61A Lecture 22

Wednesday, October 17

Closure Property of Data

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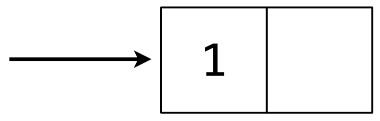

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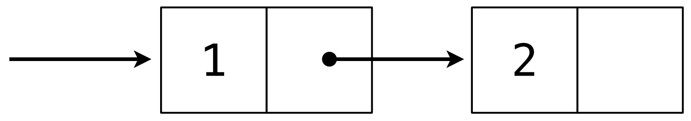

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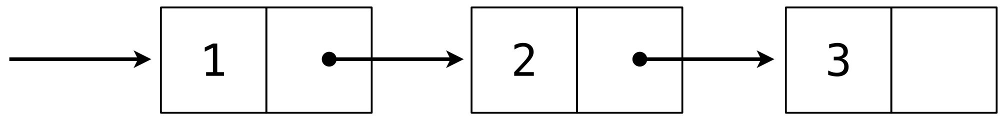

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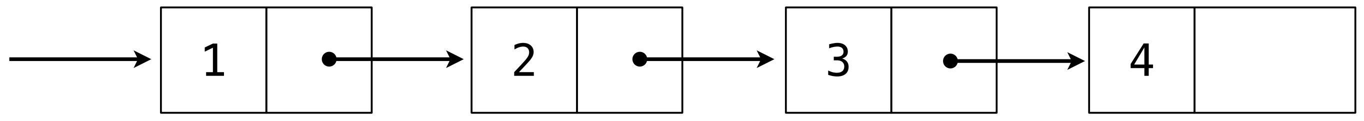

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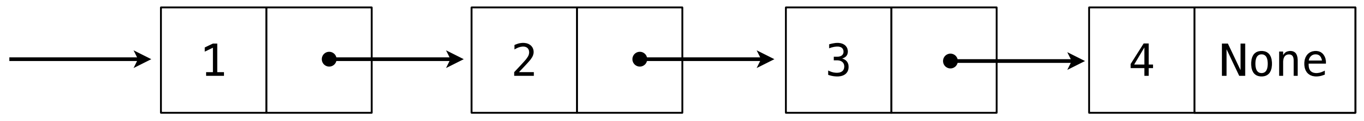

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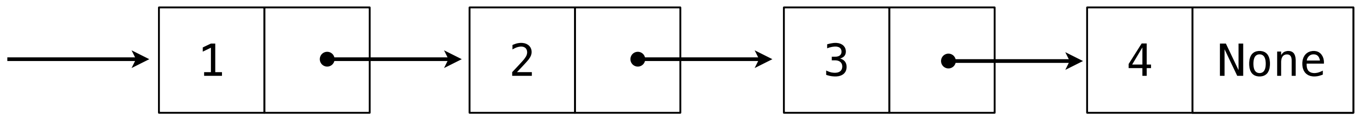

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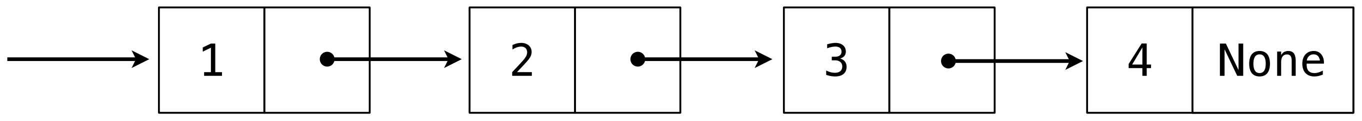

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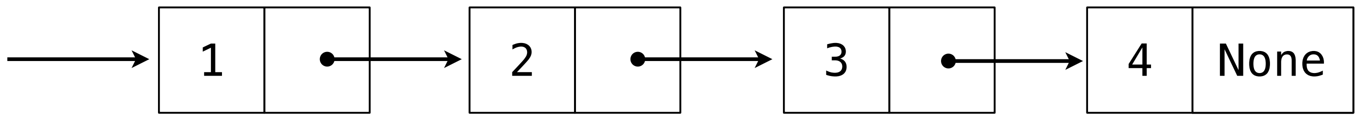

