

61A Lecture 5

Wednesday, September 11

Announcements

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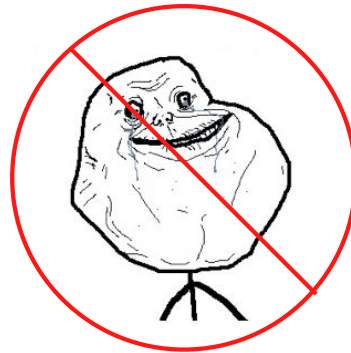
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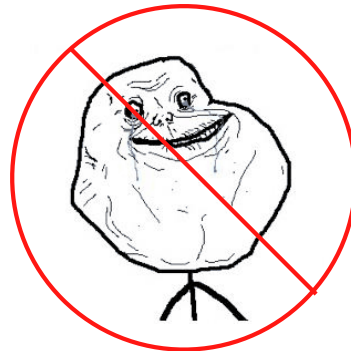
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Environments for Higher-Order Functions

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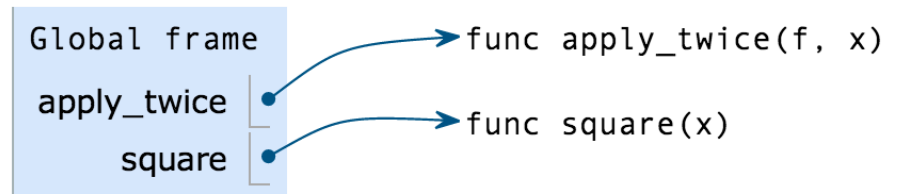
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(demo)

