61A Lecture 30

Wednesday, November 7

Functional Programming

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```
def factorial(n, k):
    while n > 0:
        n, k = n-1, k*n
    return k
```

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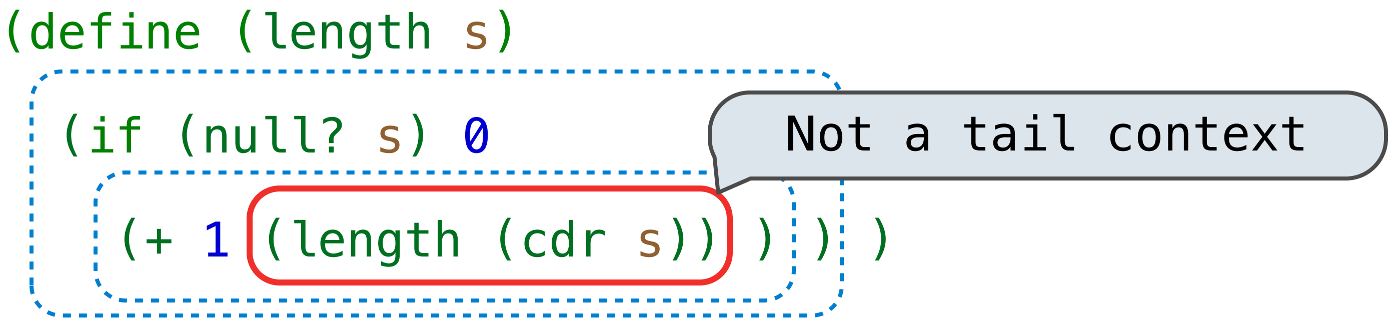

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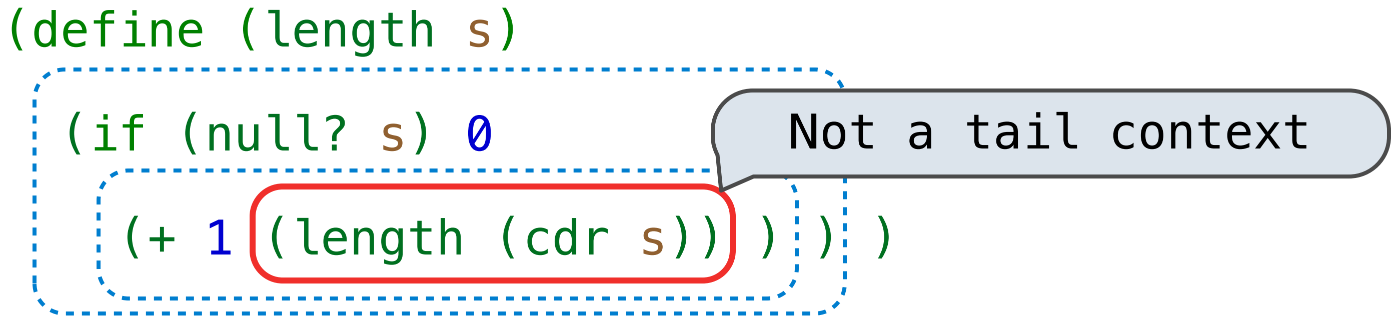

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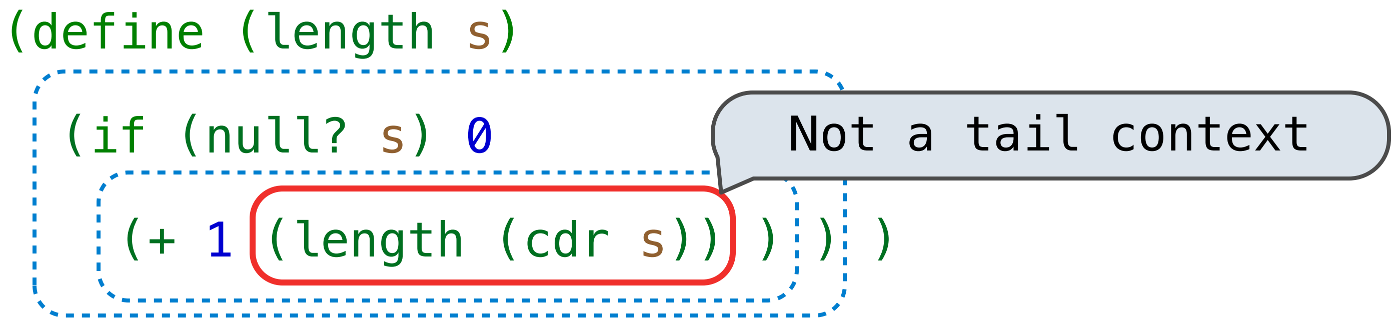

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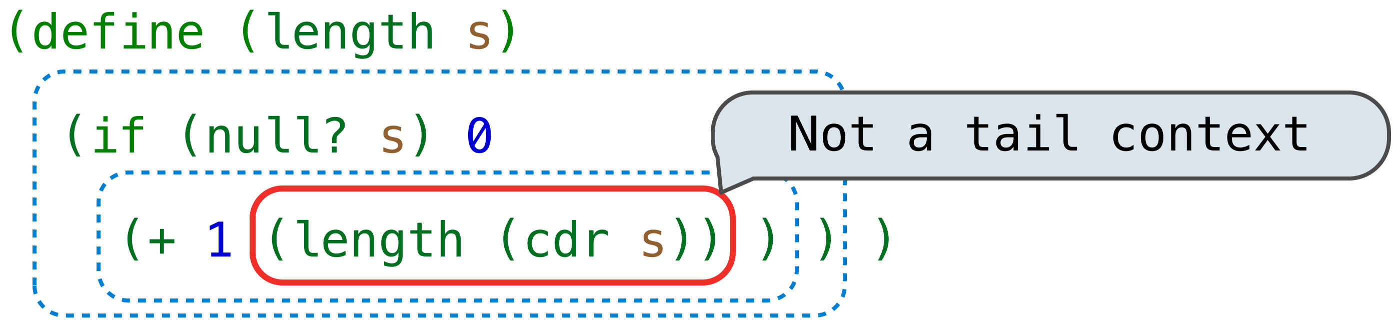

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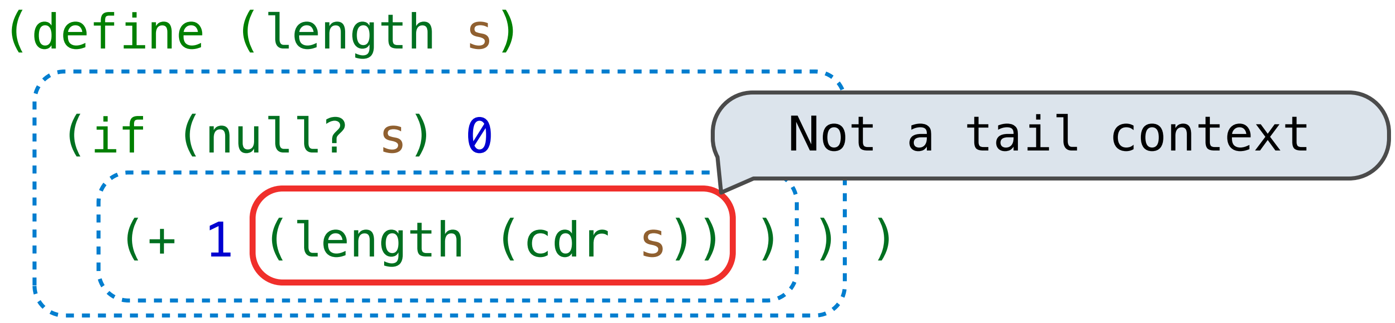