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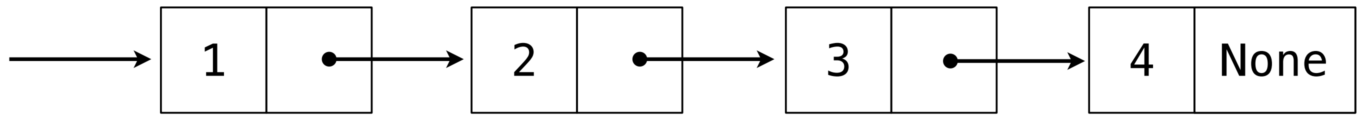

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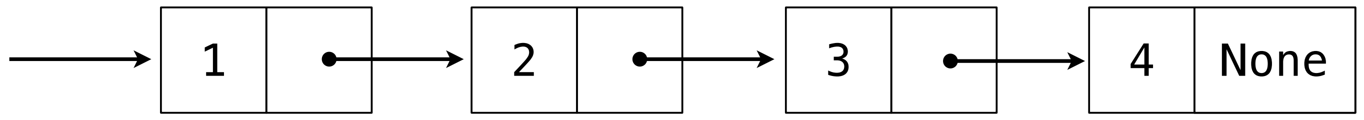

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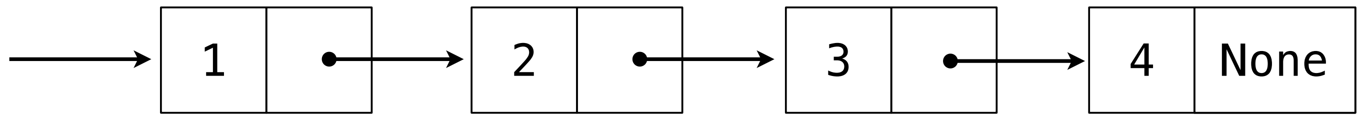

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```
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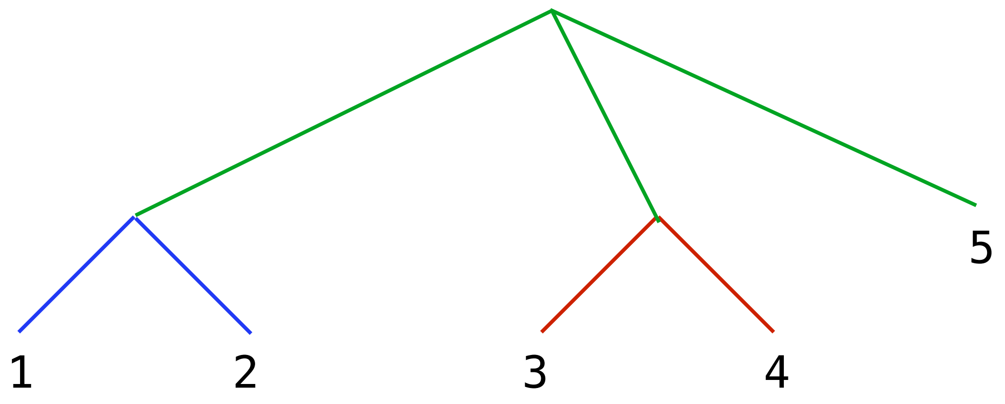