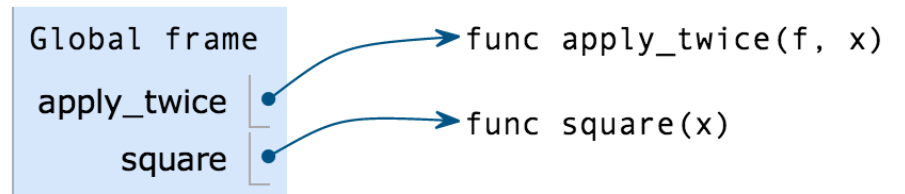
Names can be Bound to Functional Arguments

```
1 def apply_twice(f, x):  
2     return f(f(x))  
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→ 4 def square(x):  
5     return x * x  
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→ 7 result = apply_twice(square, 2)
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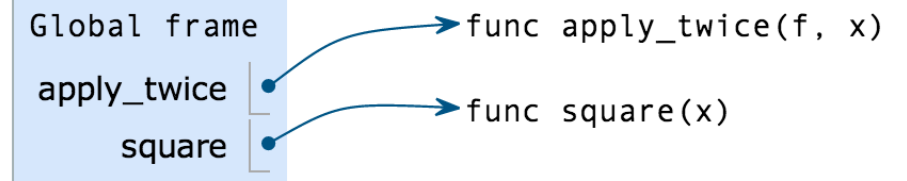
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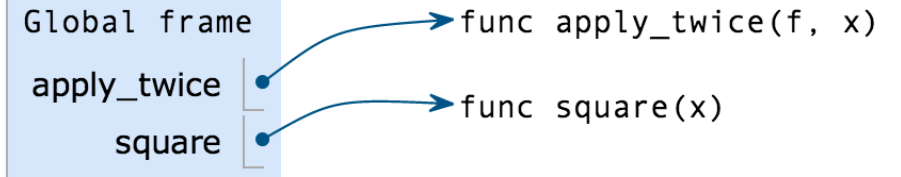


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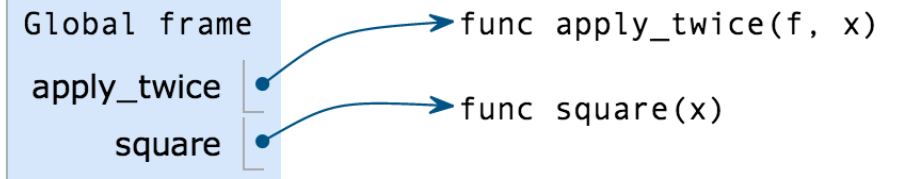
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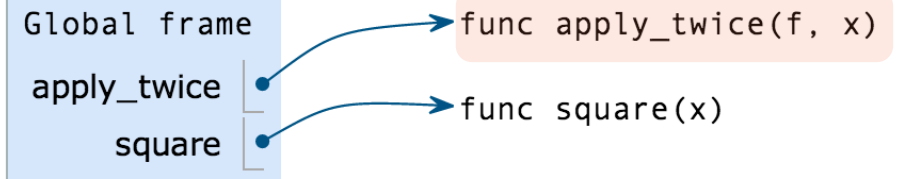
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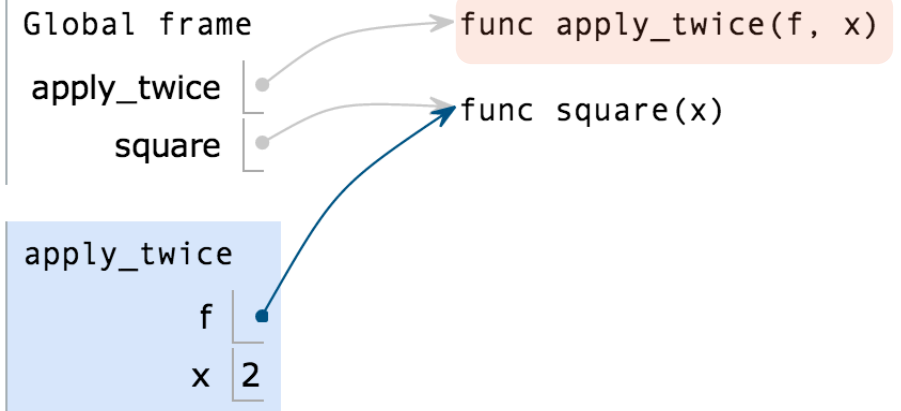
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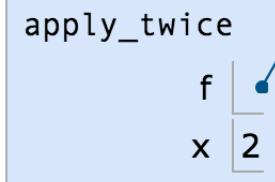
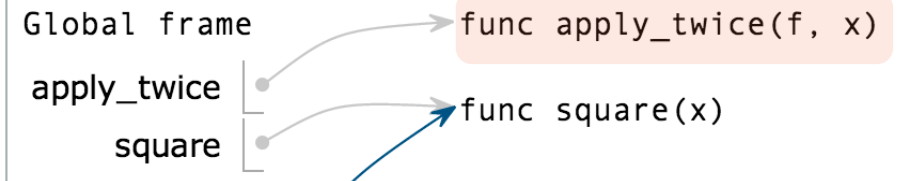
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Applying a user-defined function:

- Create a new frame
- Bind formal parameters (f & x) to arguments
- Execute the body:
return f(f(x))

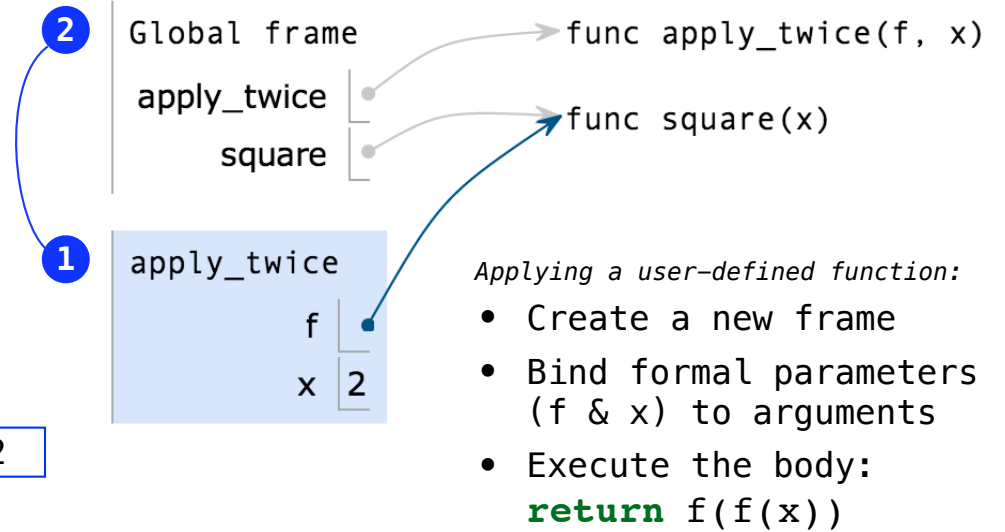