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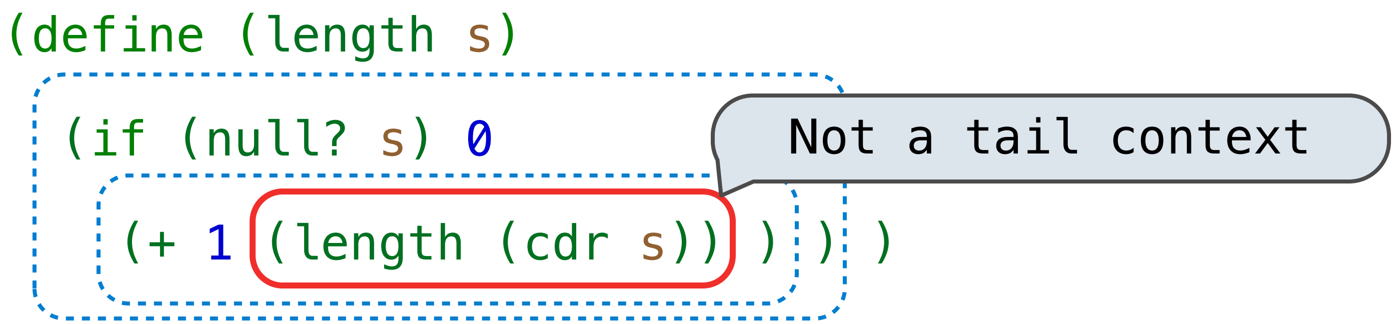

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Recursive call is a tail call

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Demo

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Logical forms may only evaluate some sub-expressions.

- **If** expression: (if <predicate> <consequent> <alternative>)
- **And** and **or**: (and <e_1> ... <e_n>), (or <e_1> ... <e_n>)
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E.g., replace (if false 1 (+ 2 3)) with (+ 2 3) and repeat.

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            (fn start (car s)) ) ) )
```