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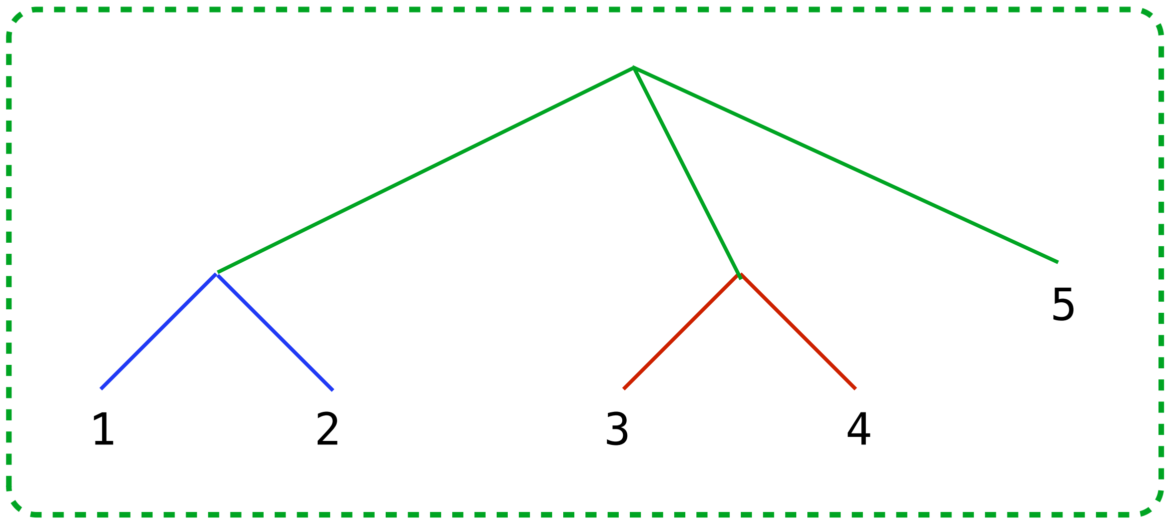

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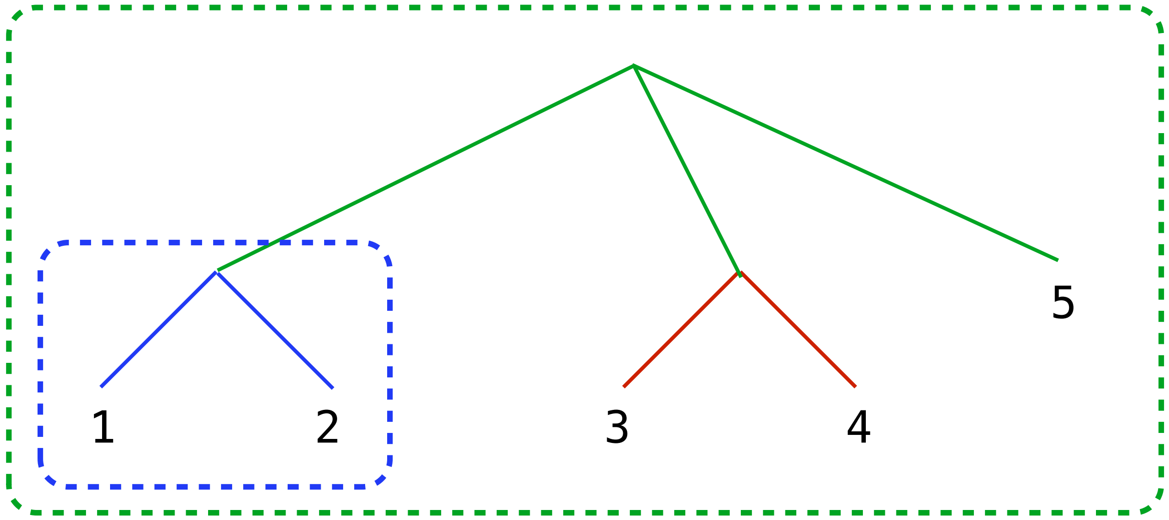