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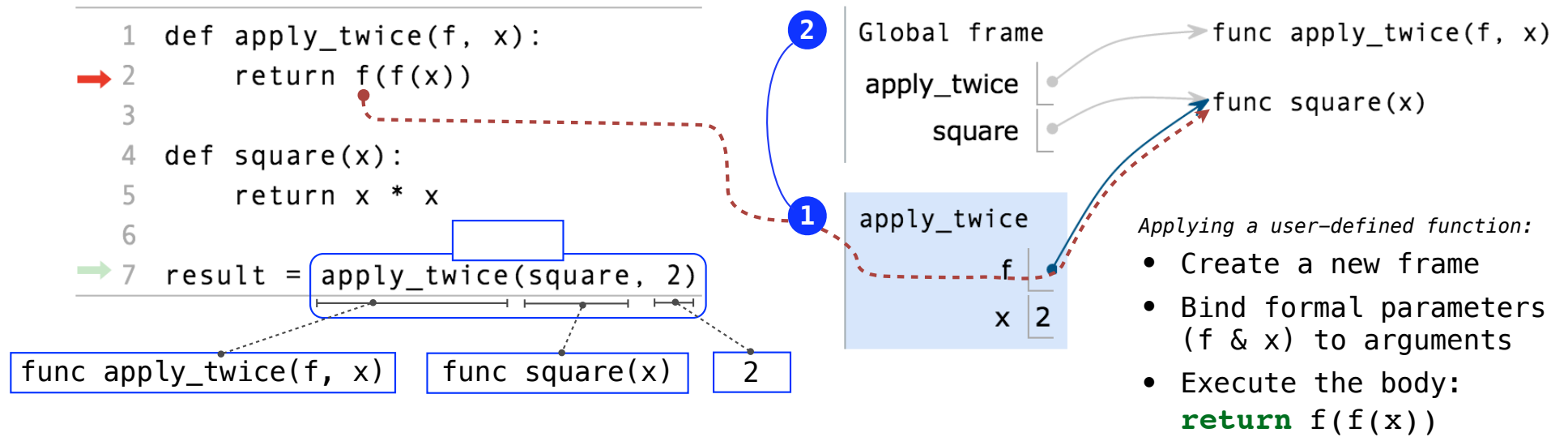
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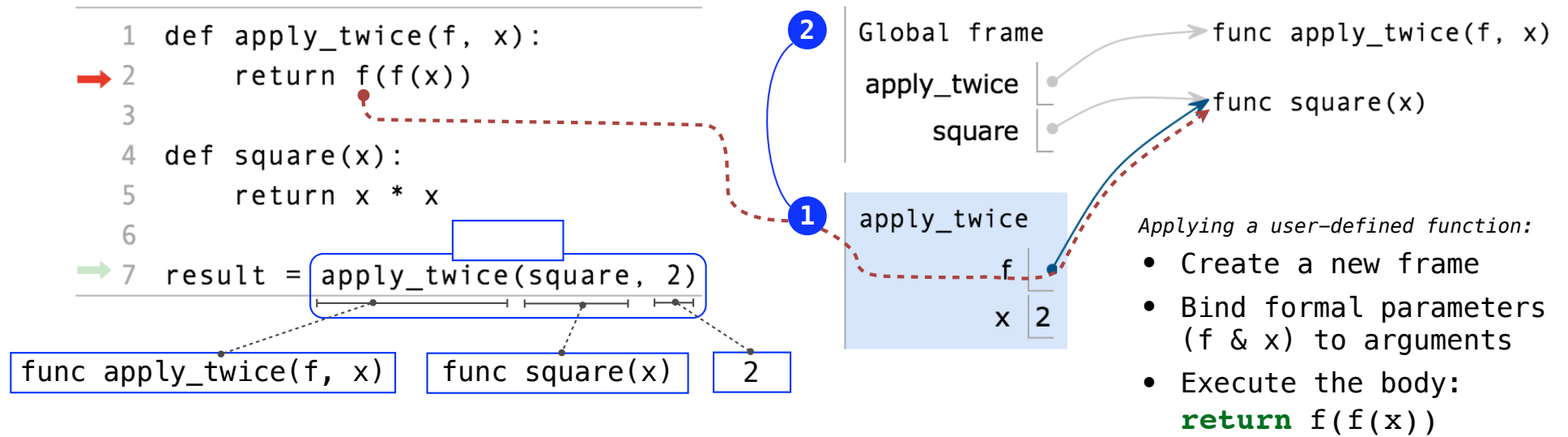
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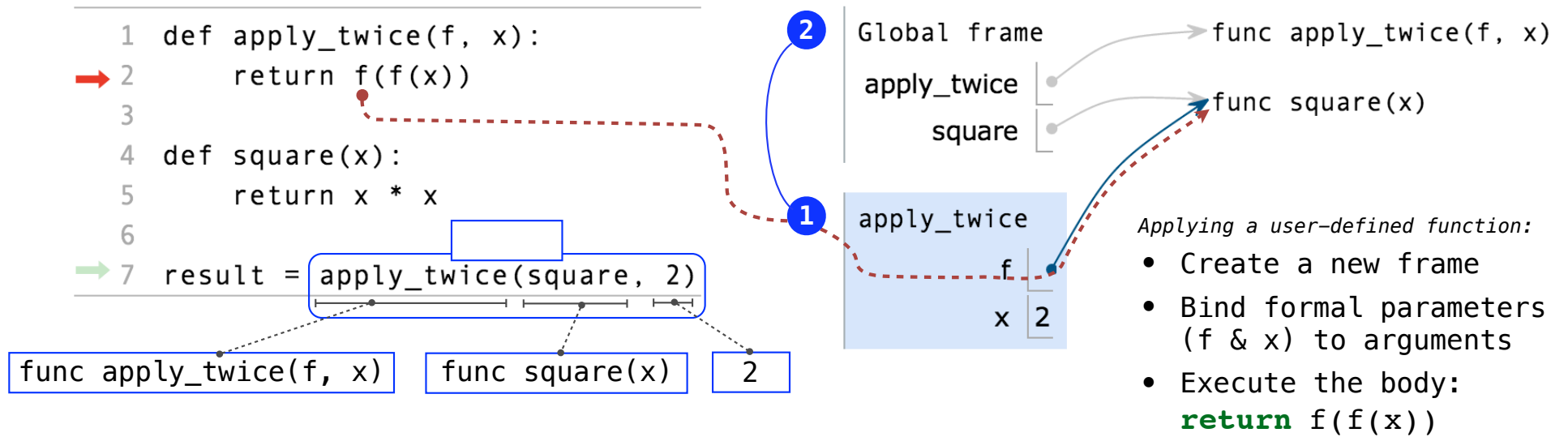


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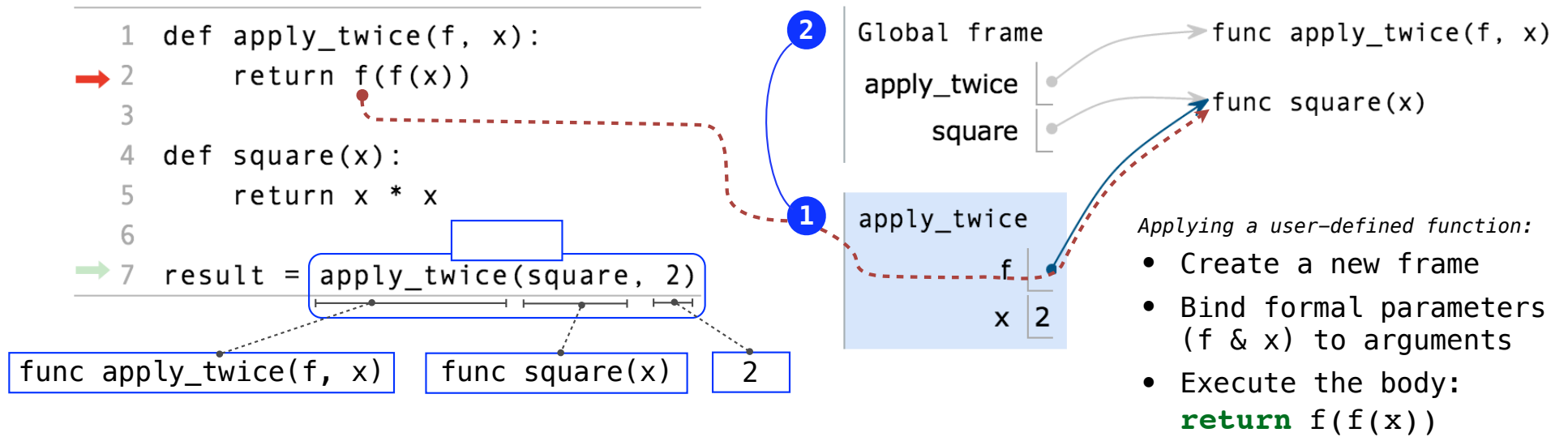
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- Multiple names can all refer to the same function, even in different frames.

Discussion Question

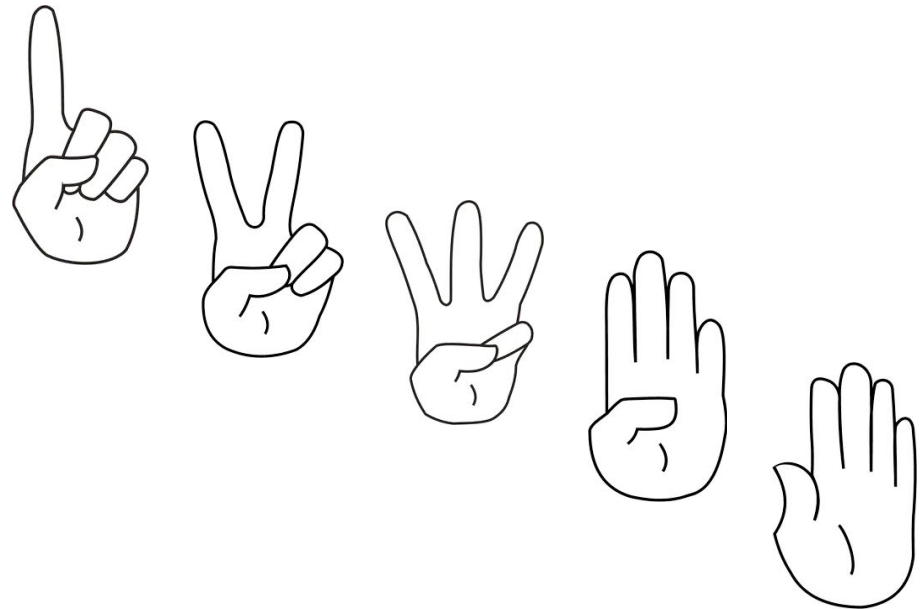
What is the value of the final expression below?

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def repeat(f, x):  
    while f(x) != x:  
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def g(y):  
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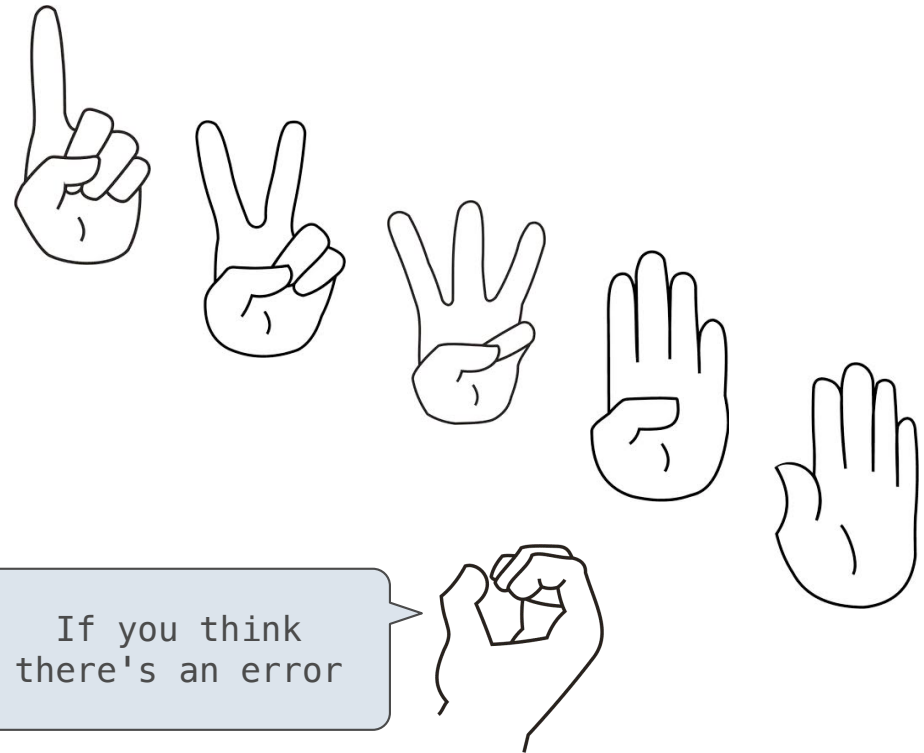
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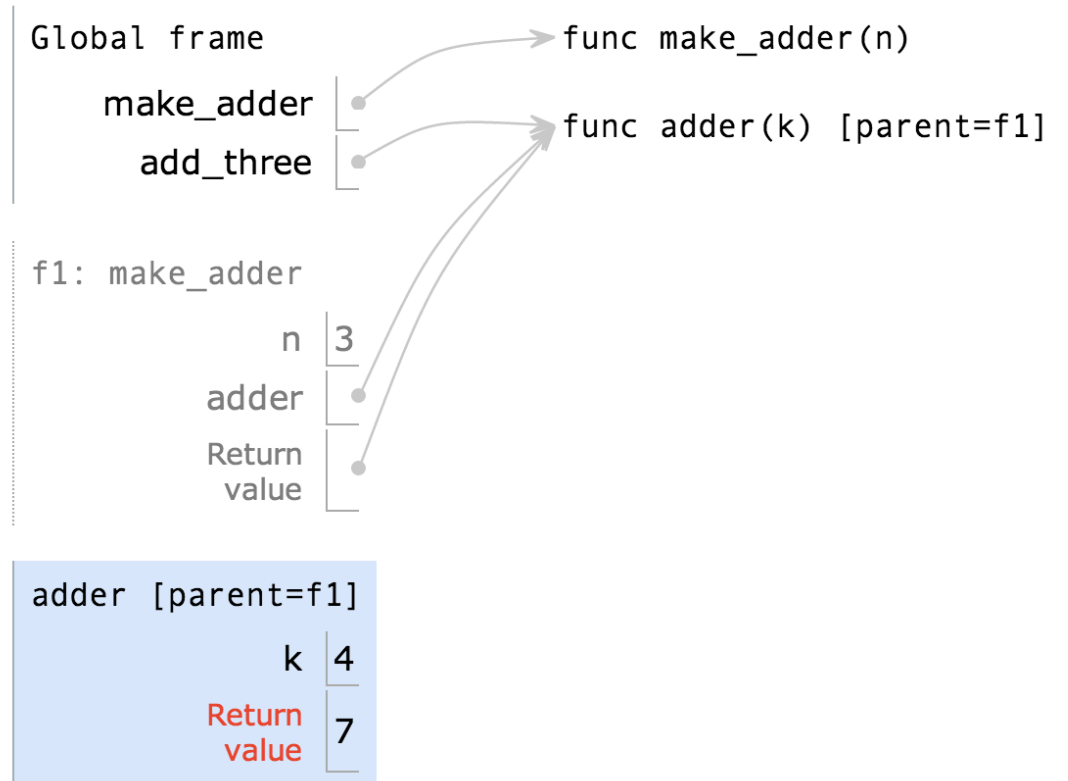
Example: <http://goo.gl/EDi0Ir>

Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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1 def make_adder(n):  
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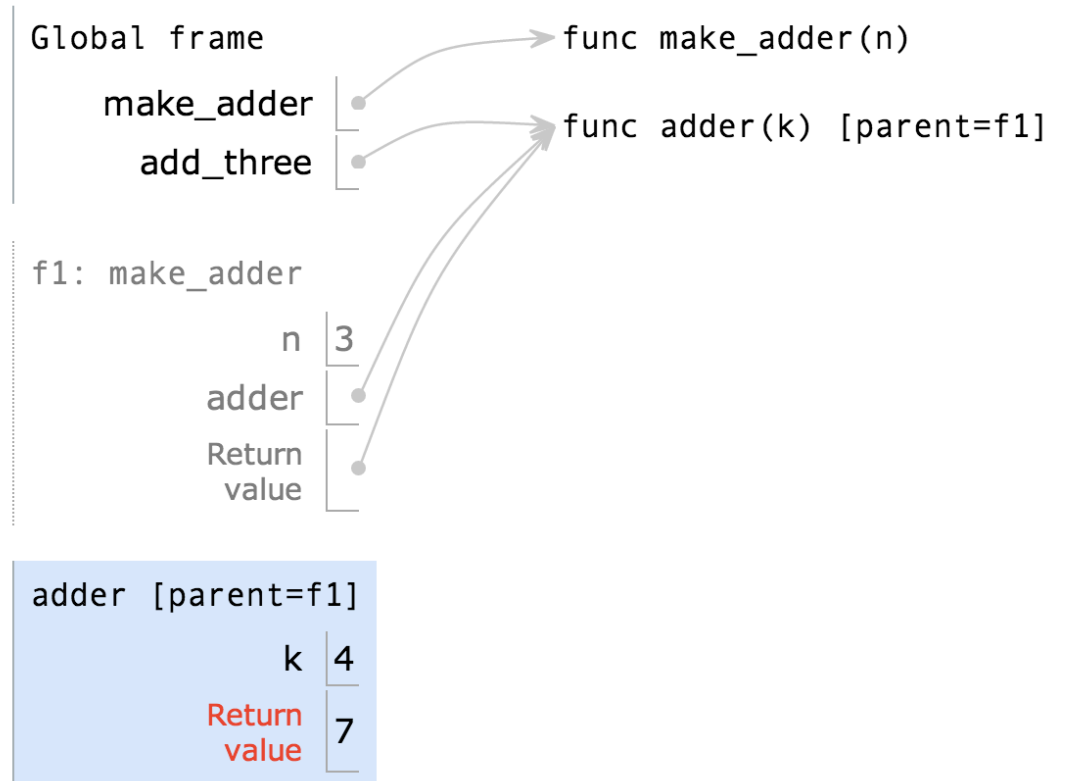