Recursive call is a tail call.

```
 (reduce * '(3 4 5) 2)                                     120

 (reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))          (5 4 3 2)
```

Example: Reduce

```
(define (reduce fn s start)
  (if (null? s) start
    (reduce fn
            (cdr s)
            (fn start (car s)) ) ) )
```

Recursive call is a tail call.

Other calls are not; constant space depends on fn.

```
(reduce * '(3 4 5) 2)                                   120

(reduce (lambda (x y) (cons y x)) '(3 4 5) '(2))        (5 4 3 2)
```

Example: Map

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(define (map fn s)
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```
(define (map fn s)
  (define (map-iter fn s m)
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An Analogy: Programs Define Machines

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Programs specify the logic of a computational device

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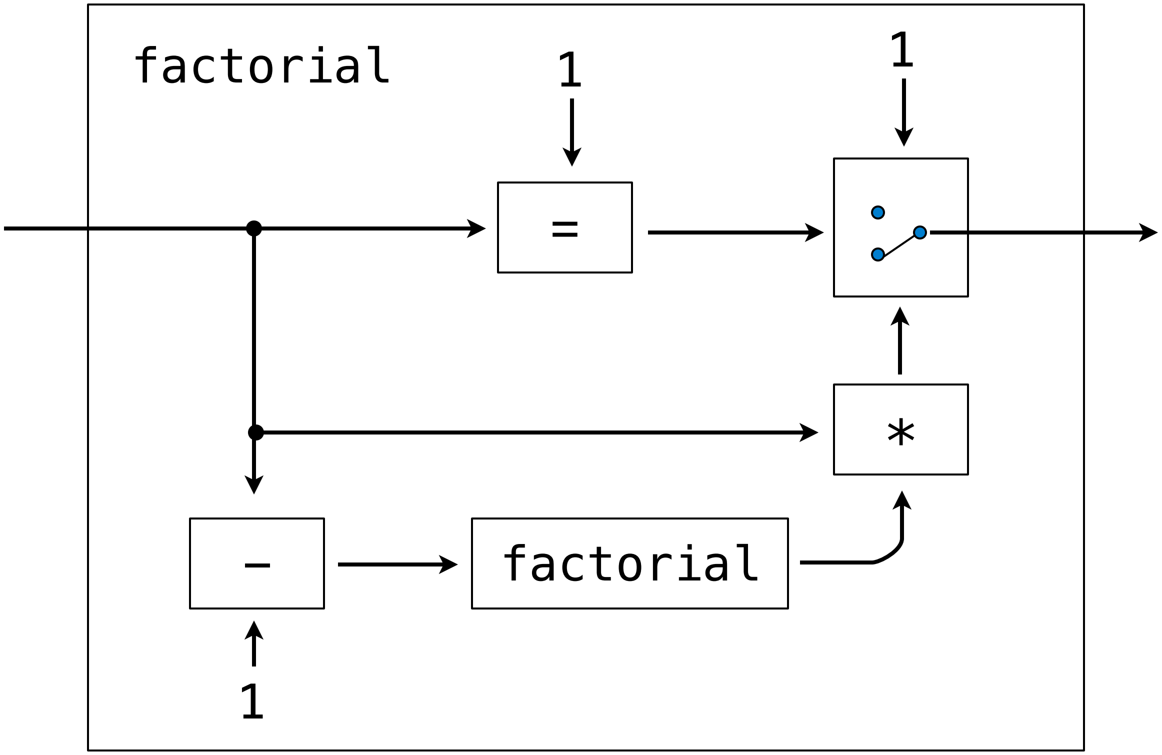
Programs specify the logic of a computational device