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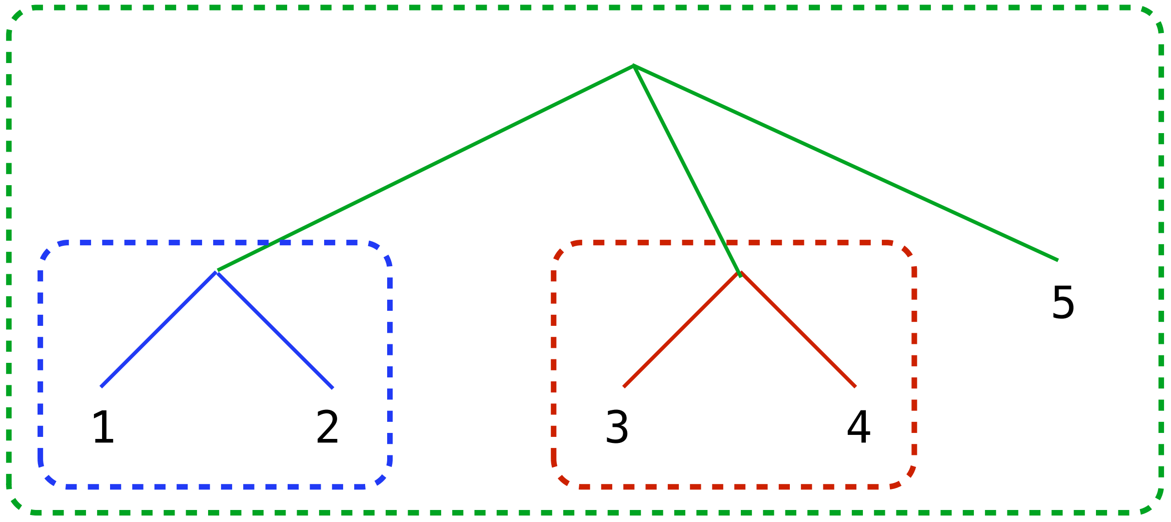

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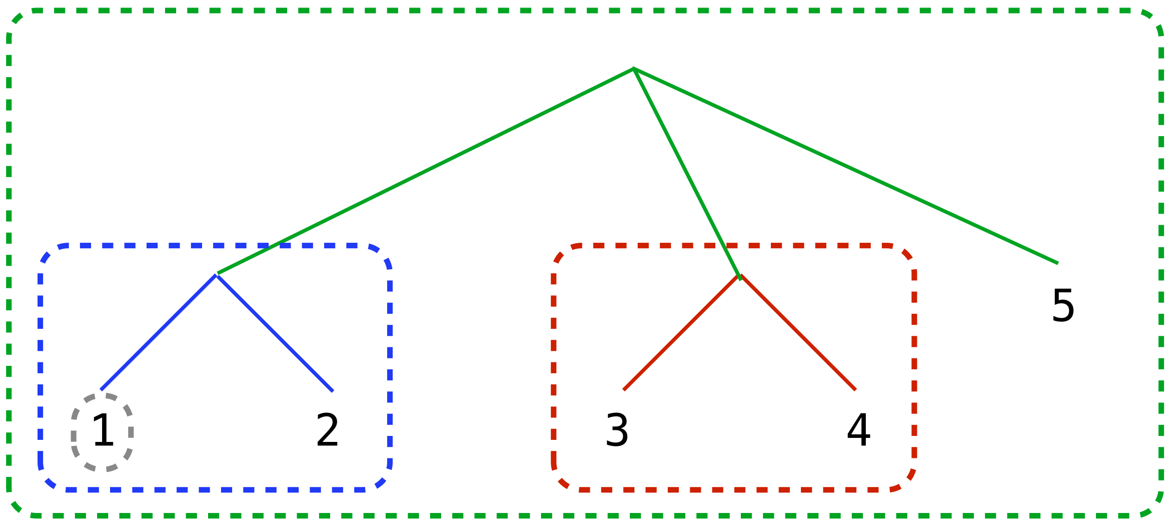