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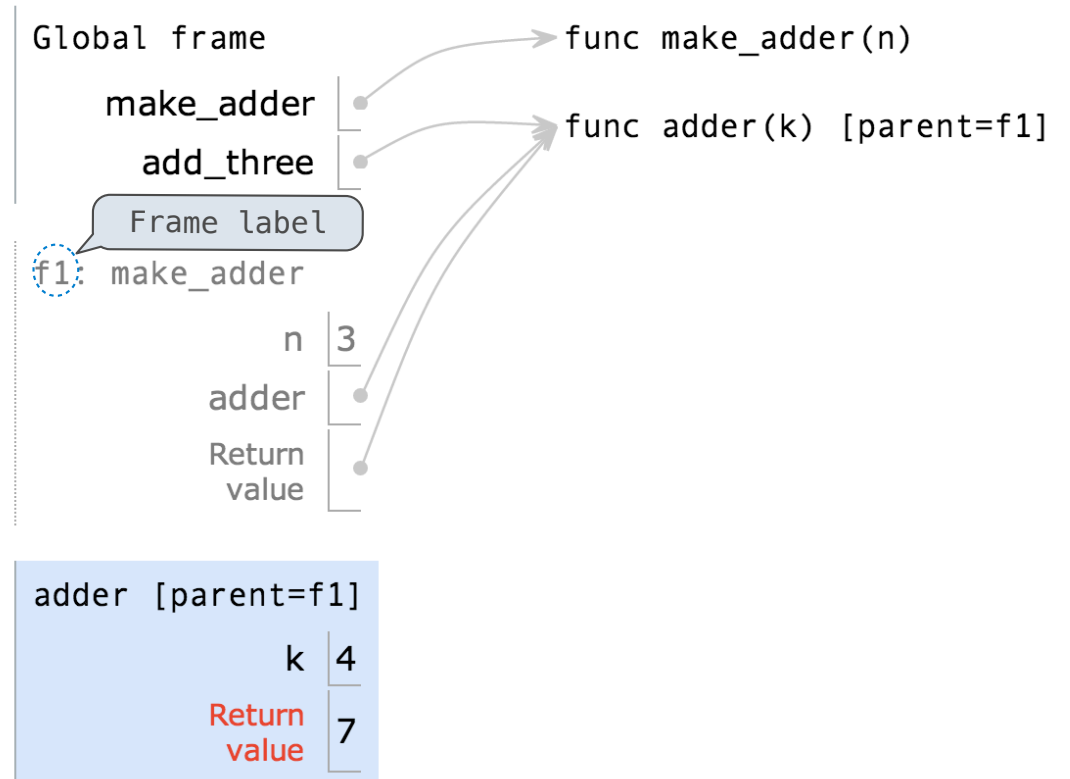
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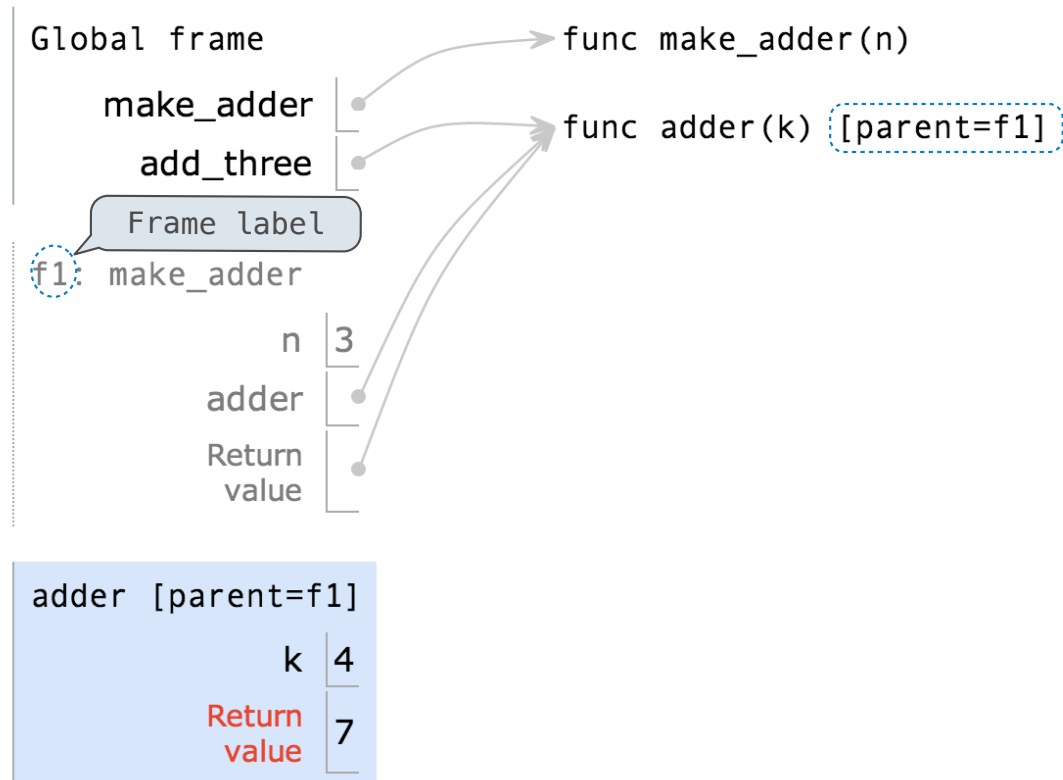
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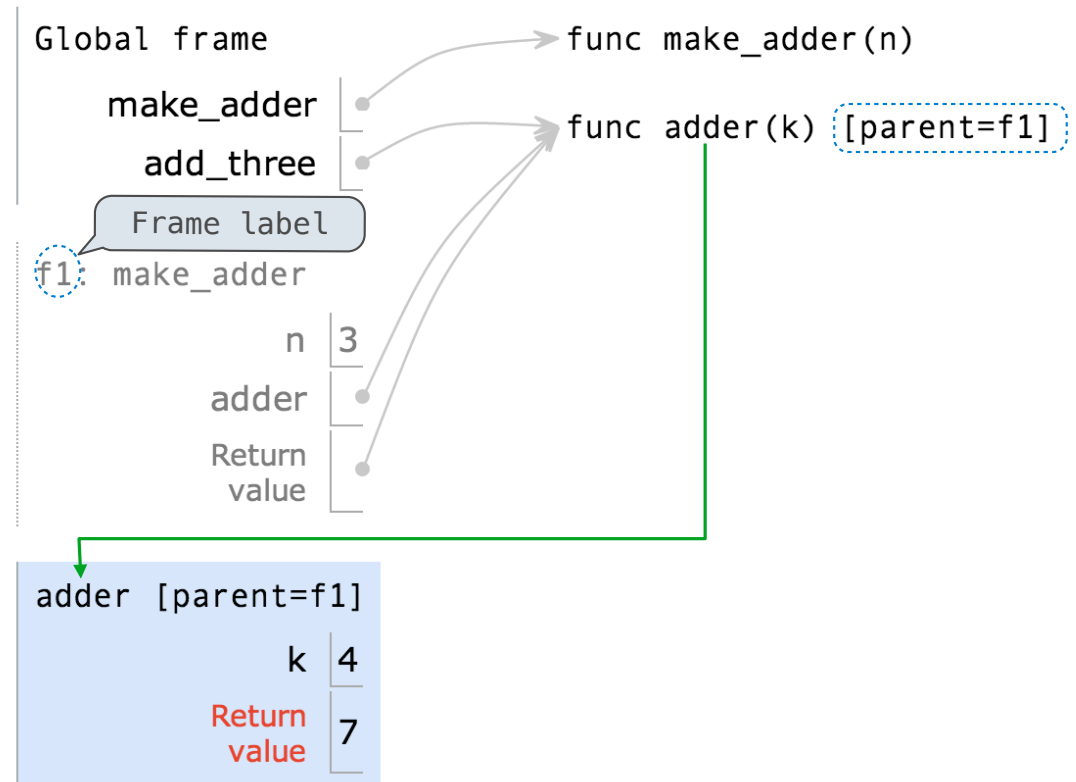
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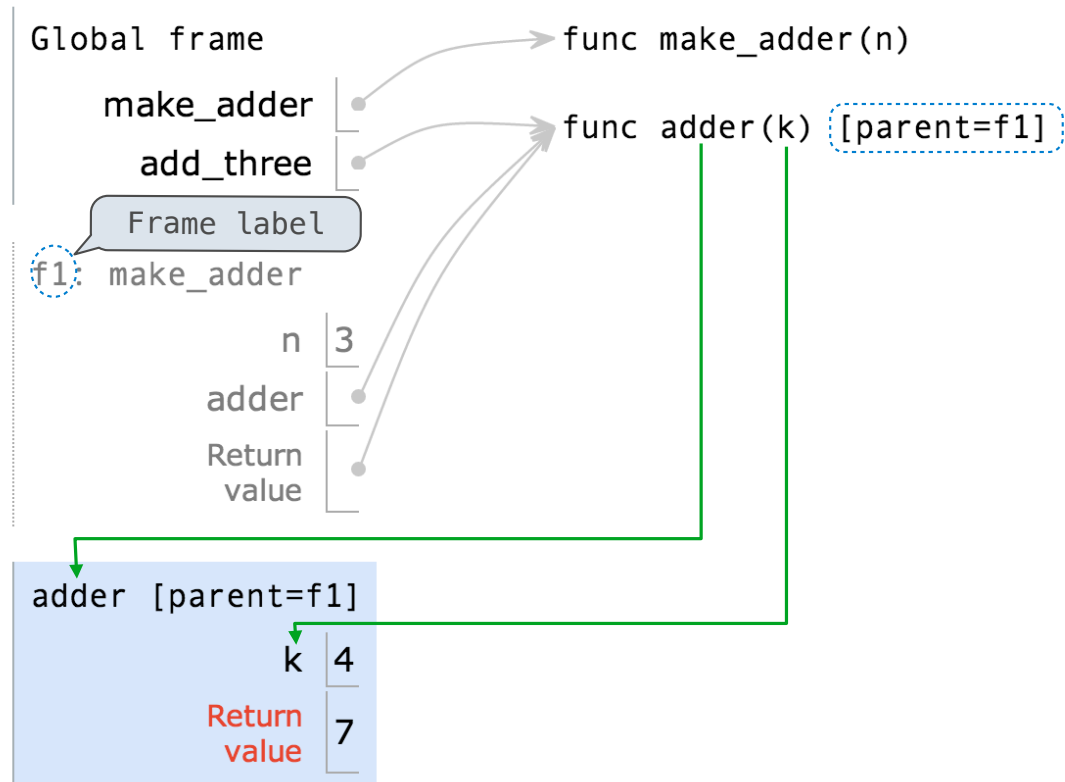


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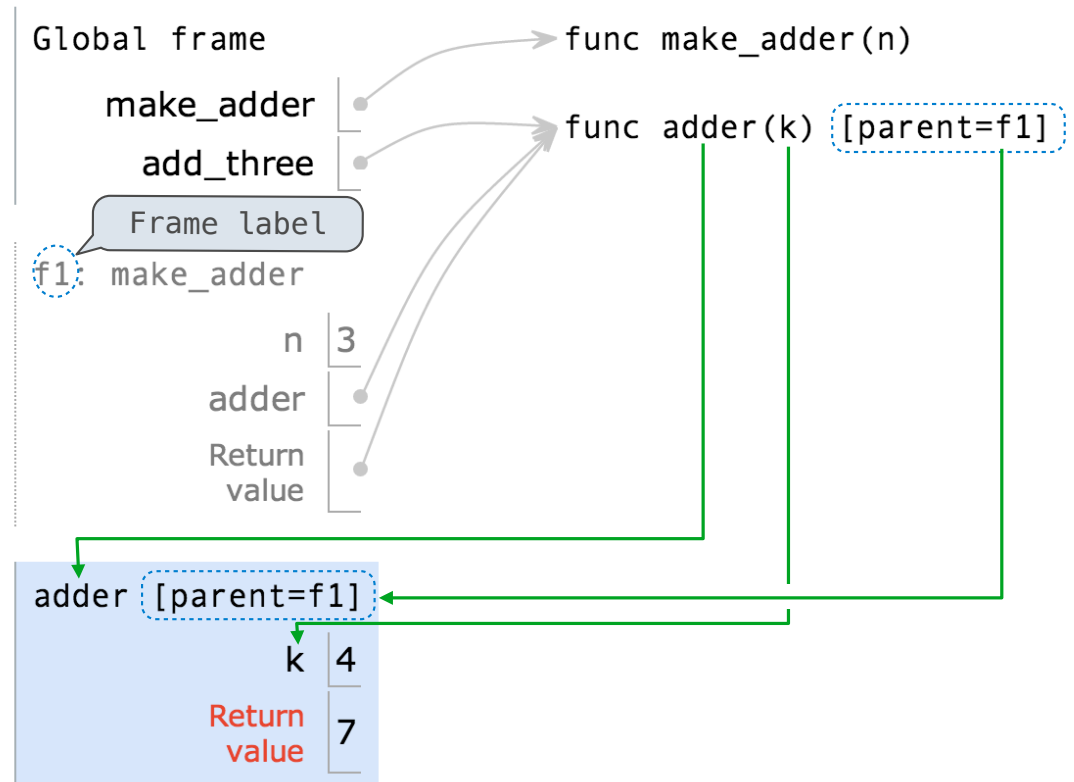
(Note: In the original image, a blue oval highlights the 'def' keyword in line 1, and a blue oval highlights the 'def' keyword in line 2. A green arrow points to line 3, and a red arrow points to line 4.)



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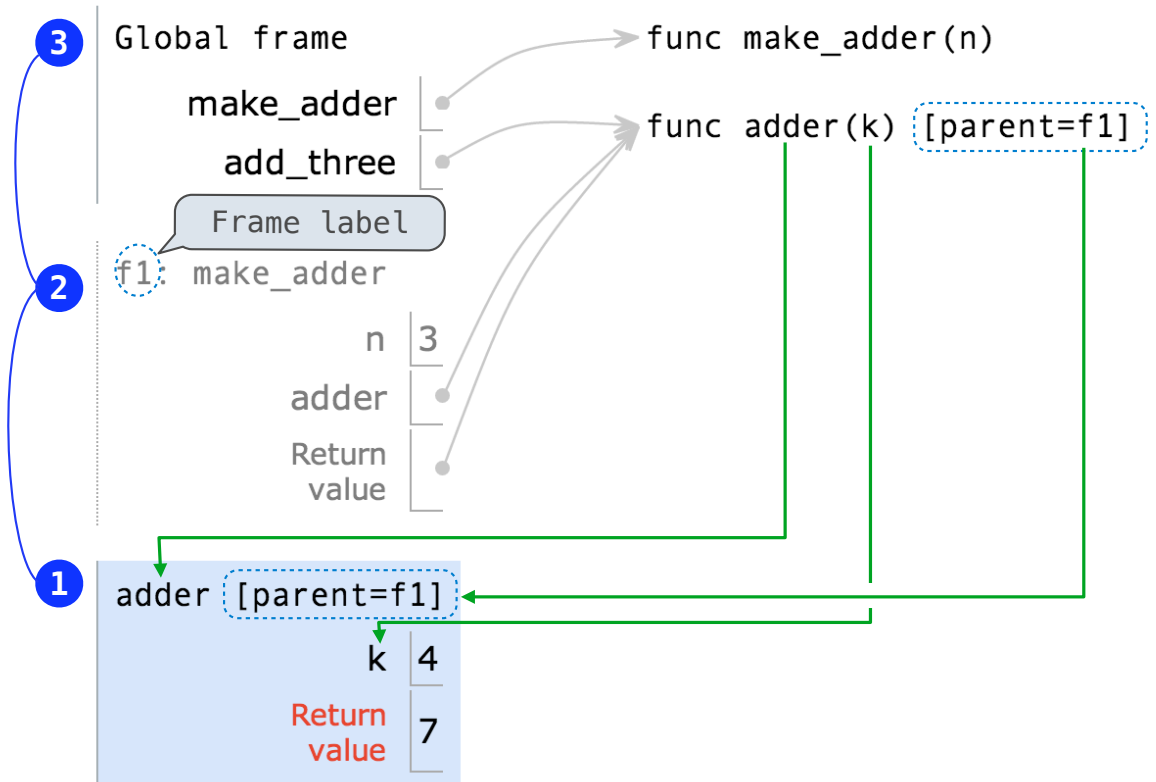


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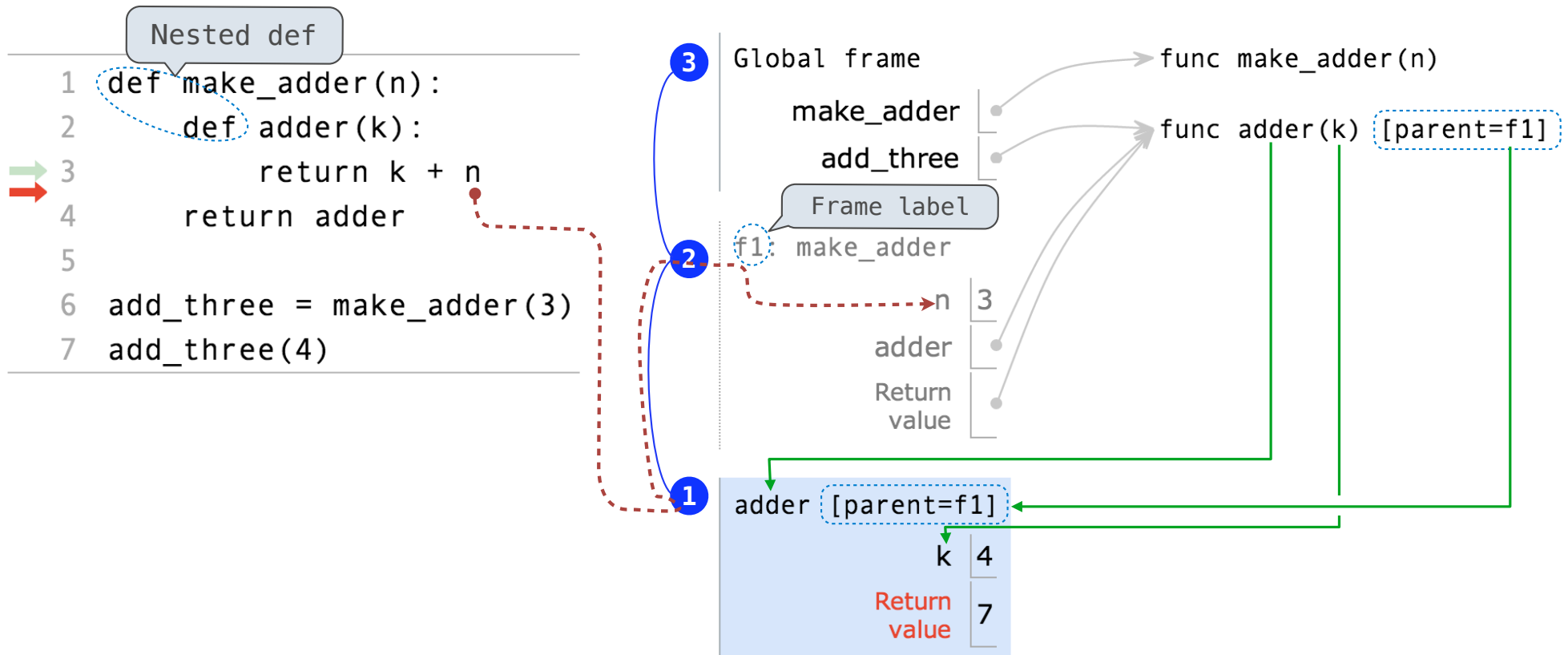
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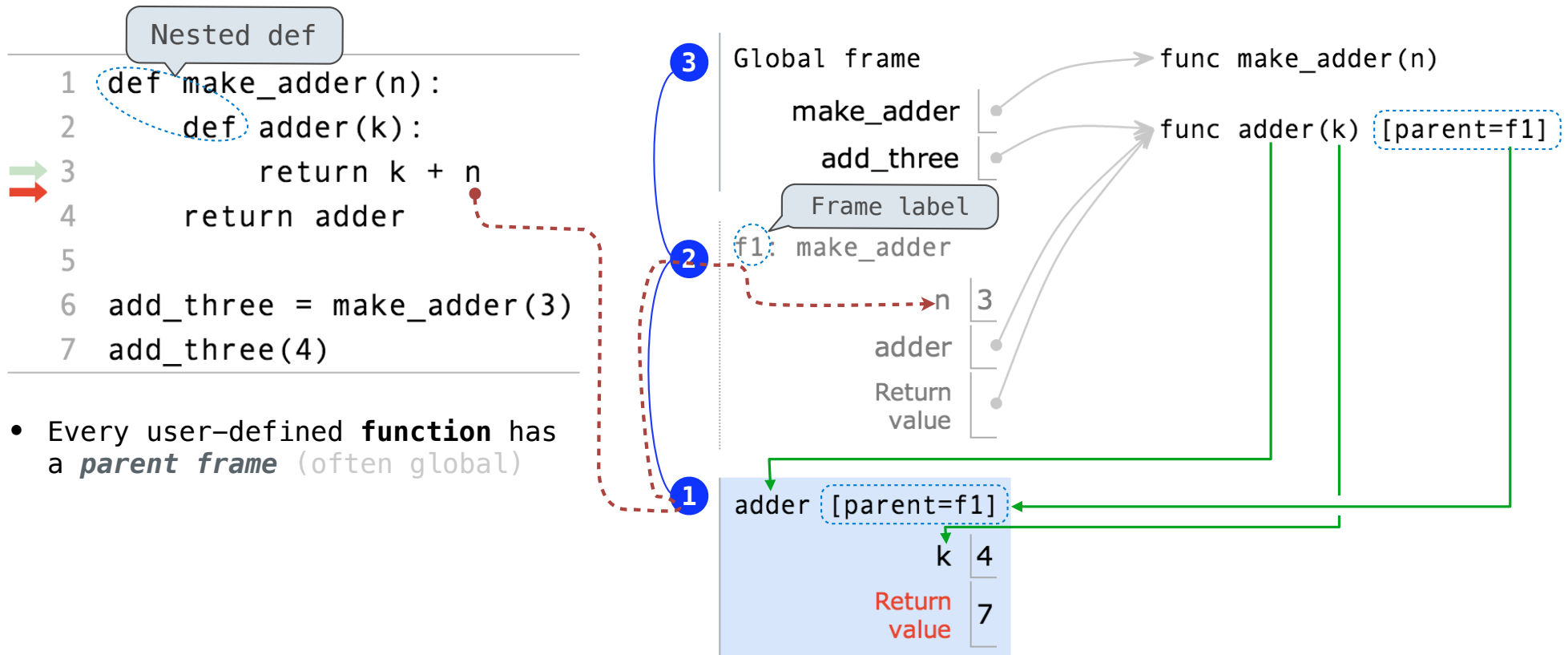
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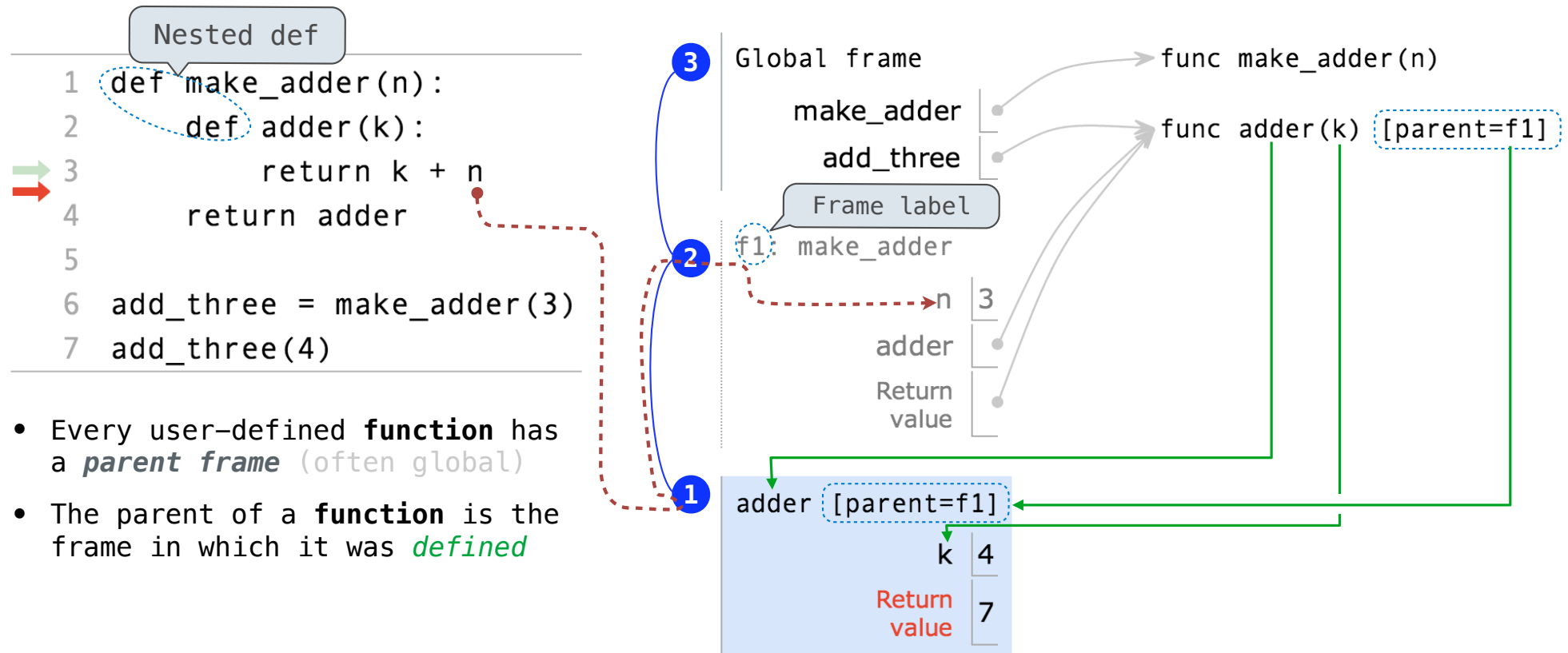
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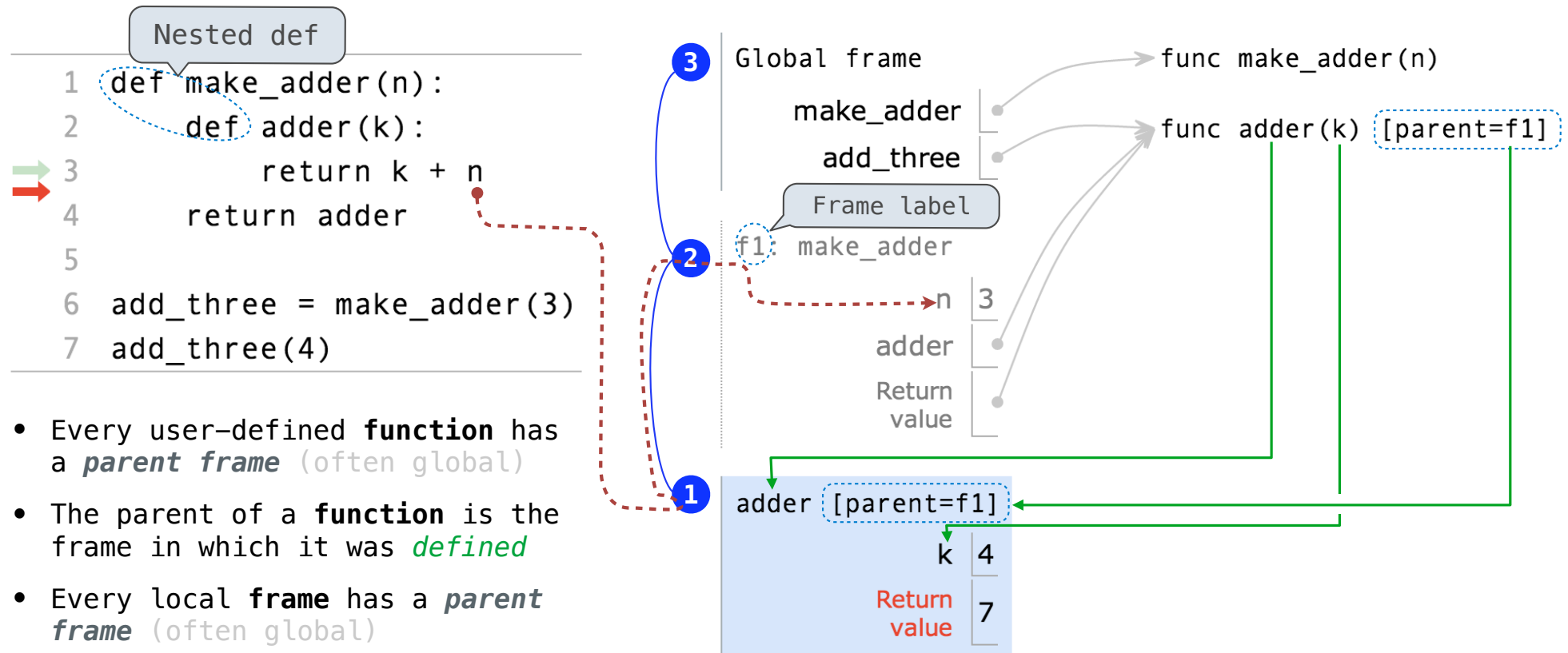
