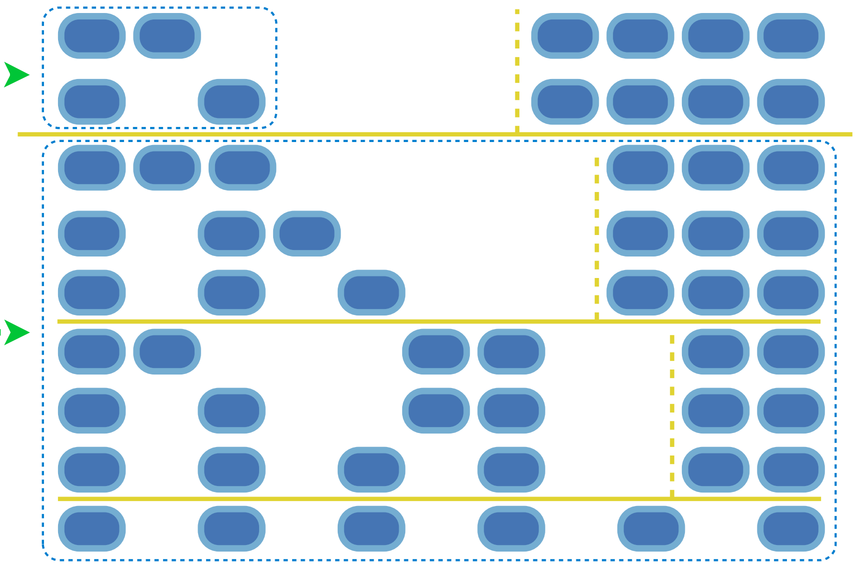


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

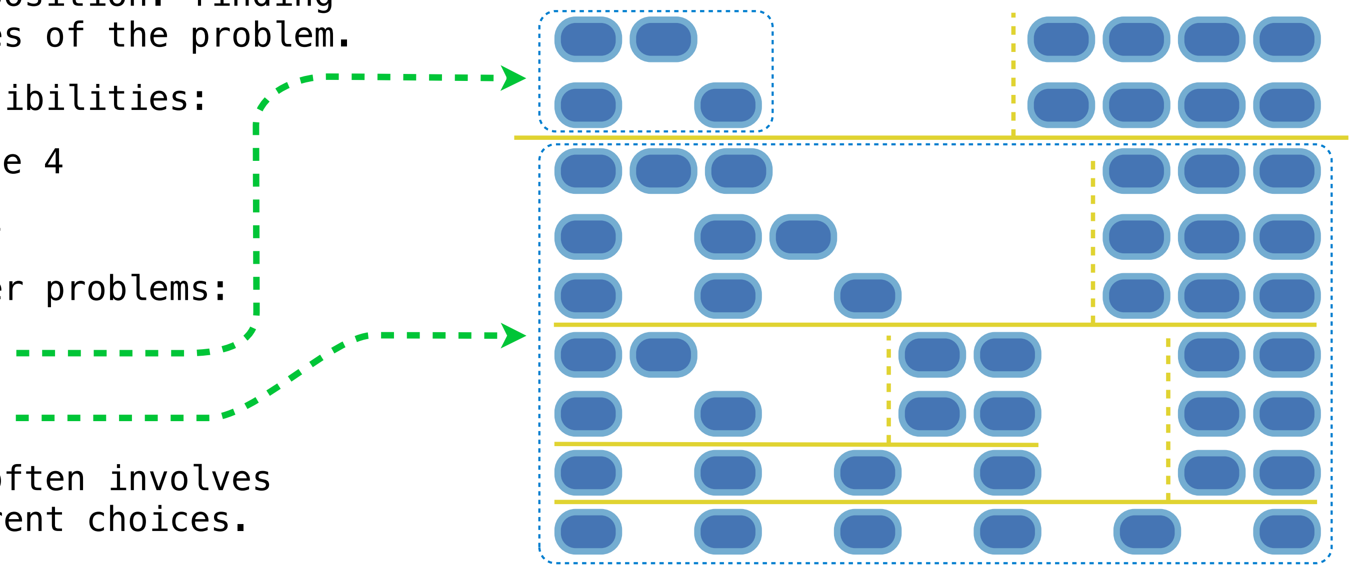


Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

partition(6, 4)

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.



Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):



    else:
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):



    else:
        with_m = count_partitions(n-m, m)
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):




    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):




    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):




    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4) ⟶ with_m
  - partition(6, 3) ⟶ without_m
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:



    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4) → with_m
  - partition(6, 3) → without_m
- Tree recursion often involves exploring different choices.

```python
def count_partitions(n, m):
    if n == 0:
        return 1


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n,** using parts up to size **m,** is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0


    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4)
  - partition(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:

    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4) → with_m
  - partition(6, 3) → without_m
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

Example: http://goo.gl/25ZSGK

Counting Partitions

The number of **partitions** of a positive integer **n**, using parts up to size **m**, is the number of ways in which **n** can be expressed as the sum of positive integer parts up to **m** in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - partition(2, 4) → with_m
  - partition(6, 3) → without_m
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
```

(Demo)

Winning Hog

How to Win at Hog

How to Win at Hog

`What is the chance that I'll score at least` **`k`** `points rolling` **`n`** `six-sided dice?`

How to Win at Hog

`What is the chance that I'll score at least` **`k`** `points rolling` **`n`** `six-sided dice?`

$$\frac{\text{Number of ways to score at least } \mathbf{k}}{\text{Number of possible rolls}}$$

How to Win at Hog

What is the chance that I'll score at least **k** points rolling **n** six-sided dice?

$$\frac{\text{Number of ways to score at least } \mathbf{k}}{\text{Number of possible rolls}}$$

The number of possible rolls is **pow(6, n).**

How to Win at Hog

What is the chance that I'll score at least **k** points rolling **n** six-sided dice?

$$\frac{\text{Number of ways to score at least } \mathbf{k}}{\text{Number of possible rolls}}$$

The number of possible rolls is **pow(6, n).**

The number of ways to score at least **k** in **n** rolls can be computed using tree recursion!

How to Win at Hog

What is the chance that I'll score at least **k** points rolling **n** six-sided dice?

$$\frac{\text{Number of ways to score at least } \mathbf{k}}{\text{Number of possible rolls}}$$

The number of possible rolls is **pow(6, n).**

The number of ways to score at least **k** in **n** rolls can be computed using tree recursion!

Sum over each possible dice outcome **d** that does not *pig out*:
the number of ways to score at least **k - d** points using **n - 1** rolls.

How to Win at Hog

What is the chance that I'll score at least **k** points rolling **n** six-sided dice?

$$\frac{\text{Number of ways to score at least } \mathbf{k}}{\text{Number of possible rolls}}$$

The number of possible rolls is **pow(6, n).**

The number of ways to score at least **k** in **n** rolls can be computed using tree recursion!

Sum over each possible dice outcome **d** that does not *pig out*:
the number of ways to score at least **k - d** points using **n - 1** rolls.

Base case: The number of ways to score at least 0 is **pow(5, n).**

How to Win at Hog

What is the chance that I'll score at least **k** points rolling **n** six-sided dice?

$$\frac{\text{Number of ways to score at least } \mathbf{k}}{\text{Number of possible rolls}}$$

The number of possible rolls is **pow(6, n).**

The number of ways to score at least **k** in **n** rolls can be computed using tree recursion!

Sum over each possible dice outcome **d** that does not *pig out*:
the number of ways to score at least **k - d** points using **n - 1** rolls.

Base case: The number of ways to score at least 0 is **pow(5, n).**

Base case: The number of ways to score positive points in 0 rolls is 0.