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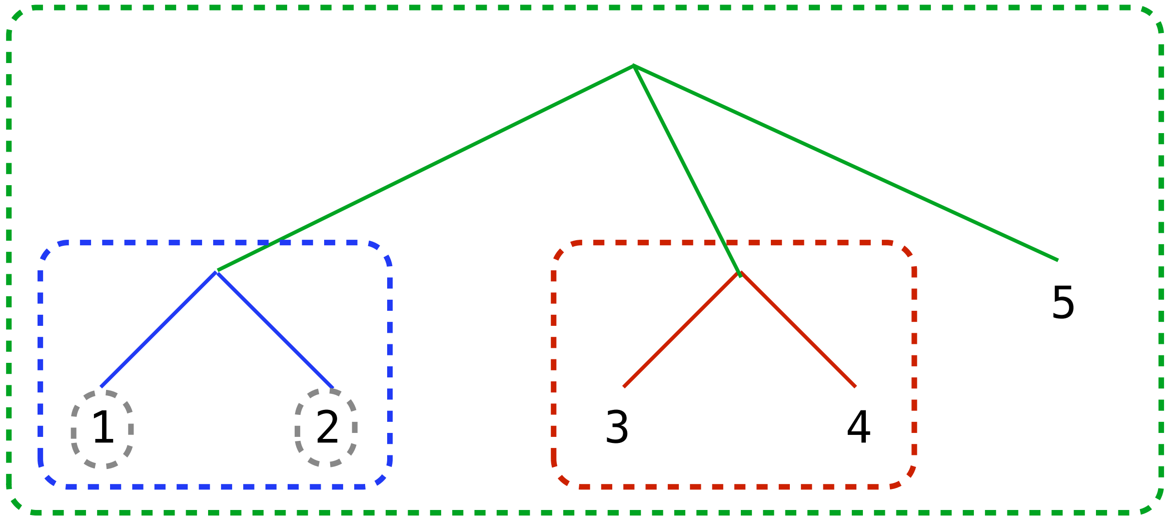

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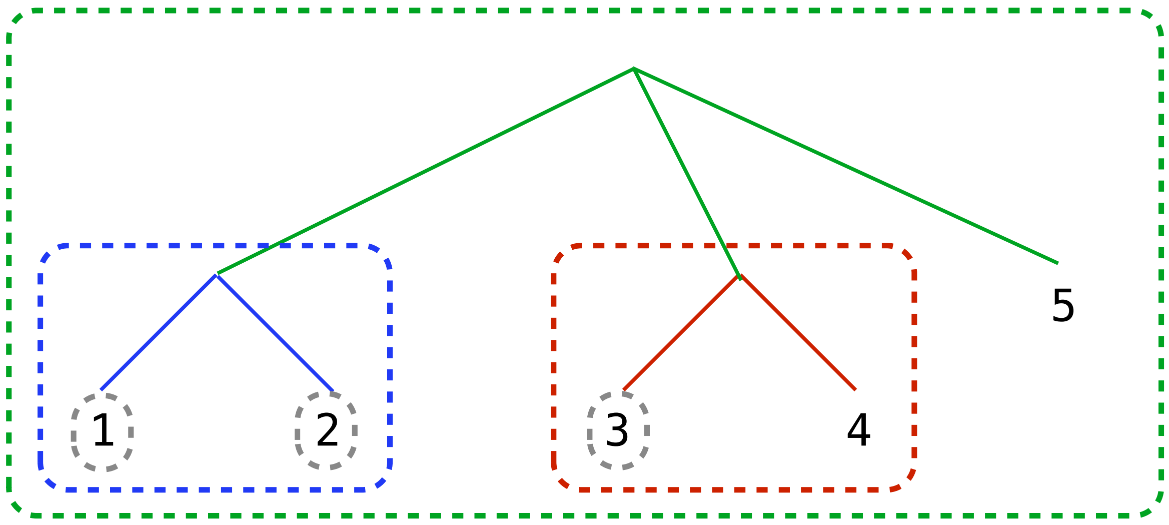