factorial

An Analogy: Programs Define Machines

`Programs specify the logic of a computational device`



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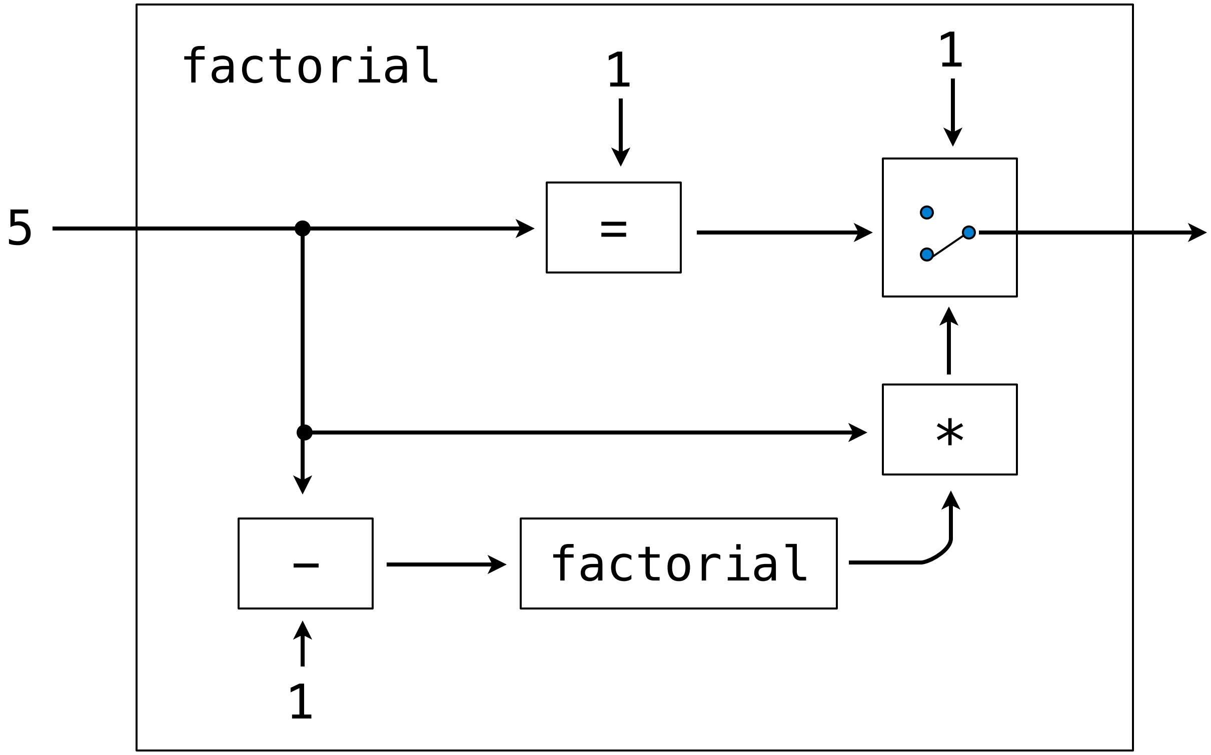