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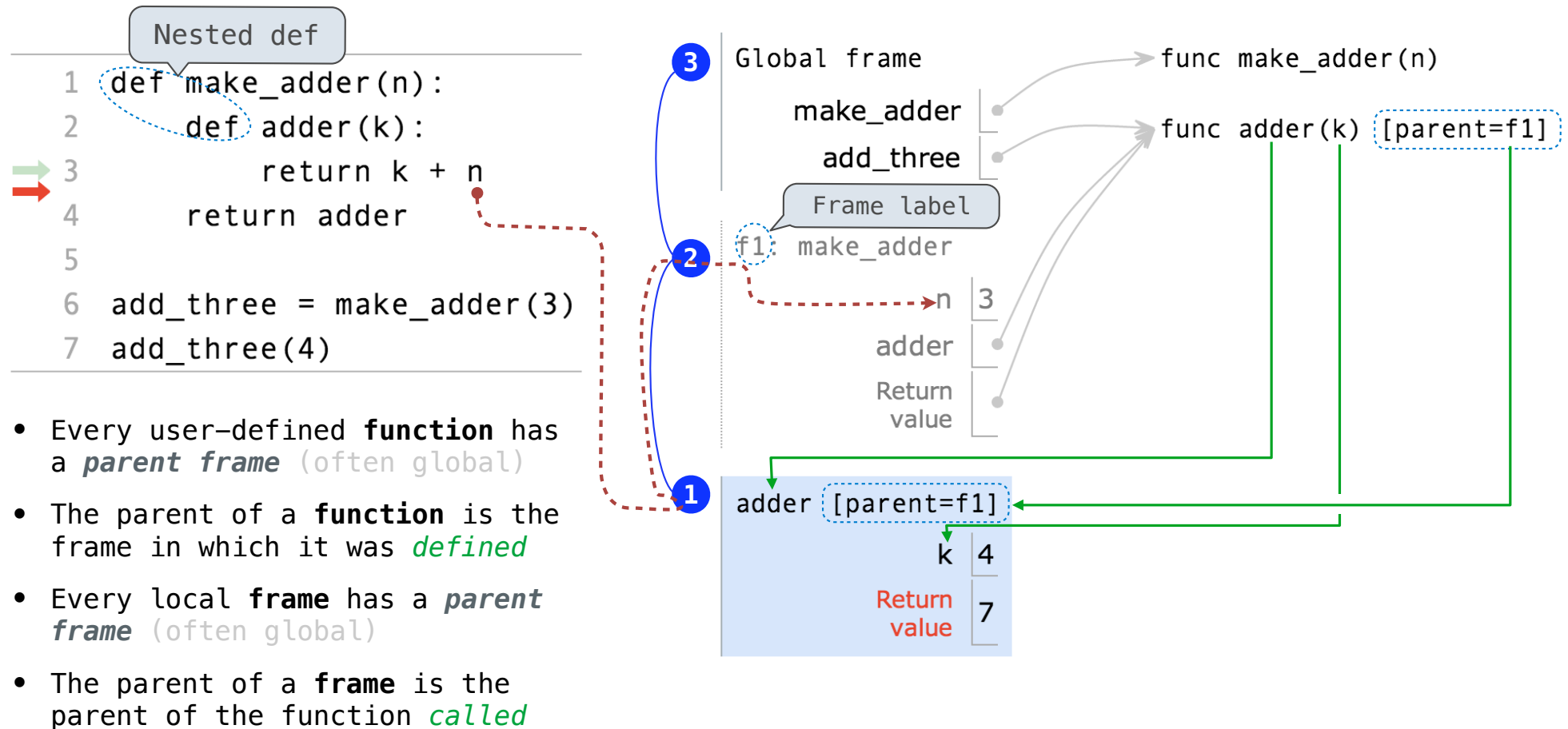
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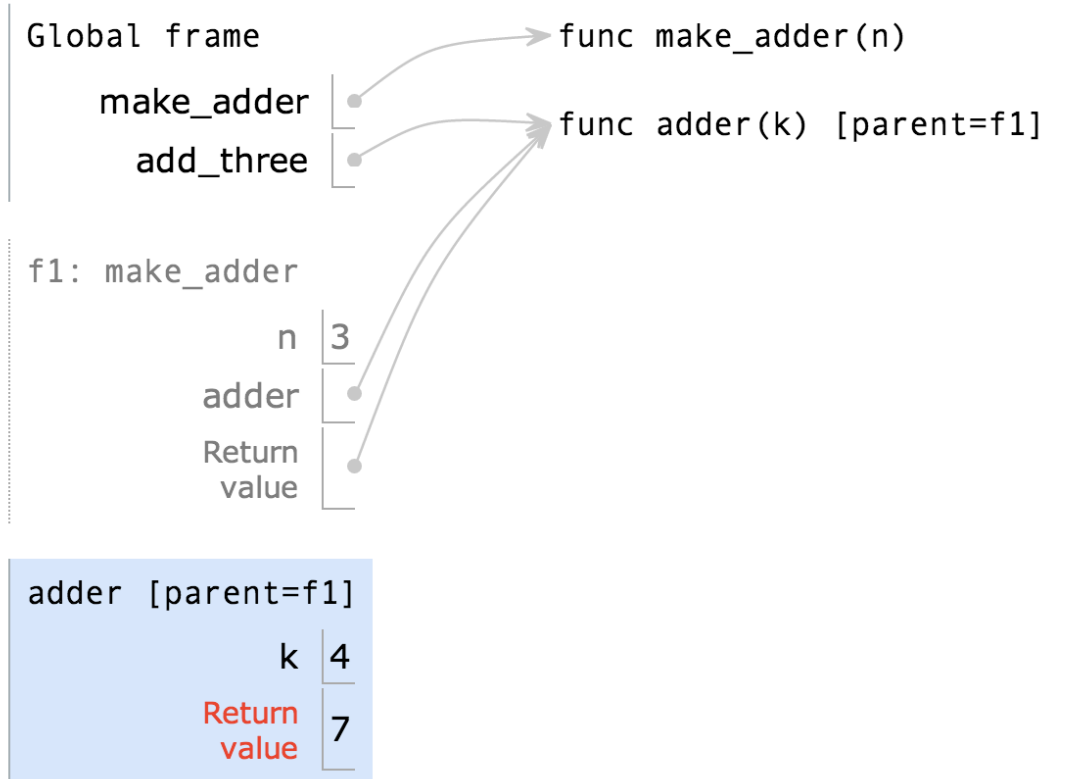
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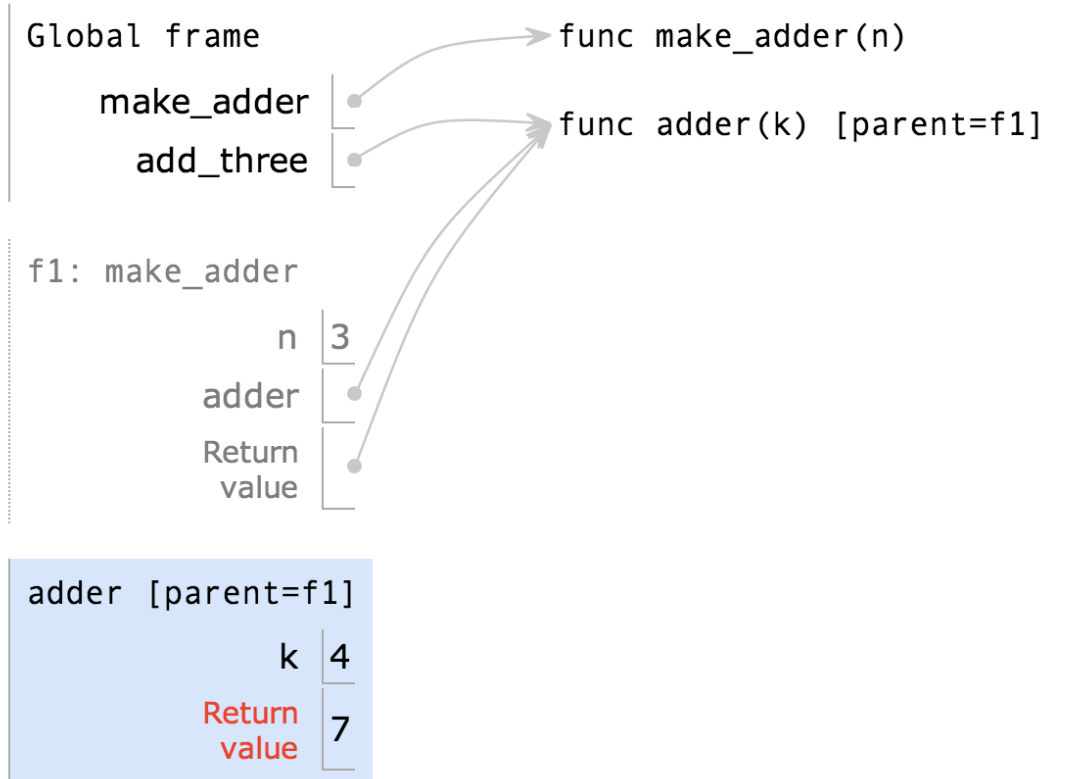
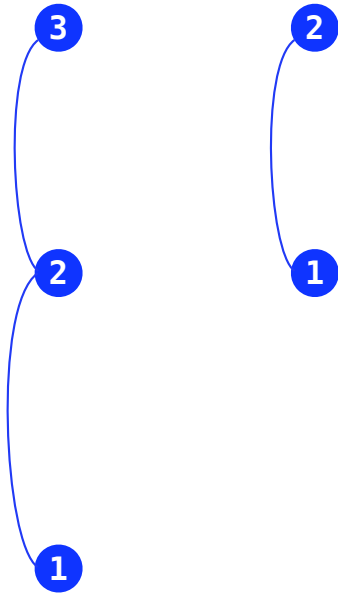


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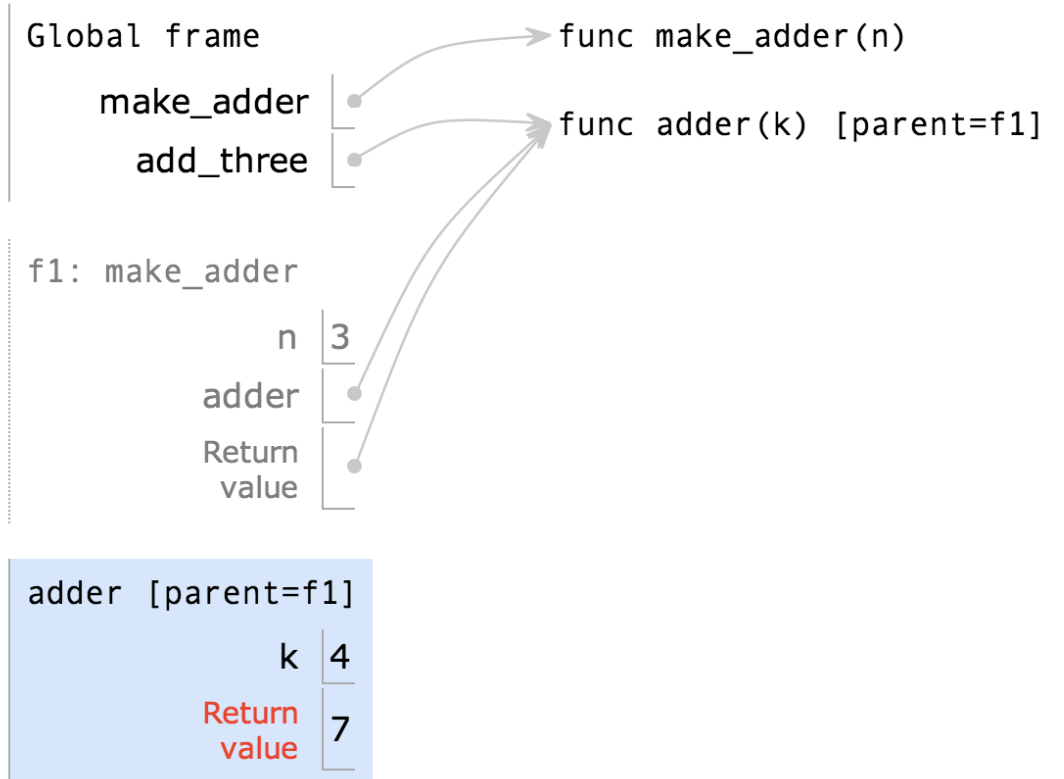
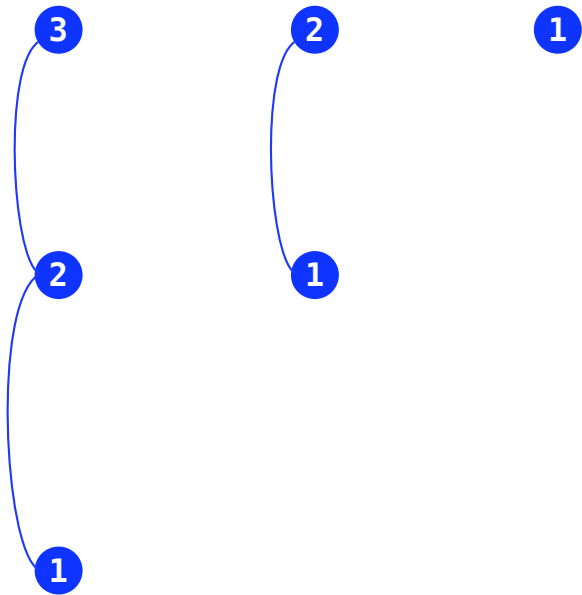
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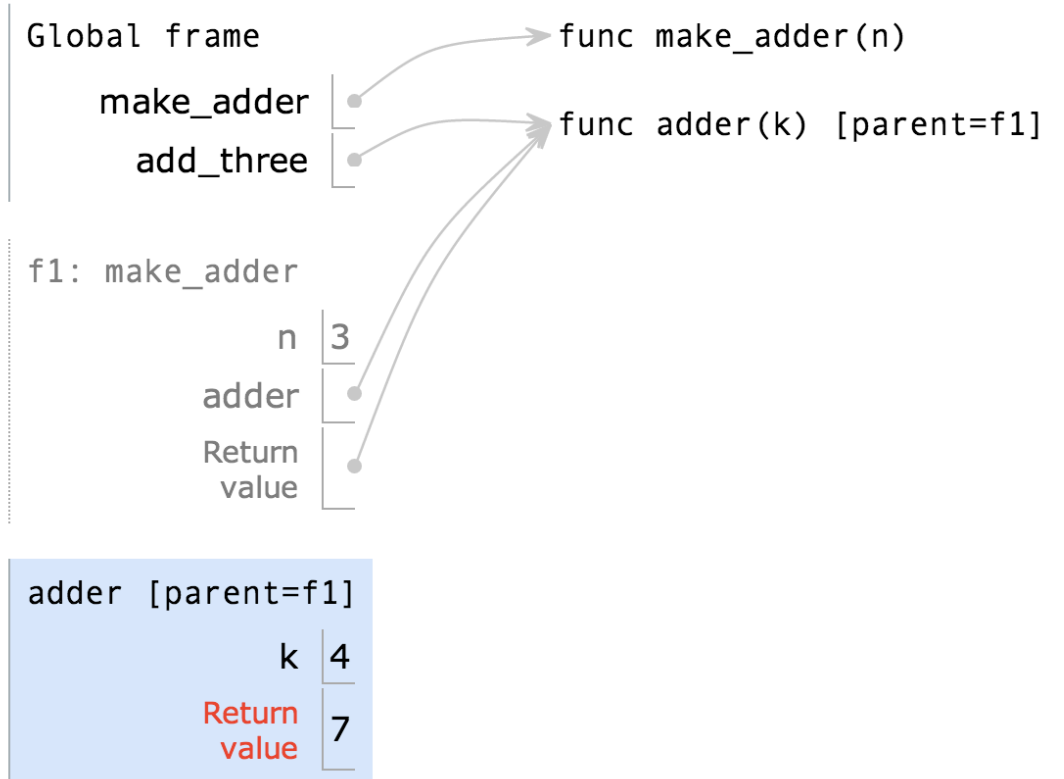
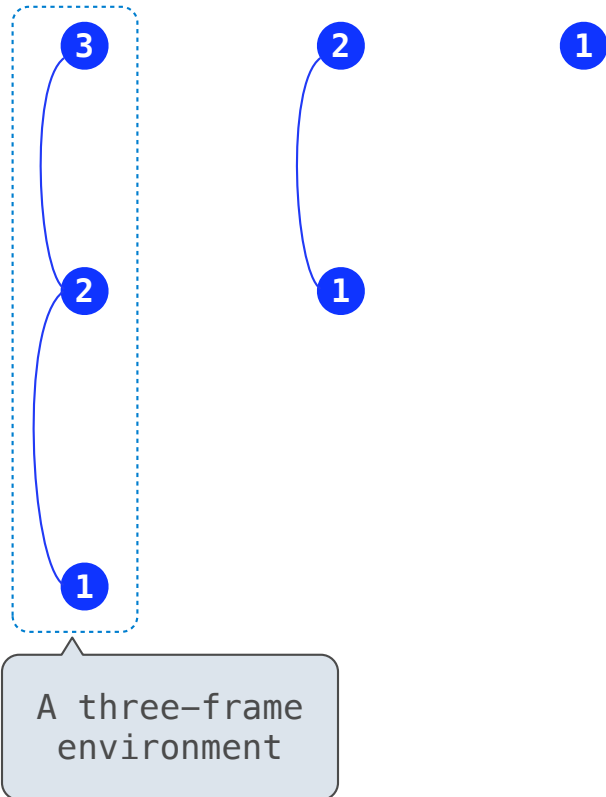
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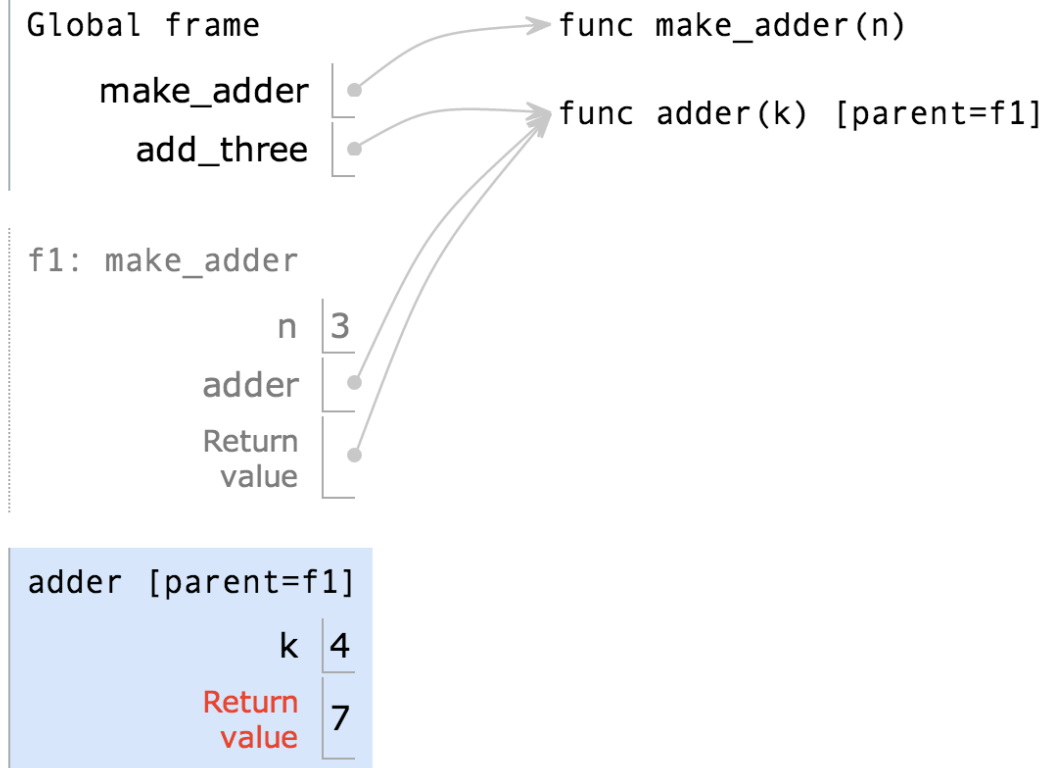
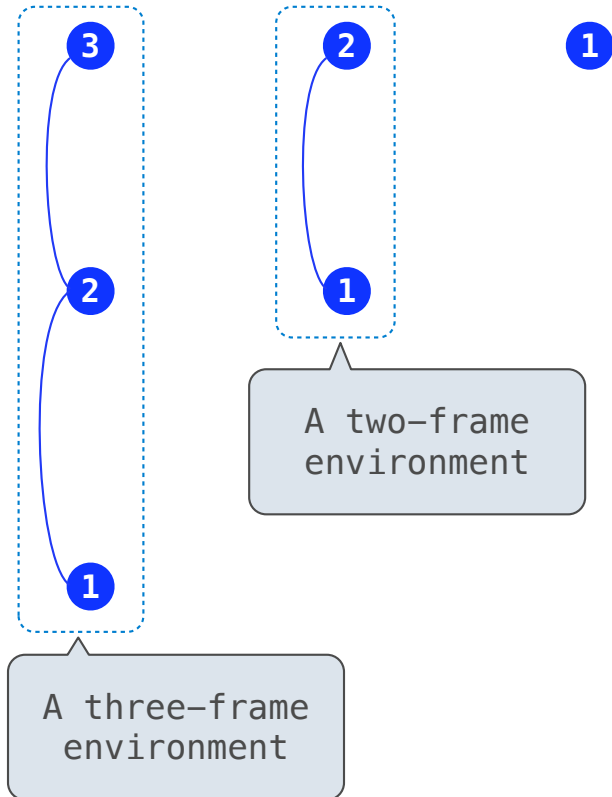
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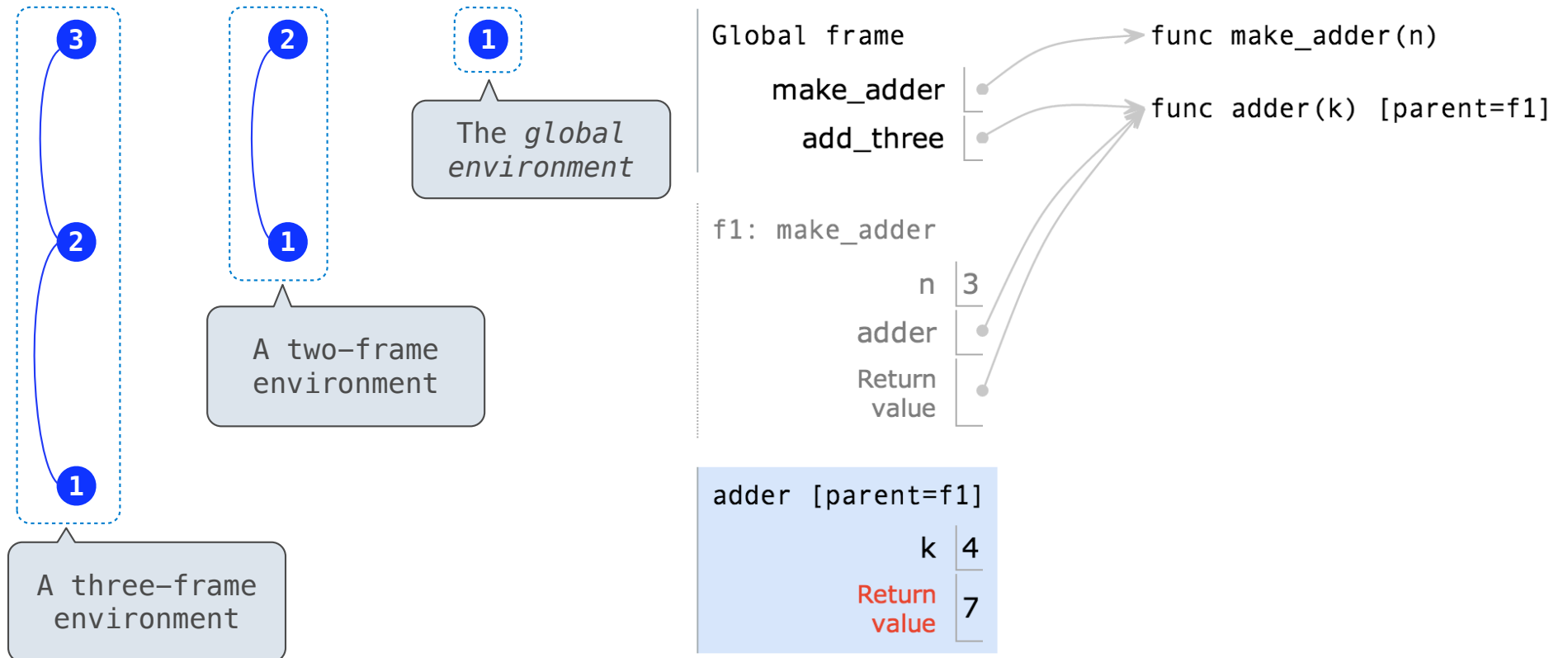
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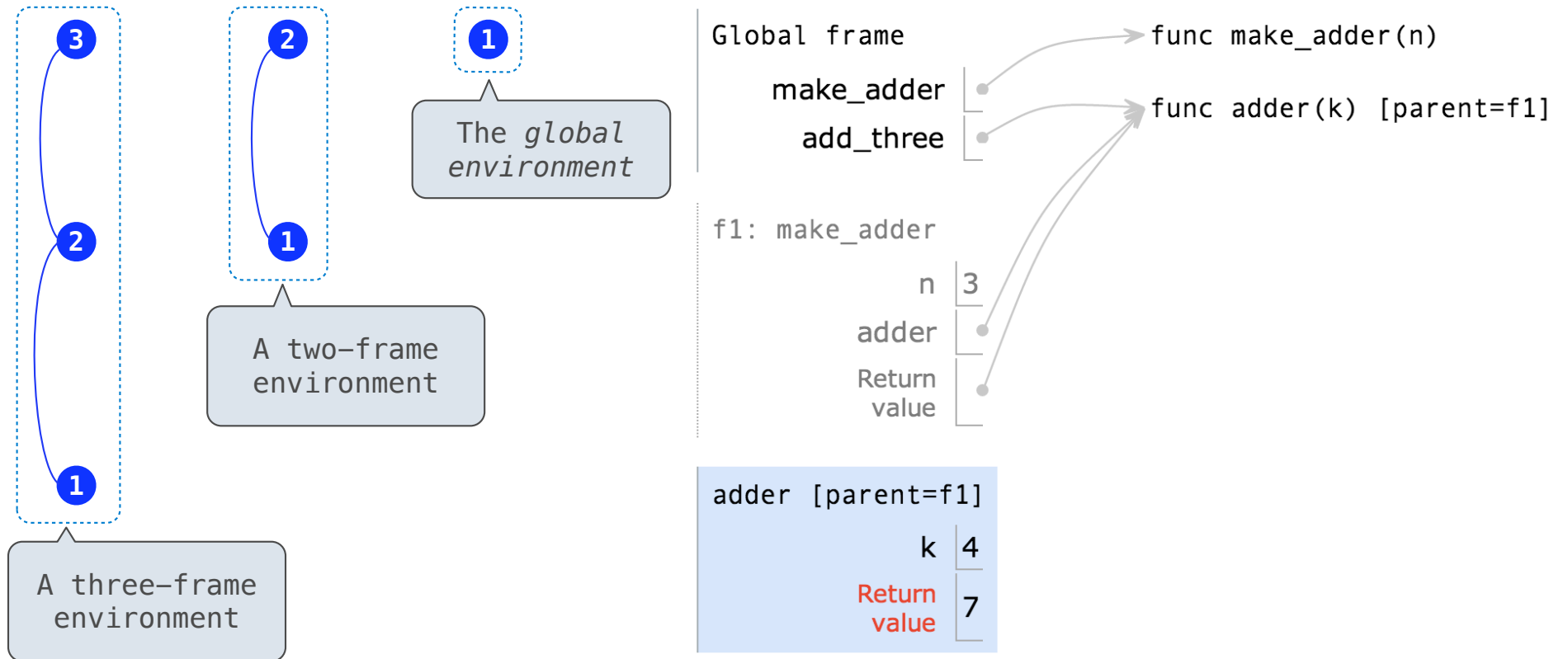
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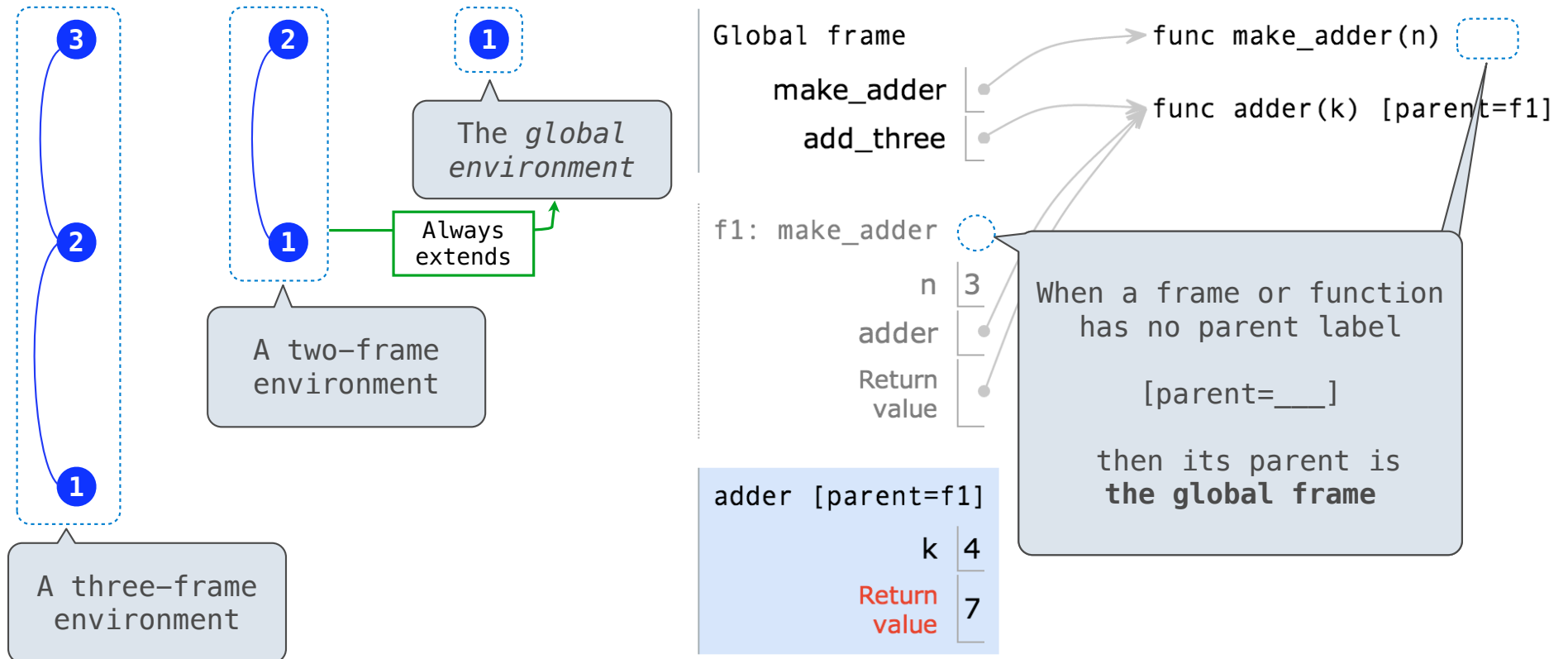


An Environment is a Sequence of Frames



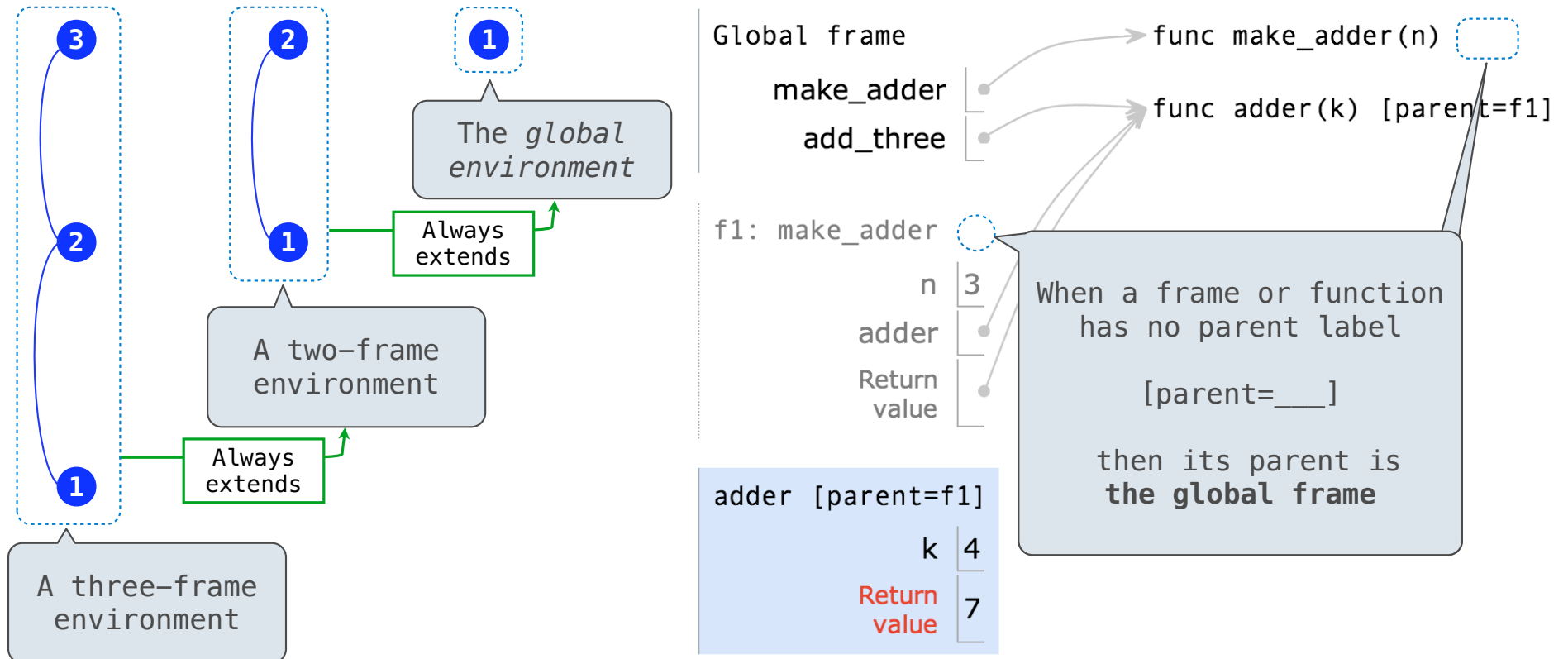
A local frame extends the environment that begins with its parent.

An Environment is a Sequence of Frames



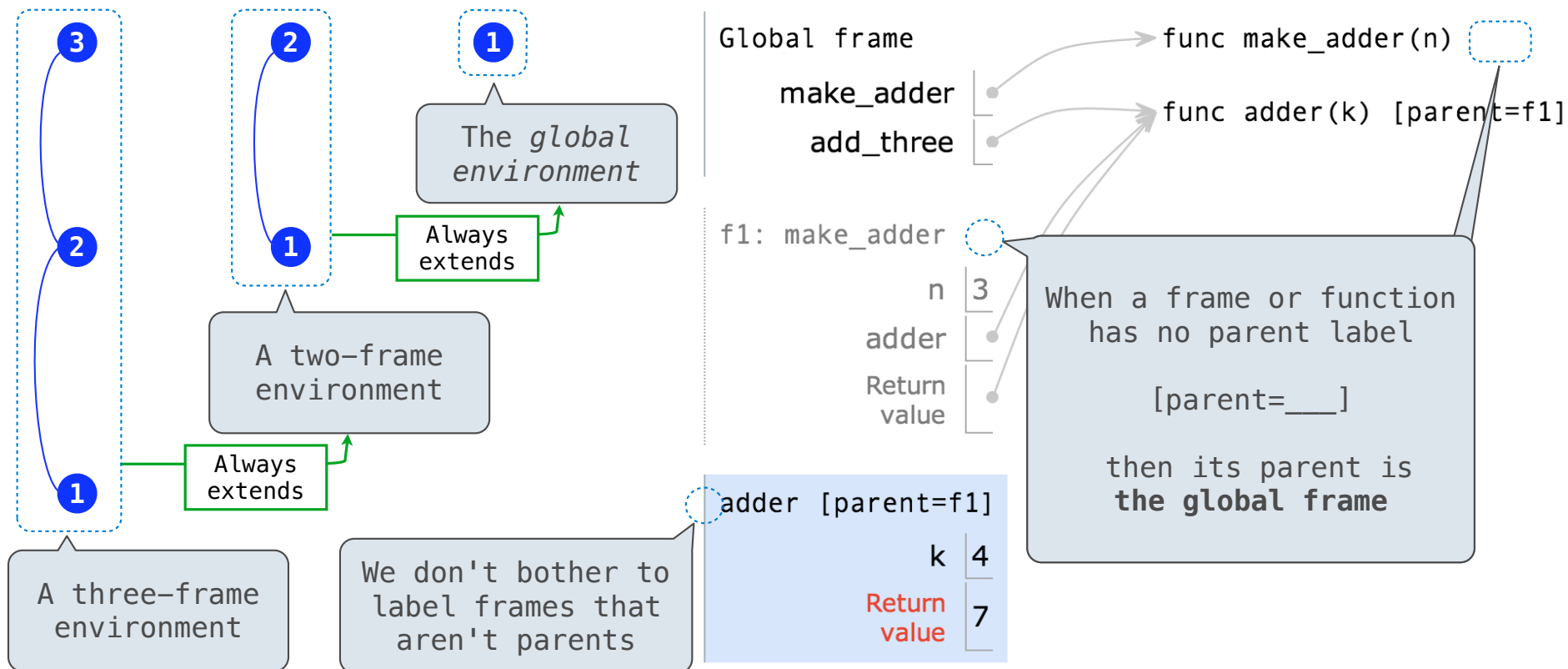