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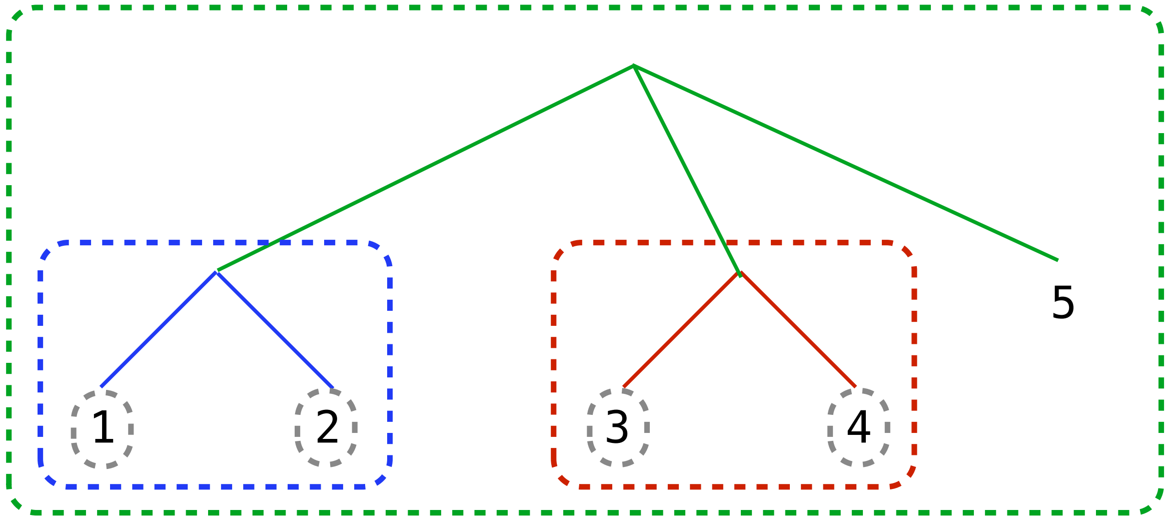

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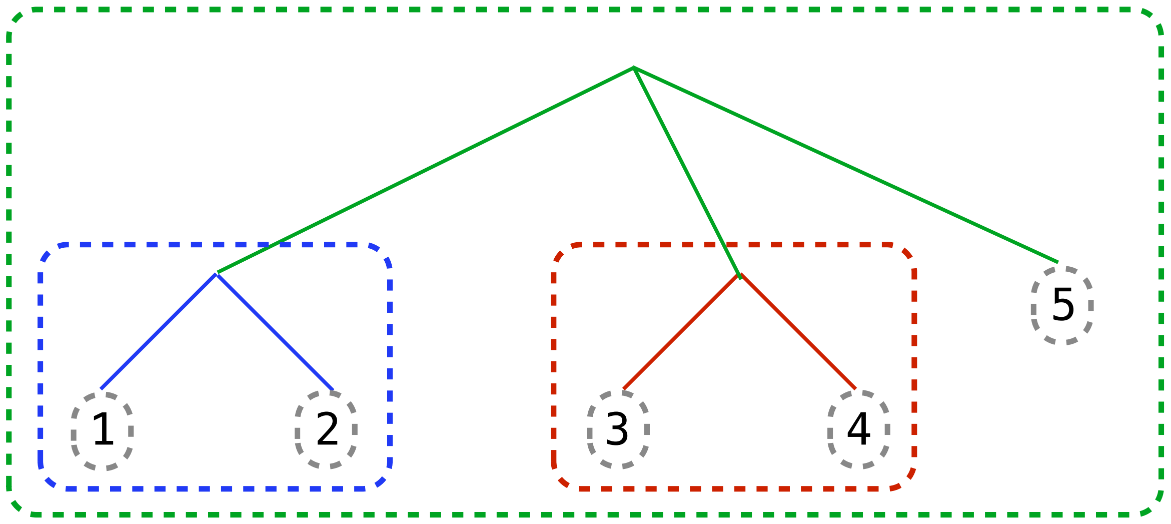