An Analogy: Programs Define Machines

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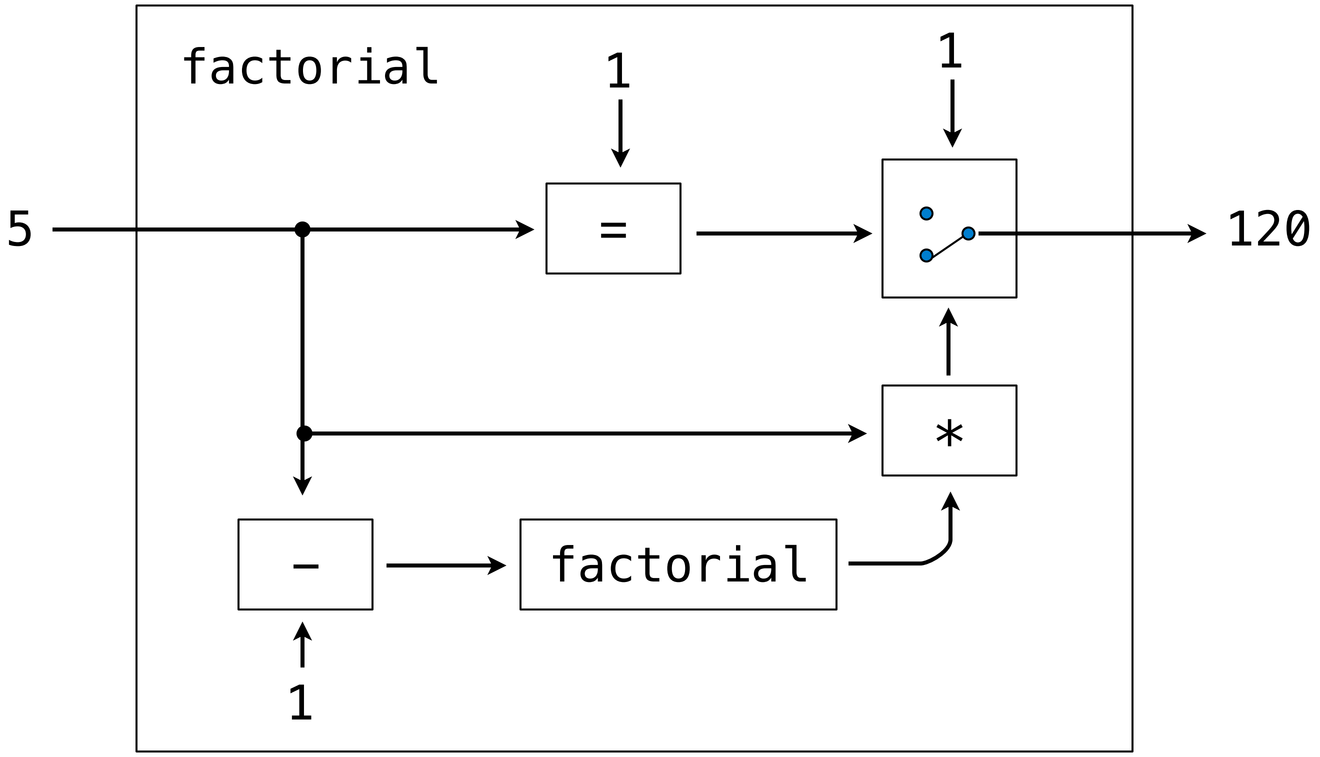

Interpreters are General Computing Machine

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An interpreter can be parameterized to simulate any machine