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An Environment is a Sequence of Frames



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How to Draw an Environment Diagram

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When a function is called:

1. Add a **local frame**, titled with the `<name>` of the function being called.

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f1: make_adder      func adder(k) [parent=f1]
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2. If the function has a parent label, copy it to the **local frame**: `[parent=<label>]`

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1. Add a **local frame**, titled with the `<name>` of the function being called.
2. If the function has a parent label, copy it to the **local frame**: `[parent=<label>]`
3. Bind the `<formal parameters>` to the arguments in the **local frame**.
4. Execute the body of the function in the environment that starts with the **local frame**.

Local Names

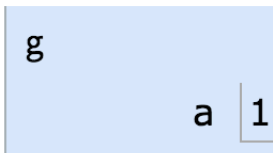
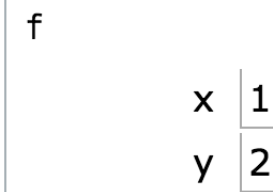
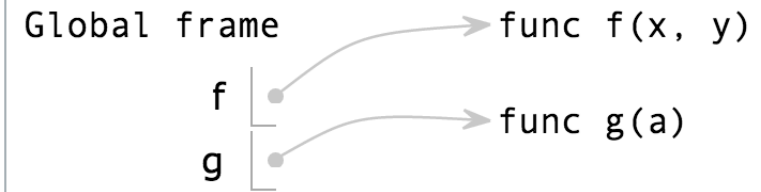
(Demo)

Local Names are *not* Visible to Other (Non-Nested) Functions