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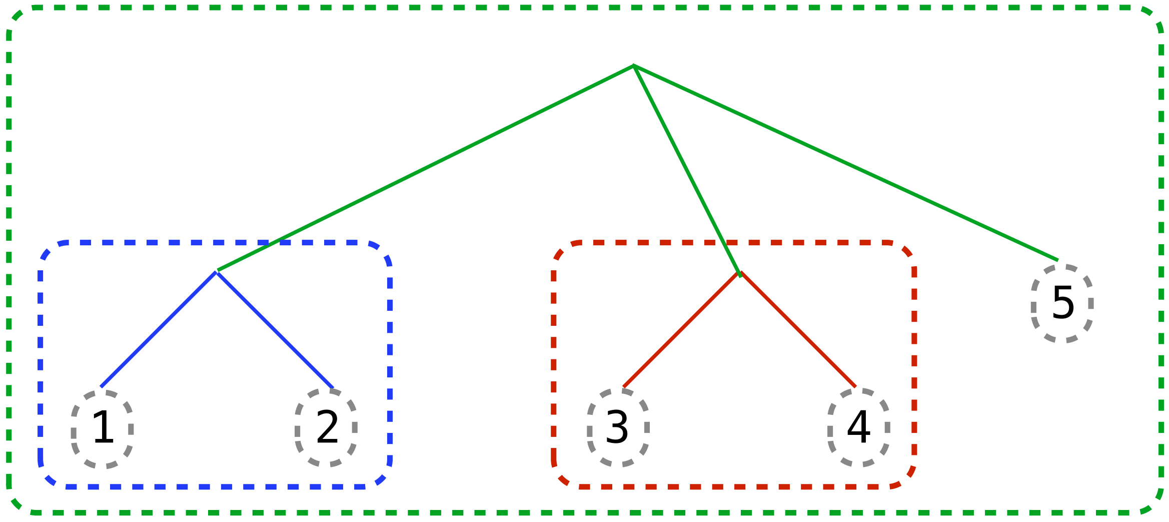

In every tree, a vast forest

Recursive Tree Processing

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Tree operations typically make recursive calls on branches

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```
def count_leaves(tree):
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def count_leaves(tree):
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def count_leaves(tree):
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def map_tree(tree, fn):
    if type(tree) != tuple:
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    return tuple(map_tree(branch, fn) for branch in tree)
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Demo