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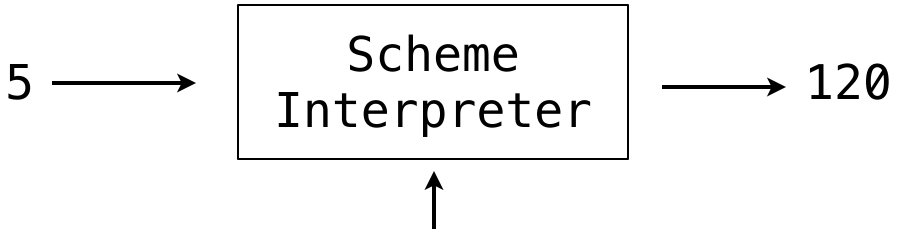

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(define (factorial n)
  (if (zero? n) 1 (* n (factorial (- n 1)))))
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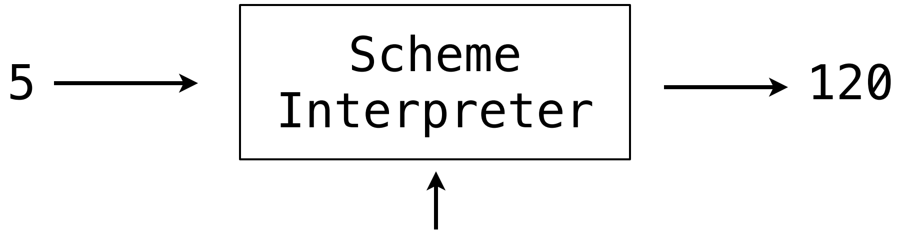

5

Scheme Interpreter

120

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Our Scheme interpreter is a universal machine

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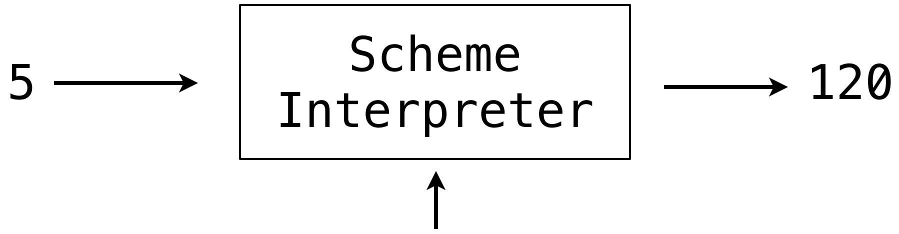

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Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

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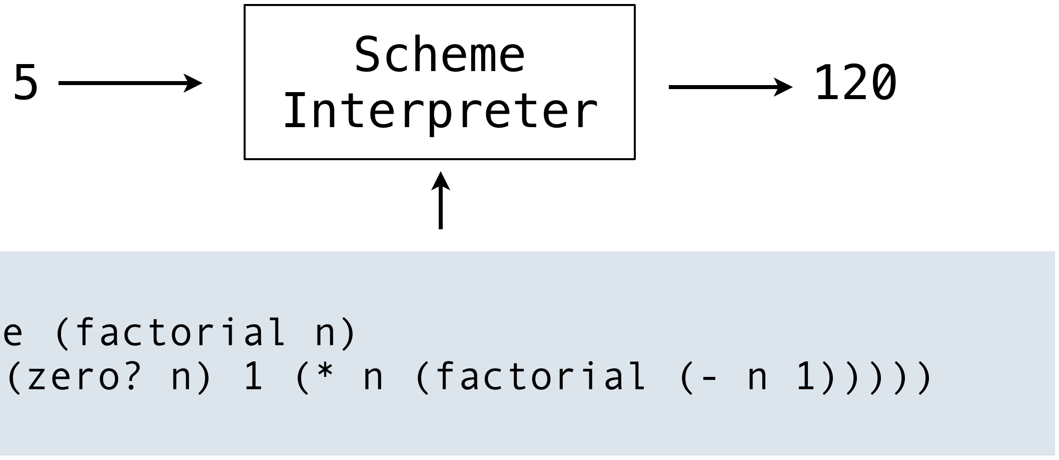

5

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Our Scheme interpreter is a universal machine

A bridge between the data objects that are manipulated by our programming language and the programming language itself

Internally, it is just a set of manipulation rules

Interpretation in Python

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Demo