```
1 def f(x, y):  
→ 2     return g(x)  
3  
4 def g(a):  
→ 5     return a + y  
6  
7 result = f(1, 2)
```

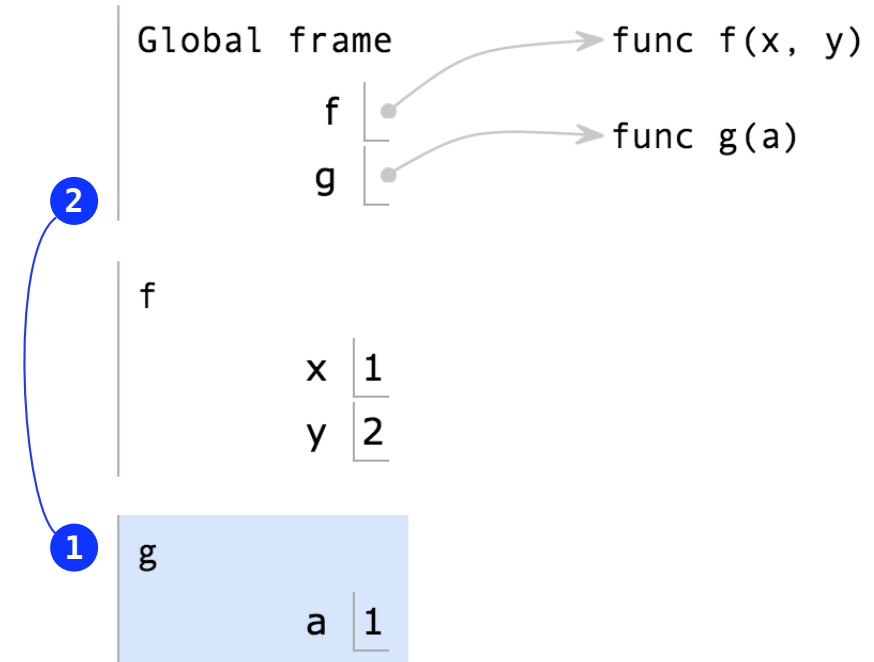
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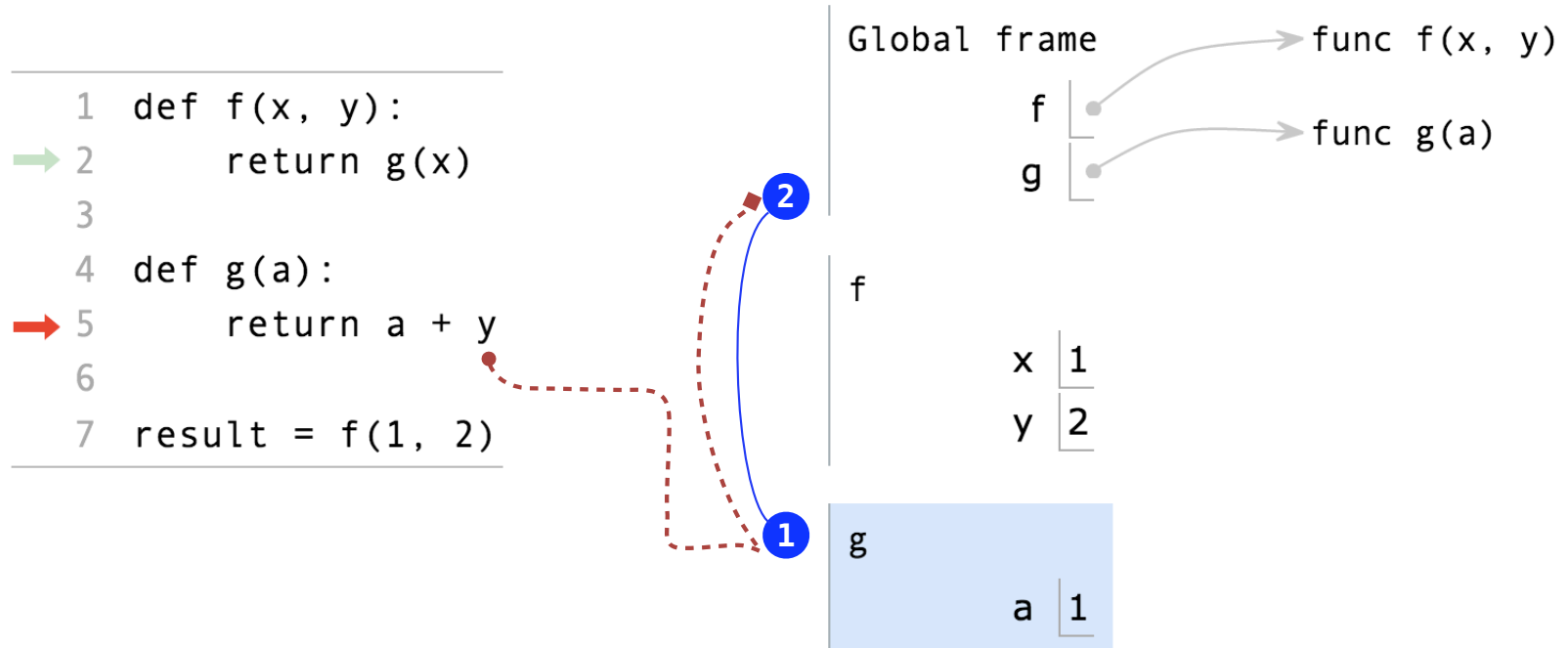


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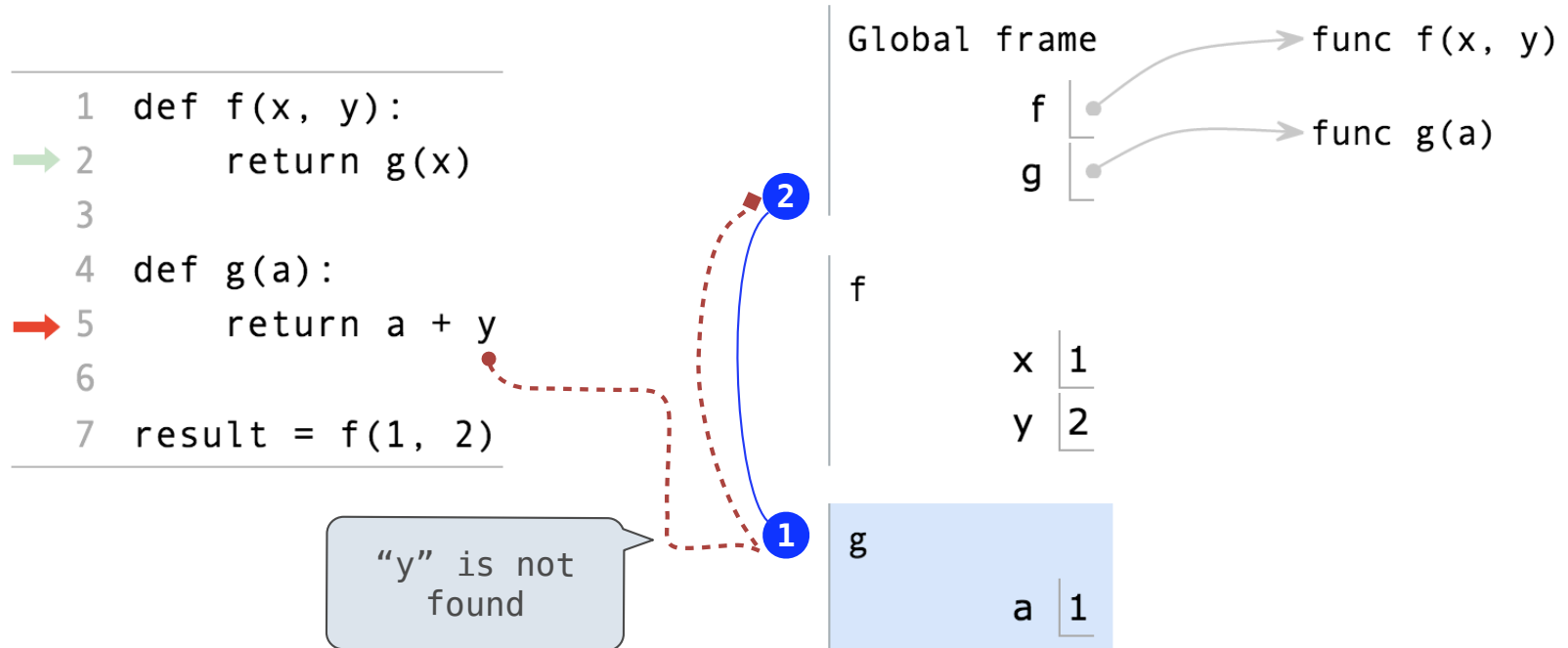
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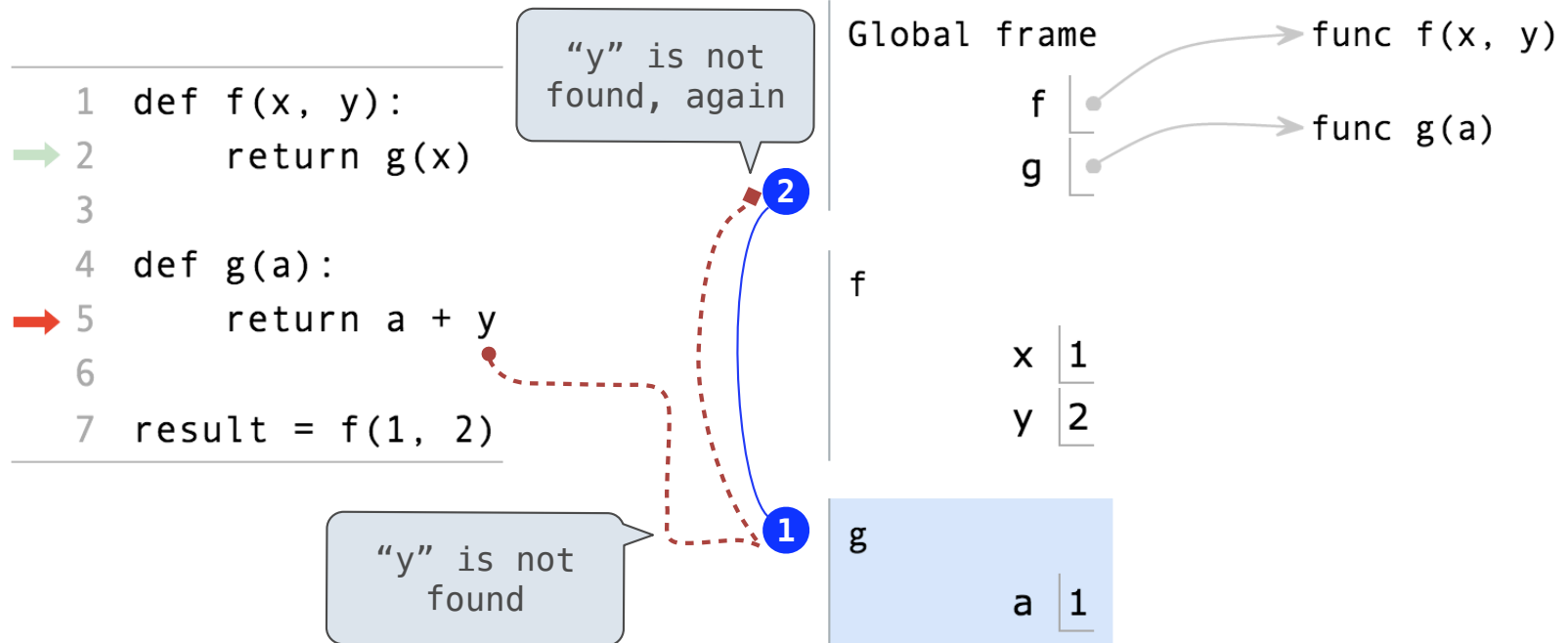
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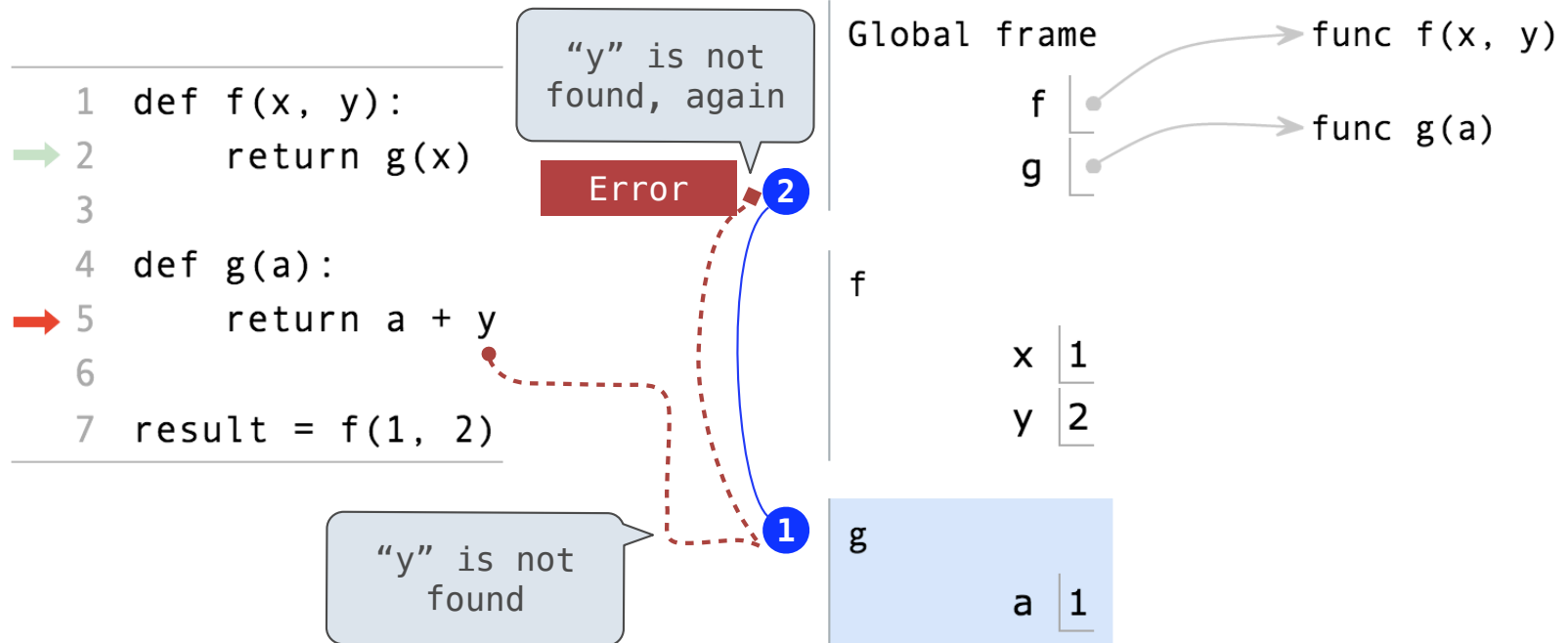
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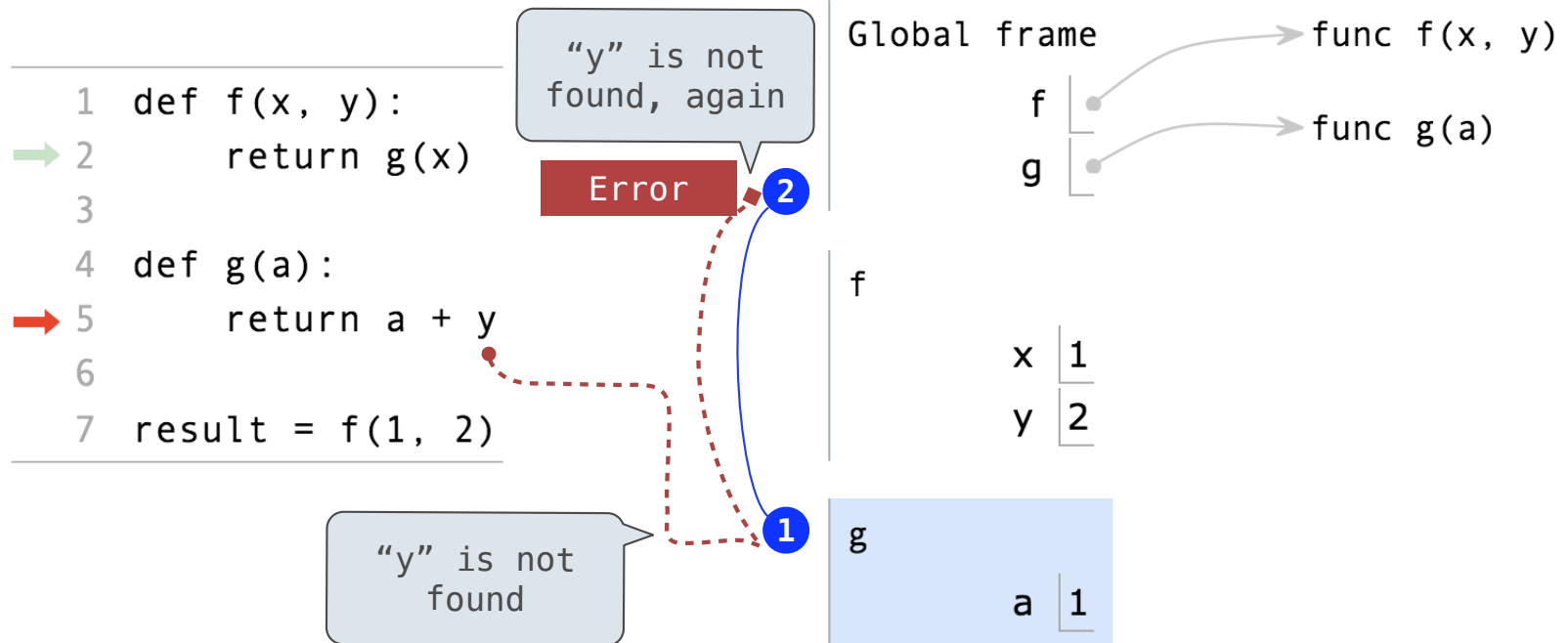
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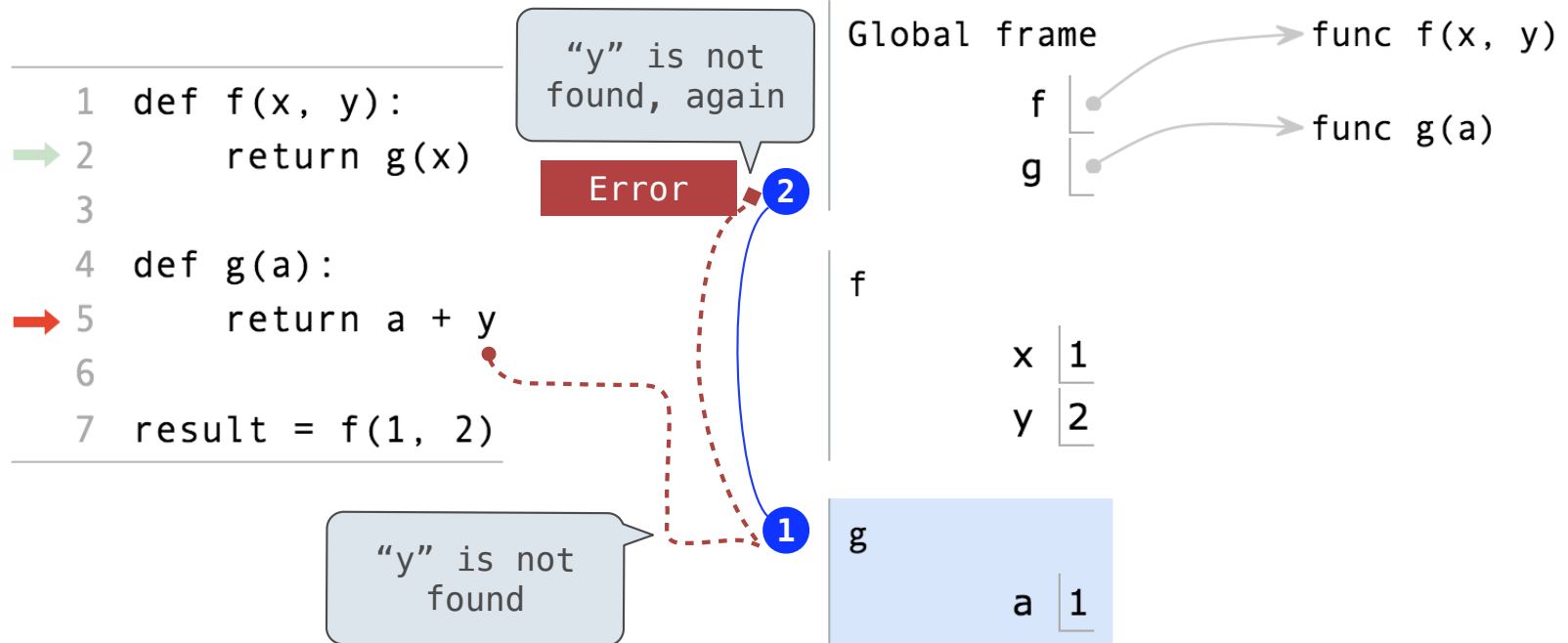


Local Names are *not* Visible to Other (Non-Nested) Functions



- An environment is a sequence of frames.

Local Names are *not* Visible to Other (Non-Nested) Functions



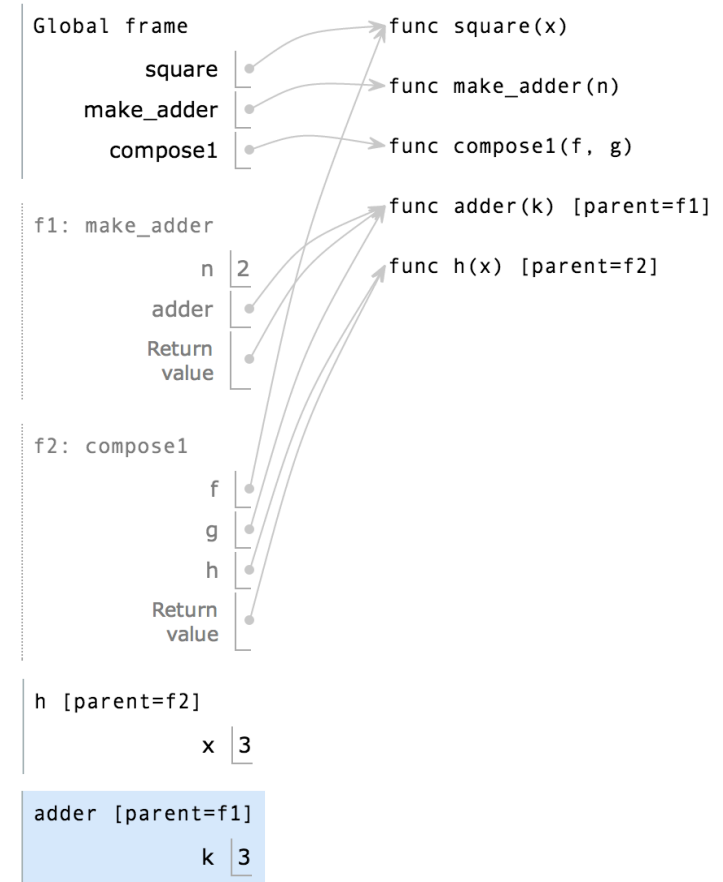
- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

Function Composition

(Demo)

The Environment Diagram for Function Composition

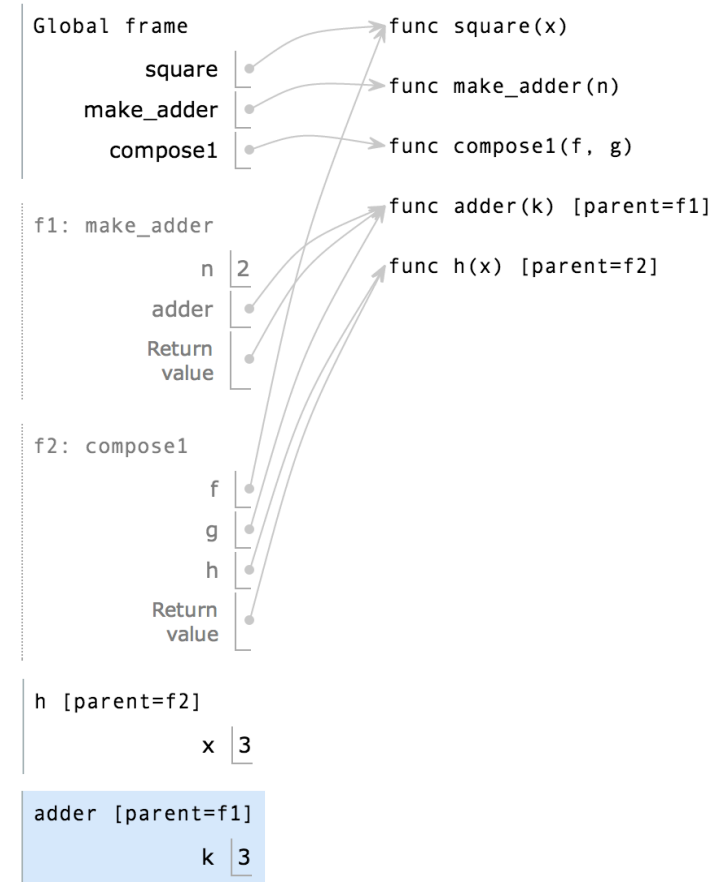
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8
9 def compose1(f, g):
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14 compose1(square, make_adder(2))(3)
```