Trees with Internal Node Values

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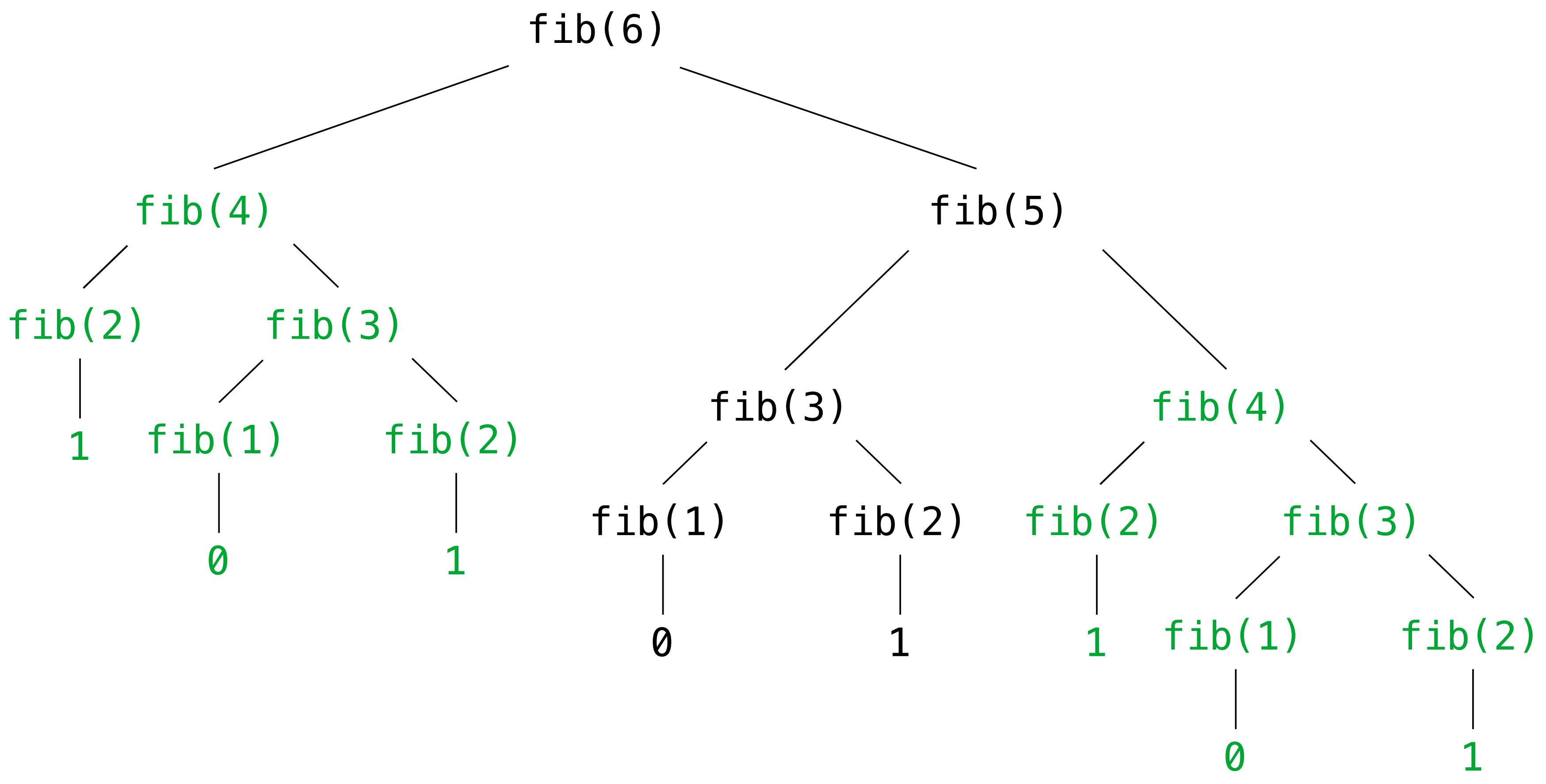

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

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class Tree(object):
    def __init__(self, entry, left=None, right=None):
```

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```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
```

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Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
```

Trees with Internal Node Values (Entries)

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```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
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        self.right = right

def fib_tree(n):
    if n == 1:
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    if n == 2:
        return Tree(1)
```

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Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
    right = fib_tree(n-1)
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
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        self.left = left
        self.right = right

def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
    right = fib_tree(n-1)
    return Tree(left.entry + right.entry, left, right)
```

Trees with Internal Node Values (Entries)

Trees need not only have values at their leaves.

```
class Tree(object):
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right
```

Demo

```
def fib_tree(n):
    if n == 1:
        return Tree(0)
    if n == 2:
        return Tree(1)
    left = fib_tree(n-2)
    right = fib_tree(n-1)
    return Tree(left.entry + right.entry, left, right)
```

Sets

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One more built-in Python container type

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```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}
```

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>>> s
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>>> 3 in s
True
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>>> 3 in s
True
>>> len(s)
4
```

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- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}


>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
```

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- Set literals are enclosed in braces
- Duplicate elements are removed on construction
- Sets are unordered, just like dictionary entries

```
>>> s = {3, 2, 1, 4, 4}
>>> s
{1, 2, 3, 4}


>>> 3 in s
True
>>> len(s)
4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Sets

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- Set literals are enclosed in braces
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```
>>> s = {3, 2, 1, 4, 4}
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True
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4
>>> s.union({1, 5})
{1, 2, 3, 4, 5}
>>> s.intersection({6, 5, 4, 3})
{3, 4}
```

Demo