Example:

The Environment Diagram for Function Composition

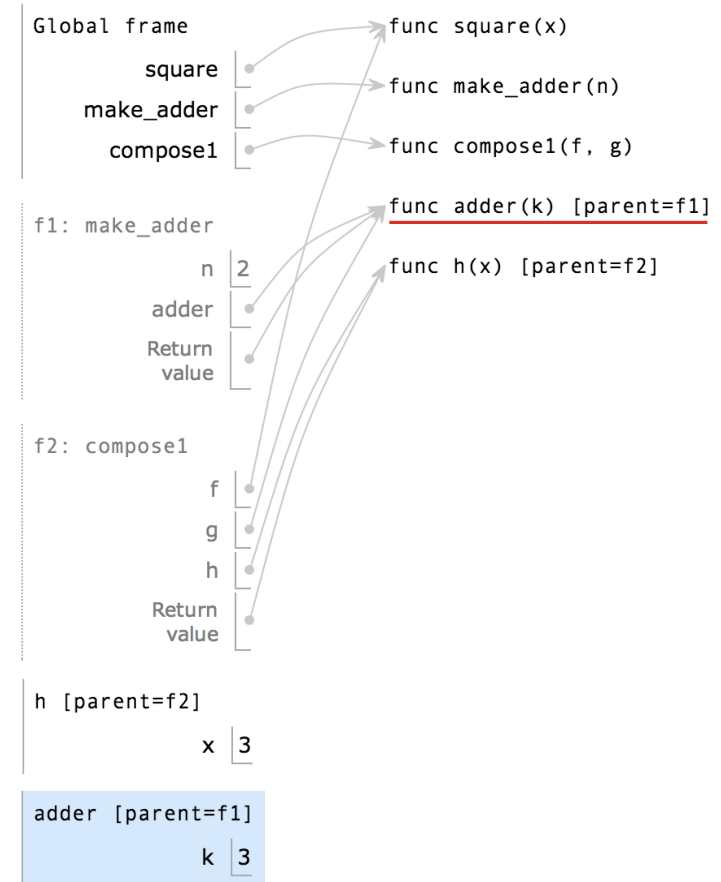
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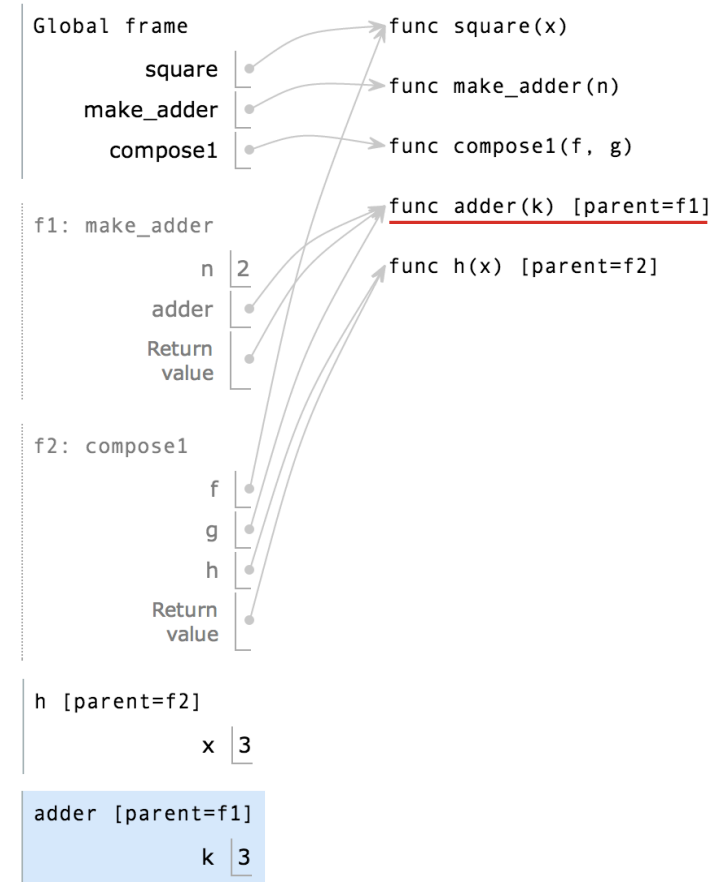
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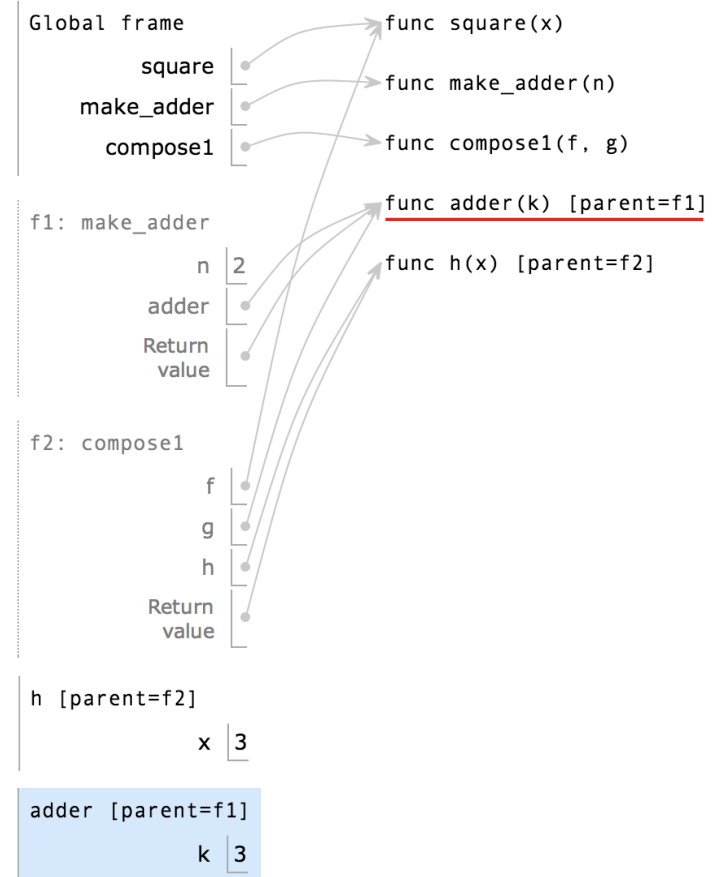


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Return value of make_adder is an argument to compose1

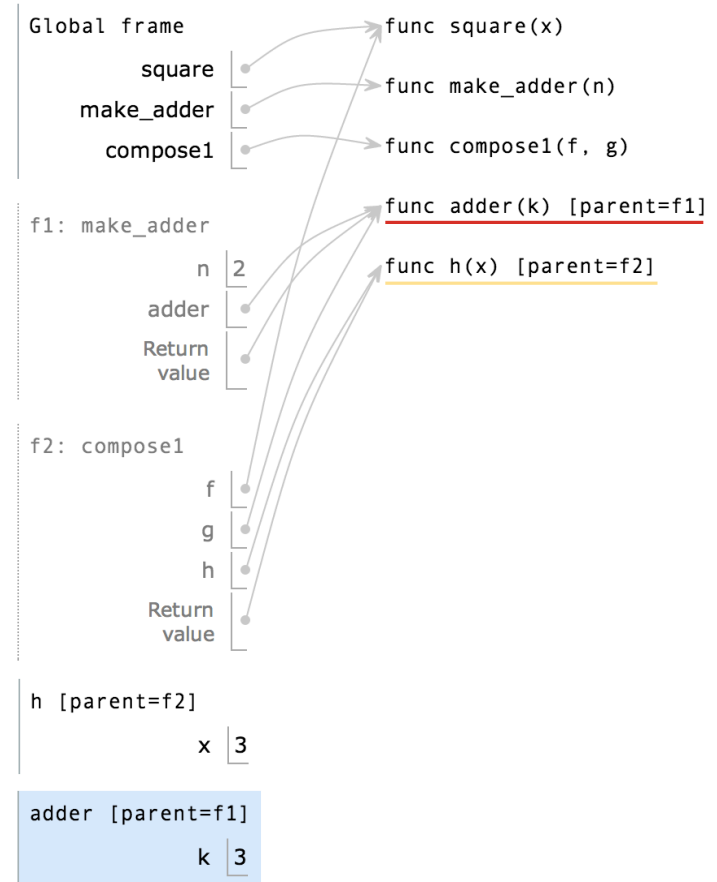


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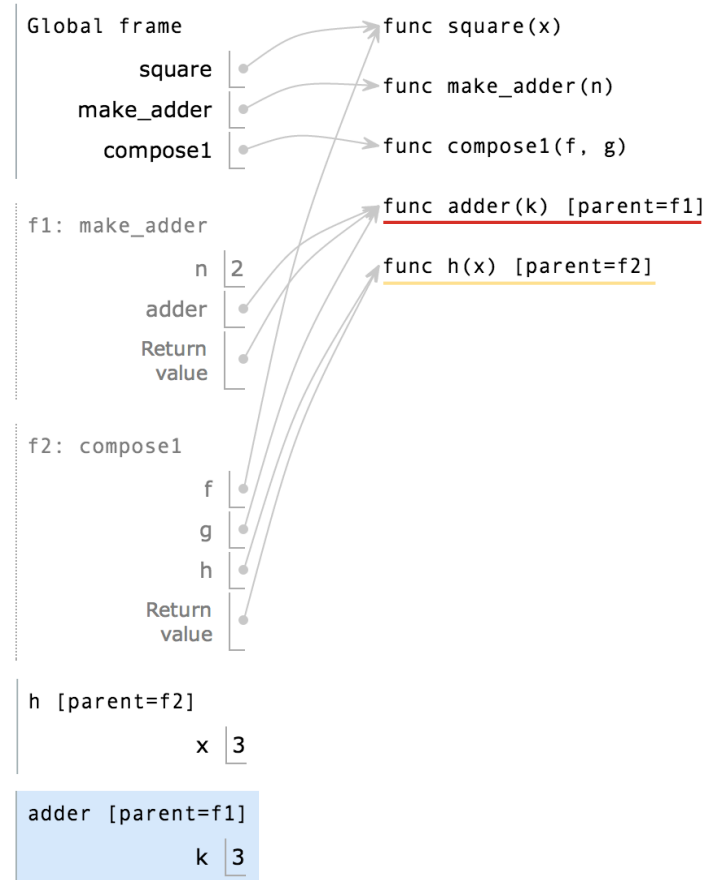


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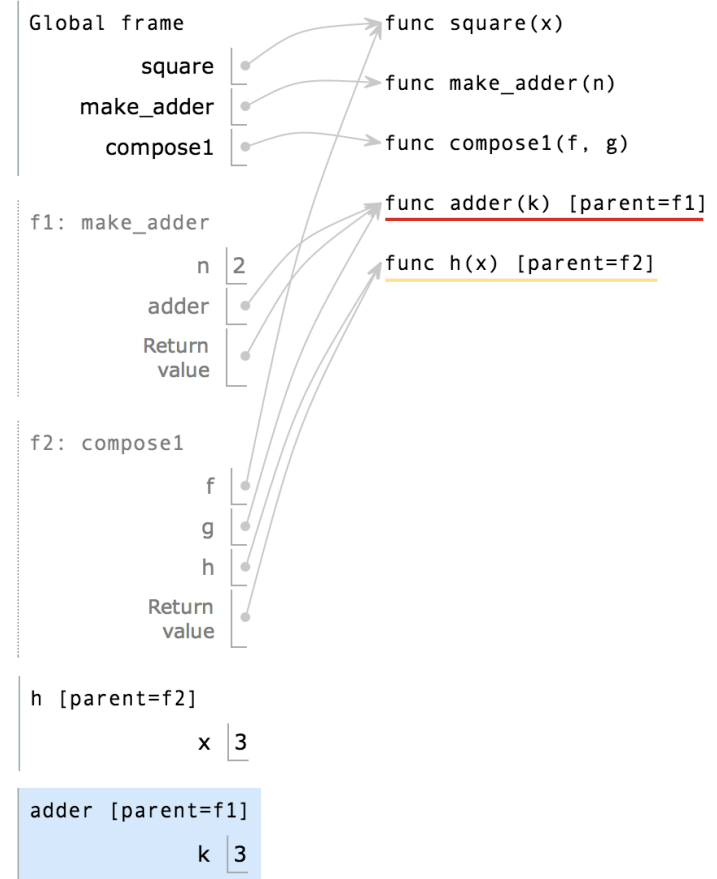


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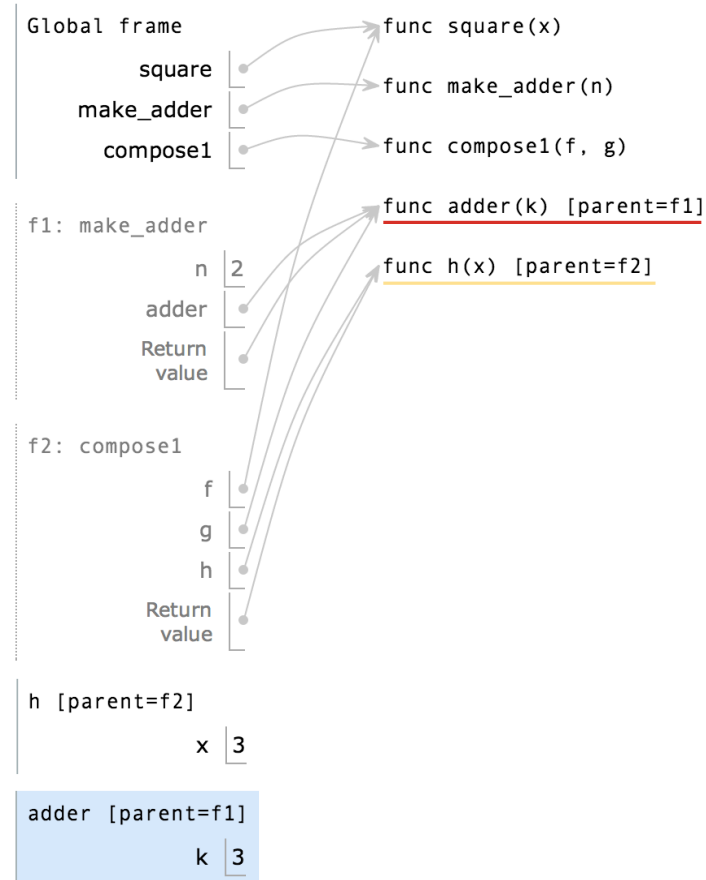
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Annotations: A green arrow points to line 6, and a red arrow points to line 7. A blue circle with the number 3 is at the end of a blue line pointing to line 3. A blue circle with the number 2 is at the end of a blue line pointing to line 11. A blue circle with the number 1 is at the end of a blue line pointing to line 14. A dashed orange box surrounds the call `compose1(square, make_adder(2))(3)`, with a red dashed box around `make_adder(2)`.

Return value of `make_adder` is an argument to `compose1`



Example:

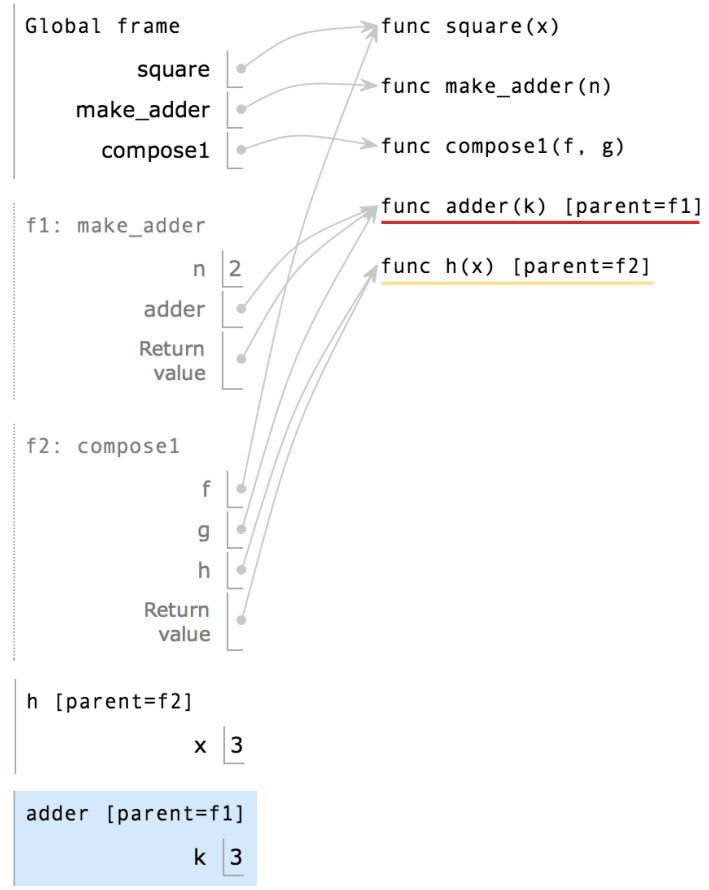
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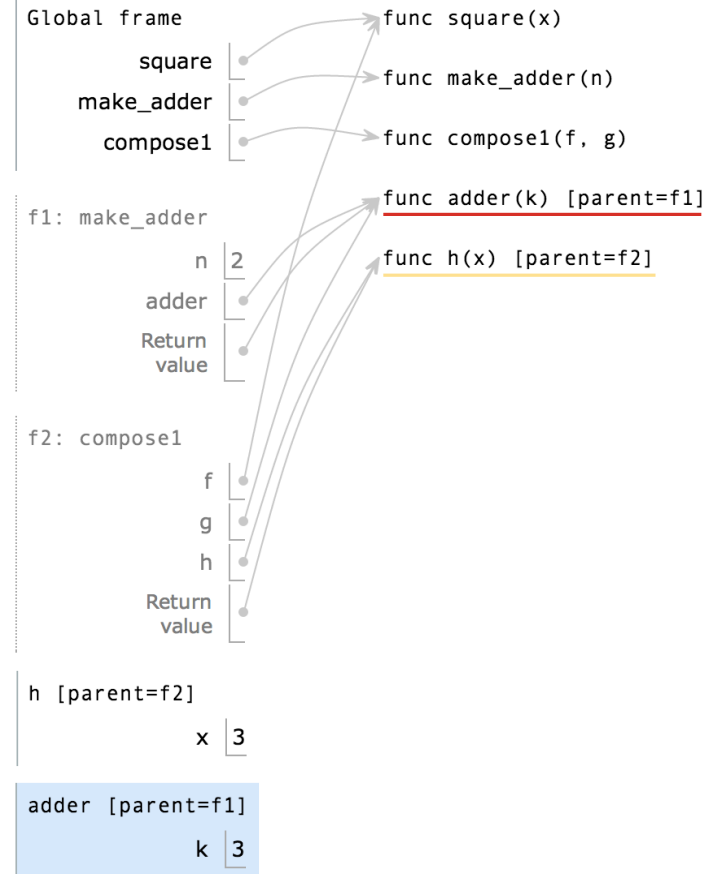
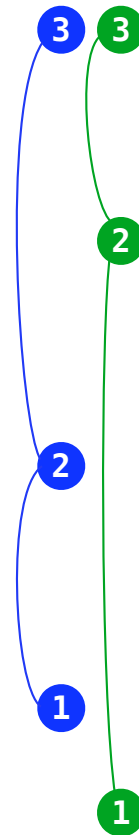
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Example:

The Game of Hog

(Demo)